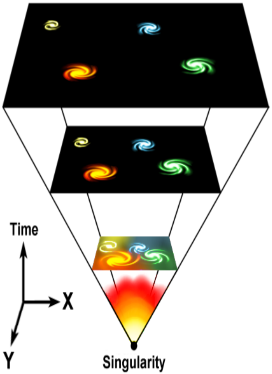
Holographic cosmology and the resolution of the initial singularity



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Quantum Gravity and Cosmology
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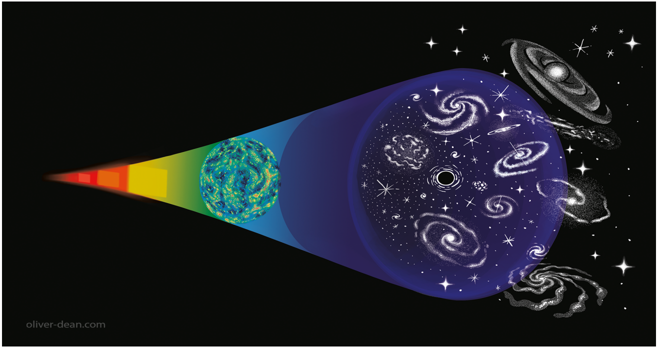
Introduction

- What are the laws of physics at the initial singularity?



- Was there something “before” the singularity? Can we formulate the laws of physics if there is no space and/or time the way we perceive them today?

- Are there possible observational signals from that era?

Outline

Holography Cosmology

Holographic cosmology is a new framework for cosmology

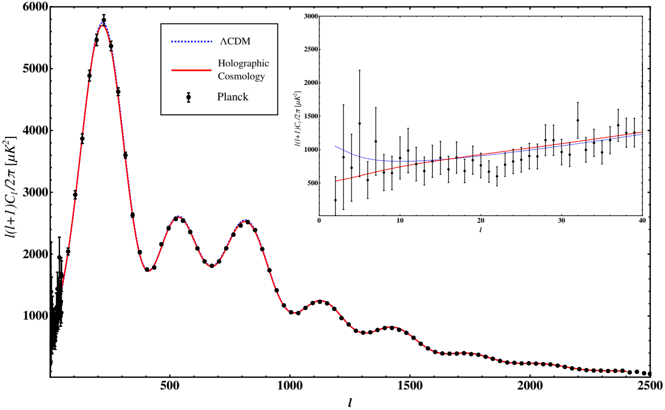
- In holographic cosmology our 4-dimensional Universe is described by providing:
- a three dimensional QFT (with no gravity)
- a dictionary that related QFT observables to 4-dimensional observables

- The new framework includes:
- conventional inflation
- qualitatively new models, describing a non-geometric very early Universe

Why holographic cosmology?

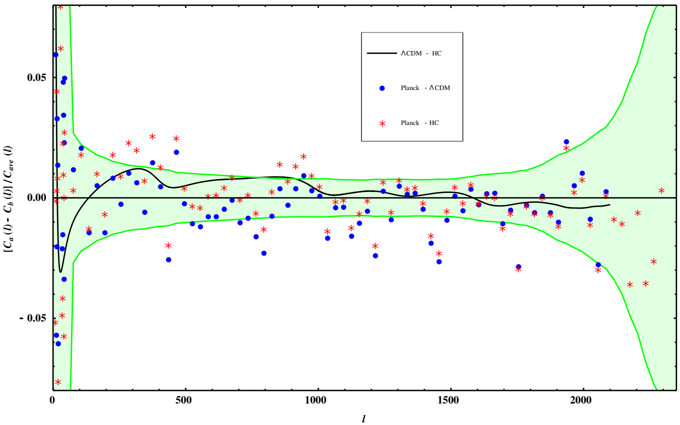
- Models are fully consistent with quantum mechanics
- General relativity emerges when the QFT dynamics becomes strongly interacting
- Models are well-defined when traditional GR breaks down
- ... including at the initial singularity

Holographic cosmology and CMB (Planck 2015)



[Afshordi, Coriano, Delle Rose, Gould, KS, PRL2017] [Afshordi, Gould, KS, PRD2017]

Planck 2015 vs ΛCDM vs holographic model (TT)



[Afshordi, Coriano, Delle Rose, Gould, KS, PRL2017] [Afshordi, Gould, KS, PRD2017]

References

- Early work

  [Witten (2001)] [Strominger (2001)] ... (dS/CFT correspondence)
  [Maldacena (2002)] ... (wavefunction of the universe)
  .....

- General setup

  [McFadden, KS (2009)]
  ....

- LatCos Collaboration:

  Southampton: A. Jüttner, B. Kitching-Morley, KS, ...
  Edinburgh: L. Del Debbio, A. Portelli, J. Lee, H. Rocha, ...
  Nonperturbative infrared finiteness in super-renormalisable scalar quantum field theory, PRL2021.

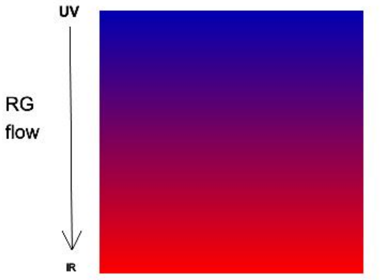
Gauge/gravity duality: general features

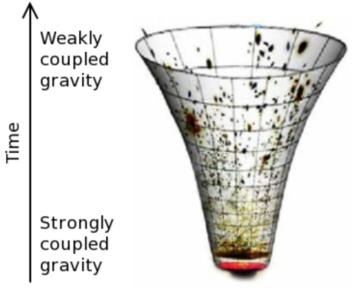
- It is a weak/strong duality: when one description is strongly coupled and the other is weakly coupled.
- There is a UV/IR connection: the UV of the QFT is the IR of gravity and vice versa.
- The holographic direction is associated with the energy scale we probe the QFT.

Time and RG-flow

- In holographic cosmology:

Cosmological evolution = inverse RG flow





Wavefunction of the Universe

- The partition function of the dual QFT computes the wavefunction of the Universe [Maldacena (2002)]:

$$\psi[\Phi] = Z_{QFT}[\Phi]$$

- Cosmological observables are computed as

$$\langle \Phi(x_1) \cdots \Phi(x_n) \rangle = \int D\Phi |\psi|^2 \Phi(x_1) \cdots \Phi(x_n)$$

- The partition has an expansion in correlation functions:

$$Z_{QFT}[\Phi] = \exp\left(\sum_n \langle O(x_1) \cdots O(x_n) \rangle \Phi(x_1) \cdots \Phi(x_n) \right)$$

Holographic formulae for power spectra

- The 2-point function of the energy momentum tensor T_{ij} in momentum space has the form

$$\langle T_{ij}(q) T_{kl}(-q) \rangle = A(q^2)\Pi_{ijkl} + B(q^2)\pi_{ij}\pi_{kl},$$

where $\Pi_{ijkl} = \frac{1}{2}(\pi_{ik}\pi_{lj} + \pi_{il}\pi_{kj} - \pi_{ij}\pi_{kl}), \quad \pi_{ij} = \delta_{ij} - q_i q_j / q^2.$

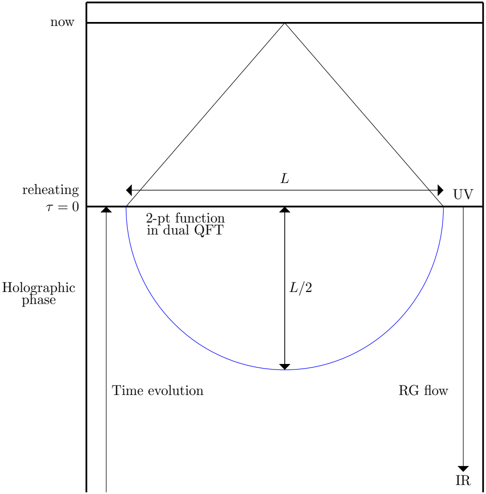
- The power spectra are given by [McFadden, KS (2009)]

$$\Delta^2_{\mathcal{R}}(q) = -\frac{q^3}{16\pi^2}\frac{1}{\text{Im } B}, \quad \Delta^2_T(q) = -\frac{2q^3}{\pi^2}\frac{1}{\text{Im } A},$$

where the imaginary part is taken after a suitable analytic continuation.
- Non-gausianities are related with higher-point functions of T_{ij}.

Holographic cosmology and the horizon problem

[Nastase, KS (2020)]



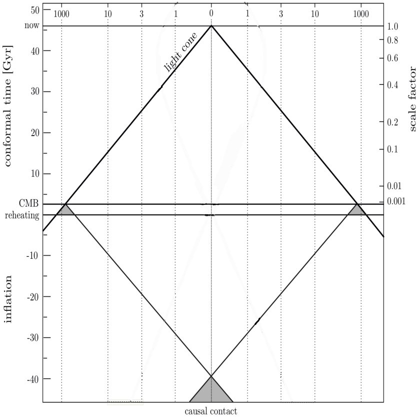


Figure taken from Baumann, McAllister, 1404.2601

Models for non-geometric universe

- Dual QFT:

$$S = \frac{1}{g_{YM}^2} \int d^3x \mathrm{tr} \left[\frac{1}{2} F_{ij} F^{ij} + \frac{1}{2} (D\phi^J)^2 + \bar{\psi}^K \not{D} \psi^K + \lambda_{J_1 J_2 J_3 J_4} \phi^{J_1} \phi^{J_2} \phi^{J_3} \phi^{J_4} + \mu_{J L_1 L_2} \phi^J \psi^{L_1} \psi^{L_2} \right].$$

  All fields are massless and in the adjoint of $SU(N)$, $\lambda_{J_1 J_2 J_3 J_4}$, $\mu_{J L_1 L_2}$ are dimensionless couplings while g_{YM}^2 has mass dimension 1.
- This class of theories appears in holographic dualities involving non-conformal branes. Maximally supersymmetric SYM theory in $d = 3$ belongs to this class.

Energy-momentum tensor

- For this class of theories, the 2-point function of T at large N takes the form,

$$\langle T(q)T(-q)\rangle = N^2 q^3 f(g_{\text{eff}}^2),$$

where $g_{\text{eff}}^2 = g_{\text{YM}}^2 N/q$ is the effective dimensionless 't Hooft coupling and $f(g_{\text{eff}}^2)$ is a general function of g_{eff}^2.

- In perturbation theory and at 2-loops,

$$f(g_{\text{eff}}^2) = f_0(1 - f_1 g_{\text{eff}}^2 \log g_{\text{eff}}^2 + f_2 g_{\text{eff}}^2 + O[g_{\text{eff}}^4]).$$

where f_0, f_1, f_2 are constants that depend on the field content etc.

Universal predictions

- The scalar power spectrum has the form

$$\Delta^2_{\mathcal{R}}(q) = \Delta_0 \frac{1}{1 + (gq_*/q)\ln|q/\beta g q_*|}$$

- The tensor power spectrum has the form

$$\Delta^2_{T}(q) = \Delta^T_0 \frac{1}{1 + (g_T q_*/q)\ln|q/\beta_T g_T q_*|}$$

- The scalar non-Guassianity is of exactly the factorisable equilateral shape with $f_{NL}^{equil} = 5/36$ plus orthogonal shape with f_{NL}^{ortho} that depends on the non-minimal coupling.

Results

- The fit to data implies that $g^2_{eff} = g^2_{YM}N/q$ is very small for all scales seen in CMB, except at very low multipoles, justifying *a posteriori* the use of perturbation theory.
- For $l < 30$ the model becomes non-perturbative and one cannot trust the perturbative prediction.
- Goodness of fit ($l > 30$)

	HC	ΛCDM
χ^2	824.0	824.5

  The difference in χ^2 indicate that the models are less than 1σ apart.
- A model that satisfies all observational constraints is: $SU(N)$ gauge theory coupled to N_ϕ non-minimal scalars with ϕ^4 self-interaction.

A very simple model and the initial singularity

- A non-minimally coupled massless scalar field in the adjoint of $SU(N)$ with ϕ^4 self-interaction

$$S = \frac{2}{g_{YM}^2} \int d^3x \mathrm{Tr} \left(\frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{4!} \phi^4 \right),$$

and energy momentum tensor

$$T_{ij} = \frac{2}{g_{YM}^2} \mathrm{Tr} \left(\partial_i \phi \partial_j \phi - \delta_{ij} (\frac{1}{2} (\partial \phi)^2 + \frac{1}{4!} \phi^4) + \xi (\delta_{ij} \Box - \partial_i \partial_j) \phi^2 \right)$$

- N is related with the smallness of the amplitude of the primordial perturbations.
- ξ is related with the tensor-to-scalar ratio,

$$r = 32(1 - 8\xi)^2$$

Is this model perturbative?

- Fit-to-data implies that perturbation theory breaks down at

$$g^2_{eff} \geq 1 \qquad \Rightarrow \qquad l < 260$$

- We cannot trust the prediction of perturbation theory below $l = 260$.

- The new model fits this portion of the data about two σ better than ΛCDM.

Singularity resolution

- Massless super-renormalizable theories have severe IR singularities in perturbation theory.
- If the IR singularities persist non-perturbatively the theory is non-predictive.
- It was argued by [Jackiw,Templeton (1981)][Appelquist, Pisarski(1981)] that these type of theories are non-perturbative IR finite:

g^2_{YM} effectively acts as an IR regulator.

- As time evolution is inverse RG flow, this corresponds to the resolution of the initial singularity.

Using Lattice QFT

- Use Lattice QFT methods to compute the observables in the low-l region.
- We need to ...
  - Discretize the continuum model
  - Find the massless point
  - Find the energy-momentum tensor
  - Compute its 2-point function
  - Compare with Planck data

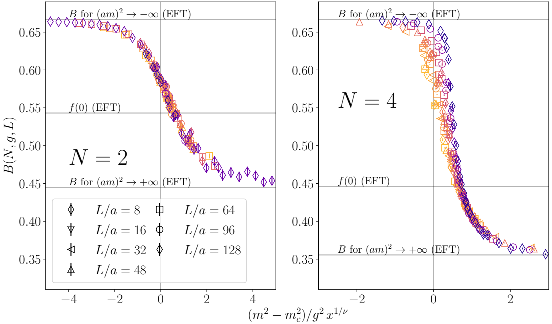
Massless point

- ➢ We want to simulate a massless theory.
- ➢ This requires introducing a bare mass m^2 and fine tuning its value so that the theory becomes massless in the continuum limit.
- ➢ In perturbation theory m^2 can be computed order by order:
- ➠ It is UV divergent up to 2-loops ➠ dealt with renormalisation.
- ➠ It is IR divergent starting from 2-loops.

- ➢ Unless the IR infinities are absent non-perturbatively, the massless theory does not exist.

Massless point: non-perturbative

- If the mass in the continuum limit is positive then $\langle \mathrm{Tr}\, M^n \rangle = 0$ for any n, where $M = \frac{1}{V} \int d^3x \phi(x)$.
- If the mass in the continuum limit is negative we are in the spontaneously broken phase, $\langle \mathrm{Tr}\, M^n \rangle \neq 0$.



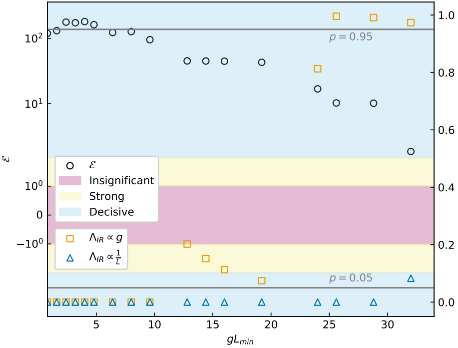
- Binder Cumulant: $B = 1 - (N/3)\langle \mathrm{Tr}\, M^4 \rangle / \langle (\mathrm{Tr}\, M^2) \rangle^2$.

IR finiteness of critical mass

On the lattice, the finite volume L^3 acts as an IR regulator.
Model 1: g acts as IR regulator; IR finiteness in the continuum limit.
Model 2: L is the IR regulator; IR divergence in the continuum limit.

- Left axis: Bayesian Evidence $\mathcal{E}$
  Circles at positive (negative) $\mathcal{E}$ indicate evidence for **model 1 (model 2)**.

- Horizontal axis: the smaller gL_{min} the more data used.

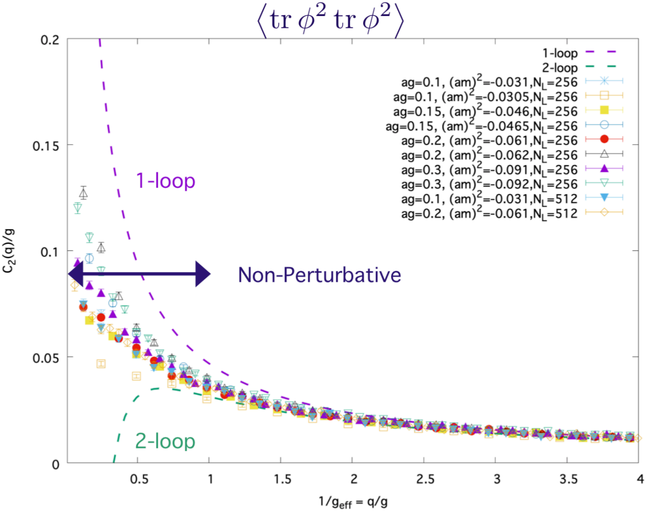


$N = 2$

We need to ...

- Discretize the continuum model ✓
- Find the massless point ✓
- Find the energy-momentum tensor ✓
- Compute its 2-point function
- Compare with Planck data

2-point function *[preliminary results]*



Conclusions

- Holography offers a unified framework for discussing the very Early Universe:
  - Strongly couple QFT: conventional inflation.
  - Weak/intermediate coupling: new non-geometric models.
- Resolution of initial singularity $\leftrightarrow$ IR finiteness of QFT
- Lattice methods instrumental in accessing the low l-region.
- Observable signatures of singularity resolution?