On polarized scattering equation for amplitudes of 10D SYM and 11D supergravity.

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Outline

- Recent decades an enormous progress in calculation of higher loop amplitudes of D=4 supersymmetric theories, in particular of maximally supersymmetric 4D $\mathcal{N}=4$ SYM and $\mathcal{N}=8$, was reached
  - [Bern, Carrasco, Dixon, Johansson and Roiban, Fortsch.Phys. 2011],
  - [Benincasa, IJMP A 2014],
  - [R. Britto, F. Cachazo, B. Feng and E. Witten, PRL2005] and refs. therein.
- The list of relevant papers certainly includes
  - Parke and Taylor, PRL 1986.
  - M.Bianchi, H.Elvang, D. Freedman, JHEP 2008 [arXiv:0805.0757 [hep-th]],
  - Drummond, Henn, Korchemsky, E. Sokatchev, NPB 2010 [arXiv:0807.1095],
  - Drummond, Henn, Plefka, JHEP 2010 [arXiv:0902.2987 [hep-th]],
  - D. Kazakov, Borlakov, Tolkachev, Vlasenko, PRD 2018 [arXiv:1712.04348 [hep-th]].

  and many others...
- Sorry for missing a lot of references!

- Many of these works used essentially spinor helicity formalism related (although not identical) to the twistor approach [Penrose 1963]
- and its superfield generalization related to the supertwistor approach [Ferber 1978],
  - Arkani-Hamed, Cachazo, Kaplan, JHEP 2010 [arXiv:0808.1446[hep-th]],
  - Brandhuber, Heslop, Travaglini, PRD 2008 [arXiv:0807.4097 [hep-th]].
  - ... .
- Thus it was natural to search for generalization of these for the case of higher dimensions, especially D=10 and D=11 interesting in String/M-theory perspective.

Higher D generalizations of spinor helicity formalism

- The generalization of spinor helicity and superamplitude formalisms to D=6 is quite straightforward [Cheung and O'Connell JHEP 2009].
- D=10 spinor helicity formalism is more complicated because the helicity and polarization spinors are constrained variables.
- This was proposed by Caron-Huot and O'Connell [JHEP 10].
- In [I.B: PRL 2017, JHEP 11(2018)] we observed that 10D spinor helicity variables of [Caron-Huot+O'Connell 2010] can be identified with
  - spinor moving frame variables
    [I.B.+ Zheltukhin 91-95], [I.B.+ Nurmagambetov 96], ... or, equivalently, with
  - D=10 Lorentz harmonics.
    [Galperin+Howe+Stelle=91, Galperin+Delduc+Sokatchev =91].
- This observation allowed us to construct the 11D spinor helicity formalism [I.B: PRL 2017, JHEP 11(2018)].

Higher D=10,11 generalizations of superamplitude formalism

- Moreover, on this basis we proposed and elaborated the generalization of the 4D superamplitude approach to D=11 supergravity and 10D SYM [I.B:PRL 2017, JHEP 11(2018)], the so-called constrained superamplitude formalism.
- Furthermore, in [IB: JHEP 05(2018)] we constructed another, 'almost unconstrained' analytic superamplitude formalism for 11D SUGRA and 10D SYM.

- More recently an apparently different approach to 11D SG and 10D SYM amplitudes was proposed by Geyer and Mason [2019=PLB2020].
- It is based on the so-called polarized scattering equation (PSE), which can be considered as a kind of square root of the CHY scattering equations [Cachazo+He+Yuan 2013] (actually present already in [Gross+Mende 87,88; Gross+Mañes 89]).

- The relation of 11D polarized scattering equation approach with ambitwistor string models was discussed and especially stressed in [Geyer+Mason:2019=PLB2020], where a modification of the 11D ambitwistor superstring action form our [I.B. JHEP 2014] was proposed for these purposes.
- This talk is based on [I.B. JHEP 11(2019)] where we
  - shown that the correct basis to derive the PSE is provided by the original 11D ambitwistor superstring of [I.B. JHEP2014] rather then its modification suggested in [Geyer+Mason:2019=PLB2020]
  - and presented a rigorous derivation of the polarized scattering equation (PSE) and other basic equations of the PSE formlism.
  - To this end we have used essentially the 11D supertwistor approach,
  - the possibility to formulate the 11D ambitwistor superstring as a system in an enlarged superspace with 528 bosonic coordinates as well as
  - the $SO(16)$ gauge symmetry of the 11D ambitwistor superstring [I.B.:2014lja].

- After a brief review of scattering equations and D=4 spinor helicity formalism and superamplitudes,
- we will introduce the 11D spinor frame variables and helicity spinors encoding momenta and polarization data of scattered particles on this basis.
- Then we describe the scattering equations and use the supertwistor form of the 11D ambitwistor superstring action with suitable vortex operators to obtain the 11D polarized scattering equation and other basic equations of the PSE formalism.

Outline

Scattering equations

- Scattering equations [Gross et al 87,88; Cachazo, He, Yuan 2013] relating massless scattered particles ($k_{\mu i}k_i^{\mu} = 0$) and points σ_i on Riemann sphere read

$$\sum_i \frac{k_i^{\mu} k_{j\mu}}{\sigma_i - \sigma_j} = 0 \ .$$

- Using this one can write a convenient expression for scattering amplitudes (see below).
- One can introduce the meromorphic 11-vector function

$$P_{\mu}(\sigma) = \sum_i \frac{k_{i\mu}}{\sigma - \sigma_i}$$

and write the scattering equation in the form

$$k_i^{\mu} P_{\mu}(\sigma_i) = 0 \ .$$

How to write amplitude with (polarized) scattering equation

- Let us present the form of 11D amplitude in CHY formalism written with the use of scattering equation $k_i \cdot P(\sigma_i) = k_i{}^{\underline{\mu}} P_{\underline{\mu}}(\sigma_i) = 0$:

$$\mathcal{A}_n^{11D} = \int d\mu_n \det{}' \mathbb{M}\, e^{\mathbb{F}} := \int \frac{1}{vol(SL(2,\mathbb{C}))} \prod_{i=1}^{n} d\sigma_i \prod_{i=1}^{n}{}' \delta(k_i \cdot P(\sigma_i)) \det{}' \mathbb{M}\, e^{\mathbb{F}} \,,$$

where $\prod_{i=1}^{n}{}' \delta(k_i \cdot P(\sigma_i)) = \sigma_{jk}\sigma_{kl}\sigma_{lj} \prod_{i=1, i\neq j,k,l}^{n} \delta(k_i \cdot P(\sigma_i))$ and $\frac{1}{vol(SL(2,\mathbb{C}))} \prod_{i=1}^{n} d\sigma_i = \prod_{i=1}^{n}{}' d\sigma_i$ are independent on choice of j, k, l, $\sigma_{ij} = \sigma_i - \sigma_j$, $\mathbb{M}$ is $2n \times 2n$ CHY matrix

$$\mathbb{M} = \begin{pmatrix} \frac{k_i \cdot k_j}{\sigma_{ij}} & \frac{U_i \cdot k_j}{\sigma_{ij}} - U_i \cdot P(\sigma_i)\delta_{ij} \\ -\frac{U_j \cdot k_i}{\sigma_{ji}} + U_j \cdot P(\sigma_j)\delta_{ij} & \frac{U_i \cdot U_j}{\sigma_{ij}} \end{pmatrix} , \qquad \det{}' \mathbb{M} = \frac{4}{\sigma_{ij}^2} \det \mathbb{M}_{ij}^{ij} \,,$$

where U_i is the polarization vector and the factor $e^{\mathbb{F}}$ determines the fermionic contribution.

- To write this $e^{\mathbb{F}}$ we need helicity spinors and **polarized** scattering equation.
- In D=4 also the measure of the amplitude simplify when written in terms of helicity spinors.

- One can also write the scattering equation as $\boxed{Res_{\sigma=\sigma_i}\frac{1}{2}P^2(\sigma)=0}$.
- This actually implies [Geyer+Mason=2019] the light-likeness of the meromorphic 11–vector function,

$$P^\mu(\sigma)P_\mu(\sigma)=0$$

  which can be considered as the third equivalent form of the scattering eq.
- The constraint $P^\mu(\sigma)P_\mu(\sigma)=0$ can be generated from the so-called ambitwistor string action [Mason+Skinner=2013].
- It can be considered as extended complexified massless particle action,
- i.e. as massless particle action the worldline of which is replaced by 2d space and time derivative replaced by holomorphic drivative.
- $P_\mu(\sigma)=\sum_i \frac{k_{i\mu}}{\sigma-\sigma_i}$ can be obtained from the deformation of this action by incorporating of the contribution to the path integral measure of the suitable vertex operators.
- Here we will use 11D supersymmetric generalization of the ambitwistor superstring action [IB:2014] to obtain the Polarized version of the 11D Scettering equation - 11D PSE proposed by Geyer and Mason [2019=PLB2020].

Outline

Bosonic spinors and spinor helicity formalism.

- An on-shell n-particle amplitude of Yang-Mills (or gravity),

$$\mathcal{A}^{(n)}(p_{(1)}, U_{(1)}; \ldots, p_{(n)}, U_{(n)}) \ ,$$

depends on n light-like momenta $p_{\mu(i)}$, $\boxed{p_{\mu(i)}p_{\mu(i)} = 0}$,

- and n polarization vectors $U_{\mu(i)}$ which obey $\boxed{p_{\mu(i)}U^{\mu}_{(i)} = 0}$.
- In $D = 4$ **spinor helicity formalism** both data are encoded in the **bosonic** Weyl spinor $\boxed{\lambda^{A}_{(i)} = (\bar{\lambda}^{\dot{A}}_{(i)})^{*}}$ $(A = 1, 2\ ,\ \dot{A} = 1, 2)$ called *helicity spinor*.
- The light-like momenta are defined through Cartan-Penrose representation

$$p_{\mu(i)}\sigma^{\mu}_{A\dot{A}} = 2\lambda_{A(i)}\bar{\lambda}_{\dot{A}(i)} \qquad \Leftrightarrow \qquad p_{\mu(i)} = \lambda_{(i)}\tilde{\sigma}_{\mu}\bar{\lambda}_{(i)}, \quad \mu = 0, \ldots, 3\ ,$$

where $A = 1, 2$, $\dot{A} = 1, 2$, $\sigma^{\mu}_{A\dot{A}}$ are relativistic Pauli matrices, obeying $\sigma^{(\mu}\tilde{\sigma}^{\nu)} = \eta^{\mu\nu}$ and $\sigma_{\mu A\dot{A}}\sigma^{\mu}_{B\dot{B}} = 2\epsilon_{AB}\epsilon_{\dot{A}\dot{B}}\ , \qquad \sigma^{\mu}_{A\dot{A}}\tilde{\sigma}^{\nu\dot{A}A} = 2\eta^{\mu\nu}$.

Bosonic spinors and spinor helicity formalism

- The polarization vectors are defined as

$$U^{(-)}_{A\dot{A}(i)} = \frac{\lambda_{A(i)}\bar{w}_{\dot{A}}}{[\bar{\lambda}_{(i)}\bar{w}]}\,, \qquad U^{(+)}_{A\dot{A}(i)} = \frac{w_A\bar{\lambda}_{\dot{A}(i)}}{< w\lambda_{(i)} >} \quad \Rightarrow \quad p^{\mu}_{(i)}\varepsilon^{(\pm)}_{\mu(i)} = 0$$

where $\bar{w}_{\dot{A}} = (w_A)^*$ is a constant reference spinor and $\pm \leftrightarrow$ helicity ± 1.

- In D=4 spinor helicity formalism the commonly accepted notation are

$$< ij > \equiv < \lambda_{(i)}\lambda_{(j)} > = \epsilon_{AB}\lambda^A_{(i)}\lambda^B_{(j)} \quad = - < ji >\,,$$
$$[ij] \equiv [\bar{\lambda}_{(i)}\bar{\lambda}_{(j)}] = \epsilon_{\dot{A}\dot{B}}\bar{\lambda}^{\dot{A}}_{(i)}\bar{\lambda}^{\dot{B}}_{(j)} \quad = -[ji]\,,$$

- where

$$\epsilon_{AB} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -\epsilon^{AB}\,, \qquad \epsilon_{\dot{A}\dot{B}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = -\epsilon^{\dot{A}\dot{B}}\,.$$

D=4 polarized scattering equation

- In D=4 $P^{\mu}(\sigma)P_{\mu}(\sigma)=0$ can be solved in terms of complex Weyl spinor function

$$P_{\mu}(\sigma)=\tfrac{1}{2}\lambda_A(\sigma)\bar{\lambda}_{\dot{A}}(\sigma)\tilde{\sigma}_{\mu}^{\dot{A}A}\,.$$

- To factorize the *meromorphic* vector function $P_{\mu}(\sigma)=\sum_i \frac{k_{i\mu}}{\sigma-\sigma_i}$ we have to use a meromorphic

$$\lambda_A(\sigma)=\sum_{i=1}^{l}\frac{u_i\epsilon_i\lambda_{Ai}}{\sigma-\sigma_i} \qquad \textit{and} \qquad \bar{\lambda}_{\dot{A}}(\sigma)=\sum_{i=l+1}^{n}\frac{\bar{u}_i\bar{\epsilon}_i\bar{\lambda}_{\dot{A}i}}{\sigma-\sigma_i}\,.$$

- Here λ_{Ai} and ϵ_i characterize the momentum and polarization of scattering particles

$$k_{iA\dot{A}}=\lambda_{Ai}\bar{\lambda}_{\dot{A}i}\,, \qquad U_{A\dot{A}i}=\epsilon_i\lambda_{Ai}\bar{w}_{\dot{A}}/[\bar{\lambda}_i\bar{w}] \qquad \textit{or} \qquad U_{A\dot{A}j}=\bar{\epsilon}_j w_A\bar{\lambda}_{\dot{A}i}/<\lambda_j w>$$

and u_i are complex numbers. The data obey the polarized scattering equation.

$$u_i\,\bar{\lambda}_{\dot{\alpha}}(\sigma_i)=\frac{\bar{\lambda}_{\dot{\alpha}i}}{\epsilon_i}\,, \qquad i=1,\ldots,l, \qquad \lambda_{\alpha}(\sigma_j)\,\bar{u}_j=\frac{\lambda_{\alpha j}}{\bar{\epsilon}_j}\,, \qquad j=l+1,\ldots,n\,.$$

D=4 amplitudes from polarized scattering equation (from ambitwistor superstring)

- The integration measure to calculate D=4 amplitudes as it was obtained from D=4 ambitwistor superstring,

$$d\mu^{4d}_{n,l} = \prod_{i=1}^{l} \delta^2 \left(u_i \bar{\lambda}_{\dot{\alpha}}(\sigma_i) - \frac{\bar{\lambda}_{\dot{\alpha} i}}{\epsilon_i} \right) \prod_{i=l+1}^{n} \delta^2 \left(\lambda_{\alpha}(\sigma_j) \bar{u}_j - \frac{\lambda_{\alpha j}}{\bar{\epsilon}_j} \right) \frac{\prod_{j=1}^{n} d\sigma_j du_j / u_j}{Vol(GL(2,\mathbb{C}))} ,$$

  contains delta functions of the polarized scattering equation and integrations over constants u_i.

- Here $GL(2,\mathbb{C}) = SL(2,\mathbb{C}) \otimes GL(1,\mathbb{C})$ with $GL(1,\mathbb{C})$ generated by

$$\sum_{i=1}^{l} u_i \partial / \partial u_i - \sum_{j=l+1}^{n} u_j \partial / \partial u_j$$

  so that

$$\frac{\prod_{j=1}^{n} d\sigma_j du_j / u_j}{Vol(GL(2,\mathbb{C}))} = \sigma_{i_1 i_2} \sigma_{i_2 i_3} \sigma_{i_1 i_3} \prod_{j=1, j \neq i_1 i_2 i_3}^{n} d\sigma_j \prod_{j=1, j \neq i_4}^{n} du_j / u_j .$$

Outline

Light-like momentum and moving frame in arbitrary D

To construct the spinor helicity formalism for arbitrary D, we first should find the counterpart of D=4 Cartan-Pernose representation which solves $\boxed{p_a p^a = 0}$.

- A particular solution of the constraint $p_a p^a = 0$ is given by

$$p_{(a)} = \rho\,(1, 0, ..., 0, -1)$$

  with arbitrary $\rho = \rho(\tau)$ describing the energy of the massless particle.

- Any other solution can be obtained from this (or from the reflected one with negative ρ) by performing an $O(1, D-1)$ Lorentz transformation

$$u_a^{(b)} \in O(1, D-1) \quad \Leftrightarrow \quad u_a^{(b)} u^{a(c)} = \eta^{(b)(c)} = diag(+1, -1, ..., -1)\,,$$

  i.e

$$p_a = u_a^{(b)} p_{(b)} = \rho\,(u_a^0 - u_a^{(D-1)}) \qquad =: \rho^{\#}\, u_a^{=}\,.$$

Solution of the light-likeness condition

- It is convenient to write the *vector frame matrix* $u_a^{(b)}(\tau) \in$ SO(1,D-1) explicitly in terms of $u_a^{=}$, its complementary $u_a^{\#}(\tau) = (u_a^0 + u_a^{(D-1)})$, and u_a^I (light-cone harmonics [Sokatchev 86,87]). Then

$$U := (u_a^{(b)}) = \left(\tfrac{1}{2}\left(u_a^{=} + u_a^{\#}\right),\, u_a^I \, , \tfrac{1}{2}\left(u_a^{\#} - u_a^{=}\right)\right) \;\in\; SO^{\uparrow}(1, D-1)$$

- implies $U^T \eta U = \eta$ which can be split into [E. Sokatchev, 86,87]

$$\boxed{u_a^{=} u^{a=} = 0}\,, \quad u_a^{\#} u^{a\#} = 0\,, \quad u_a^{=} u^{a\#} = 2\,,$$
$$u_a^I u^{a=} = 0\,, \quad u_a^I u^{a\#} = 0\,, \quad u_a^I u^{aJ} = -\delta^{IJ}\,.$$

- In particular, we see that $u^{a=}$ is light-like, $u_a^{=} u^{a=} = 0$, and this is the reason why $p_a = u_a^{(b)} p_{(b)} = \rho^{\#} u_a^{=}$ solves $p_a p^a = 0$.
- The nontrivial consequences of this simple fact appear when we "extract square root" from the vector frame by introducing the spinor frame.

Spinor moving frame $= \sqrt{\textit{moving frame}}$

- Spinor moving frame $= \sqrt{\textit{moving frame}}$ is defined by conditions of Lorentz invariance of D-dimensional Γ^a and also $C_{\alpha\beta}$ if such exists, i.e. by a matrix $V \in Spin(1, D-1)$ which obeys

$$V\Gamma_b V^T = u_b^{(a)}\Gamma_{(a)} , \qquad V^T\tilde{\Gamma}^{(a)}V = \tilde{\Gamma}^b u_b^{(a)} \qquad (\text{and } VCV^T = C \ , \ \textit{if C exists}) \ .$$

- The $SO(1,1) \times SO(D-2)$ invariant splitting of the spinor frame matrix is

$$V_\alpha^{(\beta)} = \begin{pmatrix} v_{\alpha\dot{q}}^{+}, & v_{\alpha q}^{-} \end{pmatrix} \in Spin(1, D-1) \qquad \leftrightarrow \qquad u_b^{(a)} = (u_b^{=}, u_b^{\#}, u_b^{I}) \ ,$$

where q and $\dot{q}$ are indices of the spinor representations of $SO(D-2)$, which can be different

$$D = 10 : \qquad \alpha = 1, ..., 16 \ , \qquad \dot{q} = 1, ..., 8 \ , \qquad q = 1, ..., 8 \ ,$$

or the same

$$D = 11 : \qquad \alpha = 1, ..., 32 \ , \qquad q = \dot{q} = 1, ..., 16, \qquad v_{\alpha\dot{q}}^{+} \equiv v_{\alpha q}^{+} \ .$$

- With the suitable representation for Γ−matrices, the constraints $V\Gamma_b V^T = u_b^{(a)}\Gamma_{(a)}$ $V^T\tilde{\Gamma}^{(a)}V = \tilde{\Gamma}^b u_b^{(a)}$ can be split into

$$u_a^{=}\Gamma^a_{\alpha\beta} = 2v_{\alpha q}{}^{-}v_{\beta q}{}^{-}\;, \qquad v_q^{-}\tilde{\Gamma}_a v_p^{-} = u_a^{=}\delta_{qp},$$

$$, \qquad u_a^{\#}\Gamma^a_{\alpha\beta} = 2v_{\alpha\dot{q}}{}^{+}v_{\beta\dot{q}}{}^{+}\;, \qquad v_{\dot{q}}^{+}\tilde{\Gamma}_a v_{\dot{p}}^{+} = u_a^{\#}\delta_{\dot{q}\dot{p}}\;,$$

$$u_a^{I}\Gamma^a_{\alpha\beta} = 2v_{(\alpha|q}{}^{-}\gamma^I_{q\dot{q}}v_{|\beta)\dot{q}}{}^{+}\;, \qquad v_q^{-}\tilde{\Gamma}_a v_{\dot{p}}^{+} = u_a^{I}\gamma^I_{q\dot{p}}\;.$$

- For D=11 $q, p \equiv \dot{q}, \dot{p} = 1, ..., 16$ are spinor indices of SO(9) and $\gamma^I_{qp} = \gamma^I_{pq}$.
- In our perspective an especially important among above relations are $u_a^{=}\Gamma^a_{\alpha\beta} = 2v_{\alpha q}{}^{-}v_{\beta q}{}^{-}\;, \qquad v_q^{-}\tilde{\Gamma}_a v_p^{-} = u_a^{=}\delta_{qp}$
- which allow to state that $v_{\alpha q}^{-}$ is a square root of the light-like $u_a^{=}$
- in the same sense as in D=4 one states λ_A "= $\sqrt{p_a}$" keeping in mind the Cartan-Penrose representation $p_\mu\sigma^\mu_{A\dot{A}} = 2\lambda_A\bar{\lambda}_{\dot{A}}$.

D=10 vs D=11 spinor helicity formalism

- The D=10 spinor helicity variables of Caron-Huot and O'Connell [2013] are $\lambda_{\alpha q} = \sqrt{\rho^{\#}} v_{\alpha q}^{-}$ carrying 8s index, and the polarization spinor is $\lambda^{\alpha}_{\dot{q}} = \sqrt{\rho^{\#}} v_{\dot{q}}^{-\alpha}$ which carries 8c spinor index of SO(8).
- Here $\rho^{\#}$ comes from $p_a = \rho^{\#} u_a^{=}$ which now reads $p_a = \frac{1}{8}\lambda\Gamma_a\lambda$.
- In contrast, in 11D the polarization spinor $\lambda^{\alpha}_{q} = \sqrt{\rho^{\#}} v_q^{-\alpha} = -iC^{\alpha\beta}\lambda_{\beta q}$ actually coincides with the spinor helicity variable $\boxed{\lambda_{\alpha q} = \sqrt{\rho^{\#}} v_{\alpha q}^{-}}$.
- ($p_a = \rho^{\#} u_a^{=}$ now reads $p_a = \frac{1}{16}\lambda\Gamma_a\lambda$).

- Notice that using these constrained helicity spinors one can now write an equivalent Ferber-Schirafuji-like form of the D-dimansional Brink-Schwarz superparticle action $S = \int d\tau \left(p_a(\partial_\tau X^a - i\partial_\tau\theta\Gamma^a\theta) - \frac{1}{2} e p_a p^a \right)$: substituting $p_a = \frac{1}{16}\lambda\Gamma_a\lambda$ we arrive at

$$S = \frac{1}{16}\int d\tau \lambda_{\alpha q}\lambda_{\beta q}\tilde{\Gamma}_a^{\alpha\beta}\left(\partial_\tau X^a - i\partial_\tau\theta\Gamma^a\theta\right) \, , \qquad 2\lambda_{\alpha q}\lambda_{\beta q} = \rho^{\#} u_a^{=}\Gamma^a_{\alpha\beta} \, .$$

Scattering data and helicity spinors

- Light–like momentum $k_{\mu i}$, $k_{\mu i} k_i^{\mu} = 0$ of i-th massless particle in the scattering process can be expressed in terms of helicity spinors by

$$\boxed{k_{\mu i}\delta_{qp} = \lambda_{\alpha qi}\tilde{\Gamma}_{\mu}^{\alpha\beta}\lambda_{\beta pi}} = \rho_i^{\#} v_{\alpha qi}^{-}\tilde{\Gamma}_{\mu}^{\alpha\beta} v_{\beta pi}^{-} \, ,$$

$$2\rho_i^{\#} v_{\alpha qi}{}^{-} v_{\beta pi}{}^{-} = \boxed{2\lambda_{\alpha qi}\lambda_{\beta pi} = \Gamma^{\mu}_{\alpha\beta} k_{\mu i}} \, .$$

- $\lambda_{\alpha qi}$'s also carry the information about *polarizations*, but to make it transparent we need to supply their space by an additional complex structure.
- This can be encoded in the complex polarization vector $U_{\mu i}$ which obeys

$$k_{\mu i} U_i^{\mu} = 0 \, , \qquad U_{\mu i} U_i^{\mu} = 0$$

and can be decomposed on the space-like vectors of the frame associated to the momentum $k_{\mu i}$ through $k_{\mu i} = \rho_i^{\#} u_{\mu i}^{=}$ by $\boxed{U_{\mu i} = u_{\mu i}^{I} U_i^{I}}$ with $U_i^I U_i^I = 0$.

- The complex null polarization nine-vector U^I can be related by

$$\not{U}_{qpi} := U_i^I \gamma^I_{qp} = 2\bar{w}_{qAi}\bar{w}_{pAi}$$

to the complex 16×8 matrices obeying 'purity conditions'

$$\bar{w}_{qAi}\bar{w}_{qBi} = 0 \ , \qquad A, B = 1, ..., 8 \ .$$

- Actually, $\bar{w}_{qAi}$ are internal frame variables or $SO(9)/SO(7) \times SO(2)$ *harmonics* in the sense of [GIKOS=Galperin, Ivanov, Kalitzin, Ogievetsky, Sokatchev, 1984].
- This is to say $\boxed{(\bar{w}_{qAi}, w_{qi}{}^A) \ \in \ SO(9)}$, where $w_q{}^A = (\bar{w}_{qA})^*$, which implies that

$$w_{qi}{}^A\bar{w}_{pAi} + \bar{w}_{qAi}w_{pi}{}^A = \delta_{qp} \ ,$$
$$\bar{w}_{qBi}w_{qi}{}^A = \delta_B{}^A, \quad w_{qi}{}^A w_{qi}{}^B = 0 = \bar{w}_{qAi}\bar{w}_{qBi},$$

as well as the above mentioned $U_i^I\gamma^I_{qp} = 2\bar{w}_{qAi}\bar{w}_{pAi}$ and a few similar relations with other vectors of $SO(9)$ vector frame.

- It is convenient to introduce the set of complex spinor frame variables

$$v^{\mp}_{\alpha A} := v^{\mp}_{\alpha q}\bar{w}_{qA} \; , \qquad \bar{v}^{\mp A}_{\alpha} := v^{\mp}_{\alpha q} w_q{}^A \; .$$

- By construction, $v^{-}_{\alpha A} v^{\pm\alpha}_{B} = 0 = v^{-}_{\alpha A} v^{-\alpha B}$, $v^{+}_{\alpha A} v^{-\alpha B} = \delta_A{}^B, \ldots$.
- We will use the $SO(1,1)$ invariant complex helicity spinors

$$\lambda_{\alpha A i} = \sqrt{\rho^{\#}_i}\, v^{-}_{\alpha A i} \; , \qquad \lambda^{A}_{\alpha i} = \sqrt{\rho^{\#}}\, \bar{v}^{-A}_{\alpha i}$$

which can be used to 'factorize' the on-shell momentum as

$$k_\mu \tilde{\Gamma}^{\mu\alpha\beta} = \tilde{k\!\!\!/}^{\alpha\beta} = 4\lambda^{(\alpha}_{A}\lambda^{\beta)A} \qquad \Leftrightarrow \qquad k\!\!\!/_{\alpha\beta} = 4\lambda^{A}_{(\alpha}\lambda_{\beta)A} \; .$$

The helicity spinors are solutions of the massless Dirac equations

$$\tilde{k\!\!\!/}^{\alpha\beta}_i \lambda_{\beta A i} = 0 \; , \qquad \tilde{k\!\!\!/}^{\alpha\beta}_i \lambda^{A}_{\beta i} = 0 \; .$$

Outline

Back to scattering equation $k_i \cdot P(\sigma_i) = 0 \qquad (\Rightarrow P^\mu(\sigma)P_\mu(\sigma) = 0)$

- Back to scattering equation: $P^\mu(\sigma)P_\mu(\sigma) = 0$ suggests the existence of a meromorphic function carrying 11D spinor index which plays the role of "$\sqrt{P_\mu(\sigma)}$" in the sence of

$$P_\mu(\sigma)\delta_{qp} = \lambda_q(\sigma)\tilde{\Gamma}_\mu\lambda_p(\sigma), \qquad 2\lambda_{\alpha q}(\sigma)\lambda_{\beta q}(\sigma) = \Gamma^\mu_{\alpha\beta}P_\mu(\sigma).$$

Then it is convenient to assume the existence of a spinor frame field $v^-_{\alpha q}(\sigma)$ and a (purely gauge or Stückelberg) density $\rho^\#(\sigma)$ to solve these by

$$\lambda_{\alpha q}(\sigma) = \sqrt{\rho^\#(\sigma)}v^-_{\alpha p}(\sigma)\mathcal{S}_{pq}(\sigma) , \qquad \mathcal{S}_{pr}\mathcal{S}_{qr} = \delta_{pq} .$$

- Indeed, substituting this, we find

$$P_\mu(\sigma)\delta_{qp} = \rho^\#(\sigma)v^-_q(\sigma)\tilde{\Gamma}_\mu v^-_p(\sigma) , \qquad 2\rho^\#(\sigma)v^-_{\alpha q}(\sigma)v^-_{\beta q}(\sigma) = \Gamma^\mu_{\alpha\beta}P_\mu(\sigma) ,$$

which describe the essential constraints on the spinor frame functions and their relation with the meromorphic vector $\boxed{P_\mu(\sigma) = \rho^\#(\sigma)u^=_\mu(\sigma)}$.

- The presence of $SO(16)$ valued matrix field $\mathcal{S}(\sigma) \in SO(16)$, $\boxed{\mathcal{S}\mathcal{S}^T = I}$, reflects the invariance under the $SO(16)$ gauge transformations.

Polarized scattering equation

- The 11D *polarized scattering equations* is the counterpart of

$$k_i^{\mu} P_{\mu}(\sigma_i) = 0$$

  written for the constrained $\lambda_{\alpha q}(\sigma)$ and the scattering data.
- It was proposed by Geyer and Mason [2019], who claimed its relation with a modified version of the 11D ambitwistor SSTR of [IB: JHEP2014].
- In [IB:JHEP 2019] we have shown that appropriate model for this is actually provided by the original model of [IB:JHEP 2014]
- and obtain the form of the meromorphic $\lambda_{\alpha q}(\sigma)$ and the 11D polarized scattering equation on this basis.

11D ambitwistor superstring

- The 11D ambitwistor superstring action is [IB:2014]

$$S = \int_{\mathcal{W}^2} d^2\sigma \lambda_{\alpha q}(\sigma)\lambda_{\beta q}(\sigma) \left(\bar{\partial} X^{\alpha\beta}(\sigma) - i\bar{\partial}\theta^{(\alpha}(\sigma)\,\theta^{\beta)}(\sigma)\right),$$

- where $\theta^{\alpha}(\sigma)$ are fermionic 32-component Majorana spinor functions,
- $X^{\alpha\beta}(\sigma) = \frac{1}{32}\tilde{\Gamma}_{\mu}^{\alpha\beta} X^{\mu}(\sigma)$, with bosonic 11-vector $X^{\mu}(\sigma)$, and
- $\lambda_{\alpha q} = \lambda_{\alpha q}(\sigma)$ are the above constrained spinor functions, obeying

$$\lambda^{\alpha}_{q}\lambda_{\alpha p} = 0 \qquad \text{and}$$

$$\lambda_q(\sigma)\tilde{\Gamma}_{\mu}\lambda_p(\sigma) = P_{\mu}(\sigma)\delta_{qp}, \qquad 2\lambda_{\alpha q}(\sigma)\lambda_{\beta q}(\sigma) = \Gamma^{\mu}_{\alpha\beta}P_{\mu}(\sigma),$$

where $P_{\mu}(\sigma)$ obeys $P_{\mu}(\sigma)P^{\mu}(\sigma) = 0$ and in all other respect is defined by the above constraints.

- For our purposes it is much more convenient to consider the action $S = \int_{\mathcal{W}^2} d^2\sigma \lambda_{\alpha q}\lambda_{\beta q} \left(\bar{\partial} X^{\alpha\beta} - i\bar{\partial}\theta^{(\alpha}\,\theta^{\beta)}\right)$ with an arbitrary symmetric

$$X^{\alpha\beta}(\sigma) = X^{\beta\alpha}(\sigma) \equiv \frac{1}{32}\tilde{\Gamma}_{\mu}{}^{\alpha\beta} X^{\mu}(\sigma) - \frac{1}{64} i Z^{\mu\nu}(\sigma)\tilde{\Gamma}^{\alpha\beta}_{\mu\nu} + \frac{1}{32\cdot 5!} Z^{\mu_1\ldots\mu_5}(\sigma)\tilde{\Gamma}^{\alpha\beta}_{\mu_1\ldots\mu_5}.$$

- Such a modified action is gauge equivalent to the original one, with $X^{\alpha\beta}(\sigma) =\propto \tilde{\Gamma}_{\mu}{}^{\alpha\beta} X^{\mu}(\sigma)$:
- the constraints $\boxed{\lambda_{\alpha q}(\sigma)\lambda_{\beta q}(\sigma) =\propto \Gamma^{\mu}_{\alpha\beta}}$ imposed on λ^{α}_{q} guarantee that the arbitrary $\delta Z^{\mu\nu}(\sigma)$ and $\delta Z^{\mu_1\cdots\mu_5}(\sigma)$ do not change the action
- This is the statement of gauge symmetry which can be fixed just by setting $Z^{\mu\nu}(\sigma) = 0$ and $Z^{\mu_1\cdots\mu_5}(\sigma) = 0$, which reduce arbitrary $X^{\alpha\beta}(\sigma) = X^{\beta\alpha}(\sigma)$ to $\tilde{\Gamma}_{\mu}{}^{\alpha\beta} X^{\mu}(\sigma)$.

Supertwistor form of the 11D ambitwistor superstring action

- The action $S = \int_{\mathcal{W}^2} d^2\sigma \lambda_{\alpha q}\lambda_{\beta q} \left(\bar{\partial} X^{\alpha\beta} - i\bar{\partial}\theta^{(\alpha}\, \theta^{\beta)}\right)$ can be written as

$$S = \int_{\mathcal{W}^2} d^2\sigma \left(\lambda_{\alpha q}\, \bar{\partial}\mu_q^\alpha - \bar{\partial}\lambda_{\alpha q}\, \mu_q^\alpha - i\bar{\partial}\eta_q\, \eta_q\right) \ ,$$

where $\boxed{\lambda_{\alpha q}(\sigma) = \sqrt{\rho^\#(\sigma)}v_{\alpha p}^-(\sigma)\mathcal{S}_{pq}(\sigma)}$ and

$$\mu_q^\alpha(\sigma) := X^{\alpha\beta}(\sigma)\lambda_{\beta q}(\sigma) - \frac{i}{2}\theta^\alpha(\sigma)\,\theta^\beta(\sigma)\lambda_{\beta q}(\sigma),$$
$$\eta_q(\sigma) := \theta^\beta(\sigma)\lambda_{\beta q}(\sigma) \ .$$

- These are the 11D generalizations of the 4D Penrose incidence relations imposed on the set of 16 constrained 11D supertwistors [I.B., Sorokin, de Azcarraga 2006]

$$\boxed{\mathcal{Z}_{\Lambda q} = \left(\lambda_{\alpha q}\ ,\ \mu_q^\alpha\ ,\ \eta_q\right)} \ .$$

- These 11D incidence relations

$$\mu_q^\alpha(\sigma) = X^{\alpha\beta}(\sigma)\lambda_{\beta q}(\sigma) - \frac{i}{2}\theta^\alpha(\sigma)\,\theta^\beta(\sigma)\lambda_{\beta q}(\sigma), \qquad \eta_q(\sigma) = \theta^\beta(\sigma)\lambda_{\beta q}(\sigma)$$

describe the general solution of 120 constraints

$$\mathbb{J}_{pq} := 2\lambda_{\alpha[p}\mu_{q]}{}^\alpha + i\eta_p\eta_q = 0$$

- which can be identified with generator of $SO(16)$ gauge symmetry in the Hamiltonian formalism.

- The rigid SUSY leaving invariant the original action, $\delta_\epsilon X^{\alpha\beta} = i\theta^{(\alpha}\epsilon^{\beta)}$, $\delta_\epsilon\theta^\alpha = \epsilon^\alpha$, is realized on our constrained supertwistor by

$$\delta_\epsilon\lambda_{\alpha q} = 0\ , \qquad \delta_\epsilon\mu_q{}^\alpha = -i\epsilon^\alpha\eta_q\ , \qquad \delta_\epsilon\eta_q = \epsilon^\alpha\lambda_{\alpha q}.$$

- The action is also invariant under the following gauge symmetry transformations

$$\delta\mu^{\alpha}_{q} = -\frac{1}{64} i\delta Z^{\nu_1\nu_2}(\sigma)\tilde{\Gamma}^{\alpha\beta}_{\nu_1\nu_2}\lambda_{\beta q} + \frac{1}{32\cdot 5!}\delta Z^{\nu_1\ldots\nu_5}(\sigma)\tilde{\Gamma}_{\nu_1\ldots\nu_5}{}^{\alpha\beta}\lambda_{\beta q}$$

  with arbitrary $\delta Z^{\mu\nu}(\sigma)$ and $\delta Z^{\nu_1\cdots\nu_5}(\sigma)$.
- This symmetry allows for the gauge fixing conditions reducing the general solution of the constraints $J_{qp} = 0$ with $\mu^{\alpha}_{q} = X^{\alpha\beta}\lambda_{\beta q} - \frac{i}{2}\theta^{\alpha}\,\theta^{\beta}\lambda_{\beta q}$ to

$$\mu^{\alpha}_{q} := \frac{1}{32} X^{\nu}\tilde{\Gamma}^{\alpha\beta}_{\nu}\lambda_{\beta q} - \frac{i}{2}\theta^{\alpha}\,\theta^{\beta}\lambda_{\beta q}$$

- which is the bosonic incidence relation for the case of ambitwistor superstring considered as dynamical system in the standard 11D superspace.

- The advantage of considering ambitwistor superstring as a dynamical system in enlarged superspace $\Sigma^{(528|32)}$ is that in its twistor form μ^{α}_{q} variable is restricted by the (first class) constraints $J_{qp}=0$ only.
- Furthermore, we can introduce J_{qp} with Lagrange multiplier into the action,

$$S=\int_{\mathcal{W}^2} d^2\sigma\,(\lambda_{\alpha q}\,\bar{\partial}\mu^{\alpha}_{q}-\bar{\partial}\lambda_{\alpha q}\,\mu^{\alpha}_{q}-i\bar{\partial}\eta_q\,\eta_q)+\int_{\mathcal{W}^2} d^2\sigma\bar{A}^{pq}\left(2\lambda_{\alpha[p}\mu^{\alpha}_{q]}+i\eta_{[p}\eta_{q]}\right)$$

  and consider the variables μ^{α}_{q} as unconstrained.
- It is important that the action is invariant under $SO(16)$ gauge symmetry

$$\lambda_{\alpha q}(\sigma)\mapsto\lambda_{\alpha p}(\sigma)\mathcal{O}_{pq}(\sigma),\qquad \mu^{\alpha}_{q}(\sigma)\mapsto\mu^{\alpha}_{p}(\sigma)\mathcal{O}_{pq}(\sigma)\qquad \mathcal{O}\mathcal{O}^T=\mathbb{I}\,,$$

  provided the Lagrange multiplier $\bar{A}^{pq}=\bar{A}^{[pq]}$ is transformed as a gauge field,

$$\bar{A}^{pq}\;\mapsto\;\left(\mathcal{O}^{-1}\bar{\partial}\mathcal{O}+\mathcal{O}^{-1}\bar{A}\mathcal{O}\right)^{pq},\qquad \mathcal{O}\mathcal{O}^T=\mathbb{I}\,.$$

- The fact that $\mu_q^\alpha(\sigma)$ and $\eta_q(\sigma)$ in our action can be treated as unconstrained allows to obtain immediately the equations of motion for the constrained spinor functions $\lambda_{\alpha q}(\sigma)$ and for the 16 fermionic functions $\eta_q(\sigma)$,

$$\bar{D}\lambda_{\alpha q} := \bar{\partial}\lambda_{\alpha q} - \lambda_{\alpha p}\bar{A}^{pq} = 0 \, , \qquad \bar{D}\eta_q := \bar{\partial}\eta_q - \eta_p\bar{A}^{pq} = 0 \, ,$$

where $\bar{D}$ are $SO(16)$ covariant derivatives.

- To arrive at the equation the solution of which can be related with meromorphic vector function $P_\mu(\sigma) = \sum_i \frac{k_{i\mu}}{\sigma-\sigma_i}$ by

$$P_\mu(\sigma)\delta_{qp} = \lambda_q(\sigma)\tilde{\Gamma}_\mu\lambda_p(\sigma), \qquad 2\lambda_{\alpha q}(\sigma)\lambda_{\beta q}(\sigma) = \Gamma^\mu_{\alpha\beta}P_\mu(\sigma)$$

we need to include into the action the contribution of a suitable vertex operators.

Vertex operator

- The suitable vertex operator was proposed by Geyer and Mason [2019=PLB2020].
- In the spinor frame formalism its SO(16) gauge invariant generalization reads

$$V = \int d^2\sigma_i \delta(k_i \cdot P(\sigma_i)) \, \mathfrak{W} \, \exp\left(2i\mu_q^{\alpha}(\sigma_i)\lambda_{\alpha Ai} W_{qi}^{A}(\sigma_i) + 2\eta_q(\sigma_i)\eta_{Ai} W_{qi}^{A}(\sigma_i)\right)$$

where $\mathfrak{W}$ denotes a possible additional worldsheet operator depending on polarization data the explicit form of which is not be essential for us.

- Besides this, the vertex operator is expressed in terms of fermionic and spinorial bosonic functions describing the ambitwistor string, $\eta_q(\sigma)$ and $\mu_q^{\alpha}(\sigma)$, $\lambda_{\alpha q}(\sigma)$ (the latter entering $\delta(k_i \cdot P(\sigma_i))$) and the scattering data of i-th particle.
- These latter are described by $\lambda_{\alpha Ai}$, which also defines k_i through $\tilde{k}_i^{\alpha\beta} = 4\lambda_A^{(\alpha i}\lambda_i^{\beta)A}$,
- fermionic η_{Ai} and bosonic matrix *function* $W_{qi}^{A}(\sigma)$ which obeys the purity conditions

$$W_{qi}^{A}(\sigma) W_{qi}^{B}(\sigma) = 0 \ .$$

Pure gauge nature of $W^A_{qi}(\sigma)$

- Despite of the entrance of $W^A_{qi}(\sigma)$ into the set of scattering data, we consider it as a function of σ as otherwise we break explicitly the local $SO(16)$ symmetry.
- However, it should be expressed in terms of w^A_{pi} (the square root of the polarization vector in the sense of $U\!\!\!/_{qpi} := U^I_i \gamma^I_{qp} = 2\bar{w}_{qAi}\bar{w}_{pAi}$) by

$$W^A_{qi}(\sigma) = w^A_{pi}\hat{\mathcal{O}}_{pri}\tilde{\mathcal{O}}_{rq}(\sigma) \, , \qquad \tilde{\mathcal{O}}_{pq}(\sigma) \in SO(16) \, , \qquad \hat{\mathcal{O}}_{pqi} \in SO(16) \, .$$

- This eq. implies that $W^A_{qi}(\sigma)$ is essentially a Stückelberg field for $SO(16)$ gauge symmetry and its presence implies that this gauge symmetry is actually broken by the vertex operators, i.e. at $\sigma = \sigma_i$.
- Clearly, no independent equation can be obtained by varying this Stückelberg field.

- The simplest calculations of the path integral with a vertex operator insertions can be done by searching for the saddle point of the exponent of the action multiplied by the exponential factors from the vertex operators.
- This is to say, the main contribution to the path integral will come from the extrema of the effective action with the source terms coming from vertex operator, which is essentially

$$\begin{aligned} S + S_V \;\; = \;\; & \int_{\mathcal{W}^2} d^2\sigma \left(\lambda_{\alpha q}\, \bar{\partial}\mu_q^{\alpha} - \bar{\partial}\lambda_{\alpha q}\, \mu_q^{\alpha} - i\bar{\partial}\eta_q\, \eta_q\right) + \\ & + \int_{\mathcal{W}^2} d^2\sigma \bar{A}^{[pq]} \left(2\lambda_{\alpha[p}\mu_{q]}^{\alpha} + i\eta_{[p}\eta_{q]}\right) + \\ & + \sum_i \int_{\mathcal{W}^2} d^2\sigma \delta(\sigma - \sigma_i) \left(2\mu_q^{\alpha}(\sigma)\lambda_{\alpha Ai} W_{qi}^{A}(\sigma) - 2i\eta_q(\sigma)\eta_{Ai} W_{qi}^{A}(\sigma)\right) \; . \end{aligned}$$

- Eqs of motion which follow from the variation of the eff. action with respect to the unconstrained fields, $\mu_q^\alpha(\sigma)$ and $\eta_q(\sigma)$, are

$$\bar{D}\lambda_{\alpha q}(\sigma) = \sum_i \delta(\sigma - \sigma_i)\lambda_{\alpha A i} W_{qi}^A(\sigma_i) \,, \qquad \bar{D}\eta_q(\sigma) = \sum_i \delta(\sigma - \sigma_i)\eta_{Ai} W_{qi}^A(\sigma_i) \,.$$

- The SO(16) connection $\bar{A}^{pq}$ in the covariant derivative $\bar{D}$ is a one component gauge field associated to the derivative in one (anti-holomorphic) complex direction and, as such, it can always be gauged away.
- This simplifies the search for general solution of the equation which reads

$$\lambda_{\alpha q}(\sigma) = \sum_{i=1}^{n} \frac{\lambda_{\alpha A i} W_{qi}^A(\sigma)}{\sigma - \sigma_i} \,, \qquad \eta_q(\sigma) = \sum_{i=1}^{n} \frac{\eta_{Ai} W_{qi}^A(\sigma)}{\sigma - \sigma_i} \,,$$

where $W_{qi}{}^A(\sigma) = W_{pi}^A \, \tilde{\mathcal{O}}_{pq}(\sigma)$ and $W_{qi}^A = w_{pi}^A \hat{\mathcal{O}}_{pqi}$.

Towards polarized scattering equation

- Substituting the above solution $\boxed{\lambda_{\alpha q}(\sigma) = \sum_{i=1}^{n} \frac{\lambda_{\alpha A i} W_{qi}^{A}(\sigma)}{\sigma - \sigma_i}}$ of the 11D ambitwistor superstring equations into the 11D generalization of the Cartan-Penrose relation

$$P_\mu(\sigma)\delta_{qp} = \lambda_q(\sigma)\tilde{\Gamma}_\mu \lambda_p(\sigma), \qquad 2\lambda_{\alpha q}(\sigma)\lambda_{\beta q}(\sigma) = \Gamma^{\mu}_{\alpha\beta} P_\mu(\sigma),$$

we find

$$\sum_i \frac{\lambda_{\alpha A i}}{\sigma - \sigma_i} \sum_j \frac{\lambda_{\beta B j}}{\sigma - \sigma_j} W_{qj}^{B}(\sigma) W_{qi}^{A}(\sigma) = \sum_i \frac{2\lambda_{(\alpha|Ai}\lambda^{A}_{|\beta)i}}{\sigma - \sigma_i} .$$

- The second order poles are absent in the r.h.s. $\Rightarrow$ should vanish in the l.h.s.
- This implies the purity conditions $W_{qi}^{A}(\sigma) W_{qi}^{B}(\sigma) = 0$ which is clearly obeyed for $W_{qi}^{A}(\sigma) = w_{pi}^{A} \mathcal{O}_{pqi}(\sigma)$ (as, by definition $w_{pi}^{A} w_{pi}^{B} = 0$ and $\mathcal{O}_{pqi}(\sigma) \in SO(16)$).

Polarized scattering equation

- Furthermore, the residues of the first order poles of l.h.s. and r.h.s. of $\sum_i \frac{\lambda_{\alpha Ai}}{\sigma-\sigma_i}\sum_j \frac{\lambda_{\beta Bj}}{\sigma-\sigma_j} W^B_{qj}(\sigma) W^A_{qi}(\sigma) = \sum_i \frac{2\lambda_{(\alpha|Ai}\lambda^A_{|\beta)i}}{\sigma-\sigma_i}$ coincide if the helicity spinors associated to the scattered particles are related by the condition

$$\sum_j \frac{\lambda_{\alpha Bj} W^B_{qj} W^A_{qi}}{\sigma_i - \sigma_j} = \lambda^A_{\alpha i} \, .$$

- Using $\lambda_{\alpha q}(\sigma) = \sum_{i=1}^n \frac{\lambda_{\alpha Ai} W^A_{qi}(\sigma)}{\sigma-\sigma_i}$ we can write this as

$$\lambda_{\alpha q}(\sigma_i) W^A_{qi}(\sigma_i) = \lambda^A_{\alpha i} \, .$$

  which is *11D polarized scattering equation* proposed by Geyer and Mason [2019=PLB2020] (actually, the the SO(16) covariant generalizarion of this).

- Our study in [I.B. JHEP 2019] revealed the moving frame nature of both the constrained spinors and spinor functions involved in it, and provides a rigorous derivation of the polarized scattering equation from the ambitwistor superstring action.

Polarized scattering equation

- Furthermore, using $W_i(\sigma) = w_i \hat{\mathcal{O}}_i \tilde{\mathcal{O}}(\sigma)$, we can write the PSE in the form

$$\sum_j \frac{\lambda_{\alpha Bj} w_{qj}^B w_{qi}^A}{\sigma_i - \sigma_j} = \lambda_{\alpha i}^A \qquad \left(\Rightarrow \qquad \sum \frac{k_i \cdot k_j}{\sigma_i - \sigma_j} = 0 \right)$$

which includes the scattering data described by the helicity spinors and internal harmonics (internal frame variables) only.

- It provides a polarized counterpart of the SE in its form of $\sum_j \frac{k_i \cdot k_j}{\sigma_i - \sigma_j} = 0$,
- while

$$\lambda_{\alpha q}(\sigma_i) W_{qi}^A(\sigma_i) = \lambda_{\alpha i}^A \qquad (\Rightarrow \qquad k_i \cdot \mathcal{P}(\sigma_i) = 0)$$

is a polarized counterpart of the SE in its form of $k_i \cdot \mathcal{P}(\sigma_i) = 0$.

Outline

Conclusion

- In this talk, following [I.B. JHEP 11 (2019)] we have revisited the formalism of the 11D polarized scattering equations of Geyer and Mason [2019=PLB2020] with the use of spinor frame approach (or Lorentz harmonic approach).
- In particular, we derived the equations for spinorial meromorphic function $\lambda_{\alpha q}(\sigma)$ from the (spinor frame formulation) of 11D ambitwistor superstring [I.B. JHEP2014] supplemented by the suitable vortex operator
- and then obtained the 11D polarized scattering equation on this basis.
- To this end, the (gauge equivalent) formulation of ambitwistor superstring as dynamical system in an enlarged 11D superspace $\Sigma^{(528|32)}$ with additional tensor central charge coordinates is very useful.
- Furthermore we have used extensively the 11D supertwistor approach [I.B.+de Azcarraga+Sorokin 2006]
- and $SO(16)$ hidden symmetry of the ambitwistor superstring [I.B. JHEP2014].

Outlook

- Among interesting directions of development of present results let us mention
- their use in calculations of 11D SUGRA and 10D SYM amplitudes,
- in particular, to obtain recurrent relations for 11D and 10D amplitudes, generalizing the 6D line of [Albonico, Geyer, Mason, JHEP 08 (2020)].
- It is also interesting to construct 11D generalization of the 6D rational map and symplectic Grassmannians approach of [Heydeman+Schwarz+Wen JHEP2017], [Cachazo+Guevara+Heydeman+Mizera+Schwarz+Wen JHEP2018] (see [Schwarz+Wen JHEP 2019] for 6D interrelations).