Exotic duality and higher spins

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Plan of talk

1) Introduction & motivations

2) Results on the double-dual graviton and higher duals of vector field

2.1) Double dualisation of a massless vector in $n=4$: $\square \longmapsto \begin{smallmatrix}\square\square\\ \square\end{smallmatrix}$

2.2) Double dualisation of spin-2 $h_{ab} \sim \square\square$ in $n=5$ into $D \sim \begin{smallmatrix}\square\square\\ \square\square\end{smallmatrix}$

3) Higher (or exotic) dualisation of the 3D graviton and scalar $\longrightarrow$ spin-3 gauge field

3.1) Higher dualisation of graviton in 3D $\square\square \longmapsto \square\square\square$

3.2) Higher dualisation of scalar in 3D $\bullet \longmapsto \square\square\square$

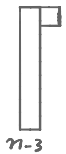
① Introduction – motivations

Duality in gravity $\rightsquigarrow$ Hull's works (circa 2000) on $\mathcal{N}_6 = (4,0)$ theory

and simultaneously West's e_{11} proposal [2001]

↳ In these works, a gauge field

$$C_{[n-3,1]} \rightsquigarrow C_{a_1 \dots a_{n-3}, b} = C_{[a_1 \dots a_{n-3}], b} \quad \text{s.t.} \quad C_{[a_1 \dots a_{n-3}, b]} \equiv 0$$

i.e. $C_{[n-2-1,1]} \sim$  of $GL(n)$ appears in Minkowski spacetime $\mathbb{R}^{1,n-1}$

that propagates the d.o.f. of Fierz–Pauli's graviton h_{ab} s.t. $\eta^{bd} K_{ab,cd}(h) = 0$.

Hull's twisted on-shell duality, relating $K_{a_1 \dots a_{n-2}, }{}^{b_1 b_2} := \partial_{[a_1} \partial^{[b_1} C_{a_2 \dots a_{n-2}], }{}^{b_2]}$

to $K_{ab,}{}^{cd} := -\frac{1}{2} \partial_{[a} \partial^{c} h^{d]}{}_{b]}$ via $\boxed{K_{[n-2,2]} = *_1 K_{[2,2]}}$.

In same work [Hull : 2001] a field

$$D_{[n-3,\,n-3]} \sim \underbrace{\begin{array}{|c|c|}\hline & \\ \hline \end{array}}_{n-3 \quad n-3} \quad \text{of } Gl(n) \,, \qquad \xrightarrow[n=5]{} \qquad D_{[2,2]} \sim \begin{array}{|c|c|}\hline & \\ \hline & \\ \hline \end{array}$$

was introduced with field equations $\boxed{\mathrm{Tr}^{n-3}\, K_{[n-2,n-2]}(D) = 0} \xrightarrow[n=5]{} \mathrm{Tr}^2\, K_{[3,3]} = 0$.

Twisted duality with h_{ab} : $\boxed{K_{[n-2,n-2]}(D) = *_1 *_2 K_{[2,2]}(h)} \xrightarrow[n=5]{} K_{[3,3]} = *_1 *_2 K_{[2,2]}(h)$.

$\underline{\text{EII}}$ $\quad d_1^+ K_{[2,2]} \equiv 0 \iff d_1 K_{[3,3]} = 0 \iff K_{[3,3]} = d_1 d_2 D_{[2,2]} =: K_{[3,3]}(D) \quad \underline{\text{BII}}$

$\boxdot$ [Dubois Violette – Henneaux 2000]

But $\quad \mathrm{Tr}\, K_{[2,2]} = 0 \quad (\text{EI}) \longleftrightarrow \mathrm{Tr}\, *_1 *_2 K_{[3,3]} = 0 \iff \mathrm{Tr}^2 K_{[3,3]} = 0 \quad (\text{EI})$ EoM for D .

Field equations mapped to Field equations (and not on Bianchi identities) .

In presence of sources, Hull argued that $D_{[2,2]}$ couples to the usual T_{ab} stress tensor confirming that $D_{[2,2]}$ is, on-shell, an avatar of h_{ab} .

Indeed, from twisted-duality $\quad K_{[3,3]}(D) = *_1 *_2 K_{[2,2]}(h)$. (*)

$$K_{[3,3]}(D) = d_1 d_2 D_{[2,2]} = *_1 *_2 d_1 d_2 h_{[1,1]} \propto \eta_{[1,1]} d_1 d_2 h_{[1,1]} \qquad (\text{since } n=5)$$

$$\Leftrightarrow \quad d_1 d_2 \left(D_{[2,2]} - a\, \eta_{[1,1]} h_{[1,1]} \right) = 0 \qquad a \in \mathbb{R}_0$$

$$\Leftrightarrow \quad D_{[2,2]} = a\, \eta_{[1,1]} h_{[1,1]} + d^{[2]} \xi_{[2,1]} \qquad (**)$$

$\Rightarrow$ When the EoMs for h_{ab} Fierz-Pauli are satisfied (since twisted on-shell duality (*) was used),

then the $D_{[2,2]}$-field is conformally flat up to a gauge transformation (**)

[Marc H., Victor Lekeu and Amaury Leonard 1909.12706]

Therefore, inverting (**) to express $h_{[1,1]} = H_{[1,1]}(\mathrm{Tr}\, D, \xi)$ and plugging in $S^{FP}[H_{ab}(\mathrm{Tr}\, D, \xi)]$ gives field equations equivalent to (**).

$\Rightarrow$ The traceless part of $D_{[2,2]}$ does not enter this action.

However, before [Marc-Victor-Amaury 2019]'s note on the double-dual graviton, an action principle had been proposed for a $D_{[2,2]}$ gauge field (plus other fields) that propagates the degrees of freedom of a single graviton:

N.B, Paul Cook, Mitya Ponomarev [2012]

"Off-shell Hodge dualities in linearized gravity and E_{11}"

- In the paper 2012.11356 with Victor Lekeu, the first thing done was to clarify the non-triviality of the action proposed earlier with P. Cook and D. Ponomarev.

② Results on the double-dual graviton and higher duals of vector field

2.1) Double dualisation of a massless vector [N.B., P. Sundell, P. West 2015]

Idea : A_b viewed as a $A_{[0,1]}$ bi-form

$$A_{[0,1]} \xrightarrow[\text{double dualize}]{} C_{[n-0-2,\, n-1-2]} \underset{n=4}{=} C_{[2,1]} \sim \begin{array}{|c|c|}\hline & \\ \hline & \\ \cline{1-1} \end{array} \quad \text{or} \quad n=3 : C_{[1,1]} \sim \begin{array}{|c|c|}\hline & \\ \hline \end{array}$$

- Starts from Maxwell and integrate by part : $S[A_\mu] = -\frac{1}{2}\int d^n x \, (\partial_a A_b \, \partial^a A^b - \partial_a A^a \, \partial_b A^b)$
- Parent action $S[Y^{\cdot\cdot|\cdot}, P_{\cdot|}^{\cdot}] = \int d^n x \, (P_{a|b} \, \partial_c Y^{ca|b} - \frac{1}{2} P_{a|b} P^{a|b} + \frac{1}{2} P^{a|}{}_a P_{b|}{}^b)$
- $\frac{\delta S[Y,P]}{\delta P_{a|b}} \approx 0 \iff \partial_c Y^{ca|b} - P^{a|b} + \eta^{ab} P^{c|}{}_c \approx 0 \xrightarrow{\mathrm{Tr}} \partial_c Y^{ca|}{}_a + (n-1) P^{a|}{}_a \approx 0 \iff P^{a|}{}_a = \frac{1}{1-n} \partial_c Y^{ca|}{}_a$

$$P^{a|b} \approx \partial_c Y^{ca|b} - \eta^{ab} \frac{1}{n-1} \partial_c Y^{cd|}{}_d$$

substitute to get

$$S[P_{a|b}] = \int d^n x \left[\frac{1}{2} \partial_c Y^{ca|b} \, \partial_d Y^d{}_{a|b} - \frac{1}{2(n-1)} \partial_a Y^{ab|}{}_b \right]$$

- From $$S[P_{a|b}] = \int d^n x \left[\tfrac{1}{2}\, \partial_c Y^{ca|b}\, \partial_d Y^{d}{}_{a|b} - \frac{1}{2(n-1)}\, \partial_a Y^{ab|}{}_{b} \right]$$

invariant under $$\delta Y^{ab|}{}_{c} = \delta^{[a}_{c}\, \partial^{b]} \lambda + \partial_d \Upsilon^{abd|}{}_{c} \,,$$

one decomposes $$Y^{ab|}{}_{c} = X^{ab|}{}_{c} + \delta^{[a}_{c}\, Z^{b]} \,, \qquad X^{ab|}{}_{b} \equiv 0$$

and

- <u>Hodge-dualise in $n=4$</u> : $$X^{ab|}{}_{c} \longleftrightarrow T_{ab|c} \sim \begin{array}{|c|c|}\hline a & c \\ \hline b & \\ \hline \end{array}$$

with gauge transformations $$\begin{cases} \delta \begin{array}{|c|c|}\hline & \\ \hline & \\ \hline \end{array} = \overset{S}{\begin{array}{|c|c|}\hline & \\ \hline \partial & \\ \hline \end{array}} + \overset{A}{\begin{array}{|c|c|}\hline & \partial \\ \hline & \\ \hline \end{array}} \\ \delta Z_a = \partial_a \lambda + \partial^b A_{ab} \end{cases}$$

- <u>Hodge-dualise in $n=3$</u> : $$Y^{ab|}{}_{c} = \varepsilon^{abd}\, h_{dc} + 2\, \delta^{[a}_{c}\, Z^{b]} \,,$$

where $h_{\cdot\cdot} \sim \begin{array}{|c|c|}\hline \cdot & \cdot \\ \hline \end{array}$ and $\delta h_{ab} = 2\, \partial_{(a} \epsilon_{b)}$, $\delta Z_a = \partial_a \lambda + \varepsilon_{abc}\, \partial^b \epsilon^c$

Ⓐ In dim. $n=3$,

$$\boxed{S[h_{ab}, Z_a] = \int d^3x \left[-\tfrac{1}{2}\partial_a h_{bc}\partial^a h^{bc} + \tfrac{1}{2}\partial_a h_{bc}\partial^b h^{ac} + \tfrac{1}{2}\varepsilon^{bcd}\partial^a h_{ab} F_{cd}(Z) + \tfrac{1}{4}F^{ab}(Z)F_{ab}(Z) \right]}$$

where $F_{ab}(Z) := 2\,\partial_{[a}Z_{b]}$ and copy the gauge transformation laws:

$$\delta h_{ab} = 2\,\partial_{(a}\epsilon_{b)} \quad , \quad \delta Z_a = \partial_a \lambda + \varepsilon_{abc}\partial^b \epsilon^c$$

One can dualise Z_a vector field into a scalar σ in $n=3$, then define

$$\phi := 2\sigma + \eta^{ab} h_{ab}$$

to obtain

$$S[\phi, h_{ab}] = \int d^3x \left[\mathcal{L}^{FP}(\partial h) + \tfrac{1}{2}\partial_a\phi\,\partial^a\phi + \partial_a\phi\,(\partial_b h^{ab} - \partial^a h) \right] \qquad (\triangle)$$

Field equations $\Rightarrow$ $\boxed{\mathrm{Tr}^2 K_{[2,2]}(h) = 0 \quad , \quad \Box\phi = 0 \quad \& \quad K_{ab}(h) = \partial_a\partial_b\phi}$

$\Rightarrow$ No doubling of physical d.o.f. !

(B) In dim. $n=4$, perform some change of field variables and dualize $Z_a \leftrightarrow \tilde{A}_a$

$$S[T_{abc}, \tilde{A}_a] = \int d^4x \left[\mathcal{L}^{\text{curt.}}(T_{ab,c}) + \frac{1}{4} F^{ab}(\tilde{A}) F_{ab}(\tilde{A}) - \frac{1}{\sqrt{2}} \tilde{A}^a K''_a(T) \right]$$

where $K^{a[3]}{}_{b[2]} := 6\, \partial^{[a} \partial_{[b} T^{aa],}{}_{b]}$ curvature, $K'' := \mathrm{Tr}^2 K$.

The gauge invariances are the ones expected for a Curtright field and a vector.

- Field equations:

$$-\partial_a F^{ab}(\tilde{A}) + \frac{1}{\sqrt{2}} K''^b = 0 \qquad (1)$$

$$K'^{abl}{}_c + \delta^{[a}_c K''^{b]} - \frac{1}{\sqrt{2}} \partial_c F^{ab} - \frac{1}{\sqrt{2}} \delta^{[a}_c \partial_d F^{b]d} = 0 \qquad (2)$$

Take the trace of (2), combine with (1) to get the EoMs and duality relation

$$\boxed{d^\dagger F_{[2]}(\tilde{A}) = 0 = \mathrm{Tr}^2 K_{[3,2]}(T) \ \& \ \mathrm{Tr}\, K_{[3,2]}(T) = \frac{1}{2} d_2 F_{[2,0]}(\tilde{A})} \Rightarrow \text{no doubling of d.o.f.!}$$

2.2) Double-dualisation of spin-2 in $n=5$.

The dualization procedure given in [N.B., P. Cook, D. Ponomarev 2012] for the double-dual graviton gives an action

$$S[D_{ab,cd}\,,\, Z^{ab|}{}_{c}] = \int d^5x\, \mathcal{L}(\partial D, \partial Z) \quad , \quad D \sim \begin{array}{|c|c|}\hline & \\ \hline & \\ \hline \end{array}$$

$$Z^{ab|}{}_{c} = -Z^{ba|}{}_{c}\,.$$

where $\mathcal{L}(\partial D, \partial Z) = \mathcal{L}(\partial Z) - \mathcal{L}(\partial D) + \mathcal{L}^{cross}$

features the complete $D_{ab,cd}$ field, including its traceless part.

The action is gauge invariant under

$$\begin{cases} \delta \begin{array}{|c|c|}\hline & \\ \hline & \\ \hline \end{array} = \begin{array}{|c|c|}\hline & \\ \hline & \partial \\ \hline \end{array} \\[2ex] \delta Z^{mn|}{}_{a} = \lambda^{mn}{}_{a} + \partial^{[m}\xi^{n]}{}_{a} - \frac{1}{2}\,\delta^{[m}_{a}\,\partial_b\,\xi^{n]b} + \delta^{[m}_{a}\,\partial^{n]}\chi \end{cases}$$

where $\lambda_{abc} = \lambda_{[abc]}$ and $\xi^{a|b} \sim \boxed{a} \otimes \boxed{b}$

Perform a change of variable

$$Y^{ab|}{}_{c} := Z^{ab|}{}_{c} + \delta^{[a}_{c} Z^{b]}$$

to get

$$S[Y^{ab|}{}_{c}, D_{ab,cd}] = \int d^5x \, \Big[\mathcal{L}^{curt.}(Y_{abic}) - \mathcal{L}(\partial D) + \tfrac{1}{2} \epsilon_{abcde} \, \partial^e D^{cd,}{}_{mn} \big(\overbrace{F^{abmlm}(Y)}^{3\,\partial^{[a} Y^{bm]|n}} - \tfrac{1}{2} F^{mnalb} \big) \Big]$$

That propagates a single graviton.

Dualize $Y^{ab|}{}_{c}$ in $n=5$ $(\epsilon_{abcde} \partial^d f^e{}_{lu} F^{abclu})$ for $f_{aic} \sim h_{ab} + B_{[ab]}$ to get

$$S[D_{ab,cd}, f_{ab}] = \int \Big[\mathcal{L}^{FP}(h) - \mathcal{L}(\partial D) - \tfrac{3}{2} h_{ab} \tilde{K}^{,ab}(D) \Big]$$

where $\tilde{K}_{[2,2]}(D) := *_1 *_2 \underbrace{d_1 d_2 D_{[2,2]}}_{K_{[3,3]}}$. Field equations:

$$\boxed{\begin{aligned} &\mathrm{Tr}\, K_{[2,2]}(h) = 0 = \mathrm{Tr}^2 K_{[3,3]}(D) \\ &\& \quad K_{[2,2]}(h) \propto \mathrm{Tr}\, K_{[3,3]}(D) . \end{aligned}}$$

③ Higher (or exotic) dualisation of the 3D graviton and scalar

3.1) Higher dualisation of graviton in 3D $\quad \begin{array}{|c|c|}\hline \ & \ \\ \hline \end{array} \longmapsto \begin{array}{|c|c|c|}\hline \ & \ & \ \\ \hline \end{array} \oplus$ more

Spin-2 Fierz-Pauli $\begin{array}{|c|c|}\hline \ & \ \\ \hline \end{array}$ viewed as $h_{[0,1,1]}$ gauge field

Curvature $\quad K_{[2,2,1]}(h) \sim \begin{array}{|c|c|c|}\hline \ & \ & \partial \\ \hline \partial & \partial & \\ \hline \end{array} \xrightarrow[*_3]{} \begin{array}{|c|c|c|}\hline \ & \ & \ \\ \hline \ & \ & \ \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline \ & \ & \ \\ \hline \partial & \partial & \partial \\ \hline \end{array} = K_{[2,2,2]}(\varphi)$

On-shell Twisted duality

$$*_3 K_{[2,2,1]}(h) = K_{abicdiu}(h)\,\varepsilon^{u}{}_{ef} \sim \begin{array}{|c|c|c|}\hline a & c & e \\ \hline b & d & f \\ \hline \end{array} \underset{TD}{=} K_{abicdief}(\varphi) := 8\,\partial_{[a}\partial_{[c}\partial_{[e}\varphi_{f]d]b]}$$

$$\mathrm{Tr}\, K_{[2,2,1]}(h) = 0 = \mathrm{Tr}\, K_{[2,2,2]} \qquad (EI)$$

Since $n=3$, $(EI) \Leftrightarrow K_{[2,2,1]}(h) = 0 = K_{[2,2,2]}(\varphi)$ Topological

To make that story <u>variationnal</u> :

1. $$S_{FP}[h_{ab}] = \int d^3x \left[-\tfrac{1}{2}\, \partial_a h_{bc}\, \partial^a h^{bc} + \tfrac{1}{2}\, \partial_a h\, \partial^a h - \partial_a h^{ab}\, \partial_b h + \partial_a h^{ab}\, \partial^c h_{cb} \right]$$

2. $$S[G_{a|bc}, D_{ab|}{}^{cd}] = \int d^3x \left[-\tfrac{1}{2}\, G_{a|bc}\, G^{a|bc} + \tfrac{1}{2}\, G_{a|a}{}^{a}\, G^{a|a}{}_{a} - G_{a|}{}^{ab}\, G_{b|a}{}^{a} + G_{a|}{}^{ab}\, G^{a|}{}_{ab} + G^{d|}{}_{bc}\, \partial^a D_{ad|}{}^{bc} \right]$$

where $G_{a|bc} \sim \boxed{a} \otimes \boxed{b|c}$, $D_{ad|}{}^{bc} \sim \begin{smallmatrix}\boxed{a}\\ \boxed{d}\end{smallmatrix} \otimes \boxed{b|c}$

3. Solve for $G_{a|bc}$ auxiliary field

$$\hookrightarrow S[D_{ab|}{}^{cd}] = \tfrac{1}{2} \int d^3x \left[\partial^a D_{ab|cd}\, \partial_e D^{eb|cd} - \partial^a D_{ab|c}{}^{b}\, \partial_d D^{de|c}{}_{e} - \partial^a D_{ab|c}{}^{c}\, \partial_e D^{eb|d}{}_{d} \right]$$

4. Decompose $\tilde{D}_{a|bc} := -\tfrac{1}{2}\, \varepsilon_{amn}\, D^{mn|}{}_{bc} \sim \boxed{a} \otimes \boxed{b|c} \xrightarrow[\downarrow\, so(3)]{} \boxed{}\boxed{}\boxed{} \oplus \boxed{}\boxed{} \oplus 2\times\boxed{}$

5. Dualise one vector into scalar $\bullet$, perform some field redefinitions to get ...

$$
\begin{aligned}
S[\varphi_{abc}, h_{ab}] = \frac{1}{2} \int d^3x \Big[& - \partial_a \varphi_{bcd}\, \partial^a \varphi^{bcd} + \partial^a \varphi^b\, \partial^c \varphi_{abc} + \partial_a \varphi^{abc}\, \partial^d \varphi_{bcd} \\
& - \tfrac{1}{7}\, \partial_a \varphi_b\, \partial^a \varphi^b - \tfrac{31}{28}\, \partial_a \varphi^a\, \partial^b \varphi_b \\
& + \tfrac{1}{2}\, \partial_a h_{bc}\, \partial^a h^{bc} + \tfrac{1}{14}\, \partial_a h\, \partial^a h - \tfrac{3}{7}\, \partial^a h_{ab}\, \partial_c h^{bc} - \tfrac{1}{7}\, \partial^a h\, \partial_c h_a{}^c \\
& + \tfrac{10}{7}\, \varepsilon_{apq}\, \partial^b h_b{}^a\, \partial^p \varphi^q - 2\, \varepsilon_{apq}\, \partial^b h^{ac}\, \partial^p \varphi^q{}_{bc} \Big]
\end{aligned}
\tag{3.32}
$$

that is invariant under

$$
\delta \varphi_{abc} = 3\, \partial_{(a} \widehat{\xi}_{bc)} - \tfrac{2}{3}\, \varepsilon_{(a}{}^{pq}\, \eta_{bc)} \partial_p \epsilon_q \,, \tag{3.33}
$$

$$
\delta h_{ab} = 2\, \partial_{(a} \epsilon_{b)} + 2\, \varepsilon_{pq(a} \partial^p \widehat{\xi}^q{}_{b)} \,. \tag{3.34}
$$

Recall that this is a topological theory.

From a general result [Grigoriev - Mkrtchyan - Skvortsov 2005], we know that

it can be put in Chern - Simons form [To appear]

3.2) Double exotic dualisation of vector in 3D : $\square \longmapsto \square\square\square \oplus$ more

Start from the first exotic dualisation of Maxwell's theory in $n = 3$

$$S[h_{ab}, Z_a] = \int d^3x \left[-\tfrac{1}{2} \partial_a h_{bc} \partial^a h^{bc} + \tfrac{1}{2} \partial_a h_{bc} \partial^b h^{ac} + \tfrac{1}{2} \varepsilon^{bcd} \partial^a h_{ab} F_{cd}(Z) + \tfrac{1}{4} F^{ab}(Z) F_{ab}(Z) \right]$$

where $\quad \delta h_{ab} = 2 \partial_{(a} \epsilon_{b)} \quad , \quad \delta Z_a = \partial_a \lambda + \varepsilon_{abc} \partial^b \epsilon^c$

and dualise $h_{ab} \sim \boxed{a}\boxed{b}$ in empty column.

After some steps (parent action, daughter action, so(3) decomposition, field redef, dualisation, field redef.), one gets an action $S[\phi_{abc}, h_{ab}, A_a]$ invariant under entangled gauge transformations:

$$\boxed{\begin{aligned} \delta \phi_{abc} &= 3 \partial_{(a} \hat{\xi}_{bc)} - \tfrac{2}{3} \varepsilon_{(a}{}^{mn} \eta_{bc)} \partial_m \epsilon_n \\ \delta h_{ab} &= 2 \partial_{(a} \epsilon_{b)} + 2 \varepsilon_{mn(a} \partial^m \hat{\xi}^n{}_{b)} \\ \delta A_a &= \partial_a \lambda + \varepsilon_{abc} \partial^b \epsilon^c \end{aligned}}$$

④ Outlook

- In the topological case in 3D, pursuing the higher (exotic) dualisations to produce higher (undecomposable) spin fields, we expect an infinite spectrum of fields of rank $\in \{2, 3, 4, \dots\}$.

  ↳ It cannot match the well-known Blencowe model, since here we have an undecomposable structure at the level of gauge transformations.

- In the propagating case in 3D, pursuing the higher (exotic) dualisations to produce higher (undecomposable) spin fields, we expect an infinite spectrum of fields of rank $\in \{1, 2, 3, \dots\}$.

- In the case of double exotic dual of a scalar field in $n=3$, the action

$$S[h_{ab},\sigma] = \int d^3x \left[-\tfrac{1}{2}\, \partial_a h_{bc}\, \partial^a h^{bc} + \partial_a h^{ab}\, \partial^c h_{bc} + 2\, \partial_a \sigma \left(\partial^a \sigma + \partial_b h^{ab} \right) \right]$$

is obtained in [2012.11356], invariant under

$$\delta h_{ab} = 2\, \partial_{(a} \epsilon_{b)} \quad , \quad \delta \sigma = -\, \partial^a \epsilon_a \quad .$$

↳ We now have found a direct relation with the spin-2 triplet system

as studied and discussed in

"Maxwell-like Lagrangians for higher-spins", [A. Campoleoni & D. Francia, 1206.5877]

- In the higher-spin cases, other undecomposable, finite-dimensional representations are expected.