

Holographic QCD for NICA

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Quarks 2021

“Integrability, Holography, Higher-Spin Gravity and String”
dedicated to A.D. Sakharov’s centennial

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Theoretical studies of the formation and properties of quark-gluon matter under conditions of high baryon densities

- By theoretical I mainly mean the holographic studies
- Results from holography for heavy ions collisions:
 - Fits experimental data
 - Predicts new results
- What is special for NICA (Nuclotron Based Ion Collider fAcility)

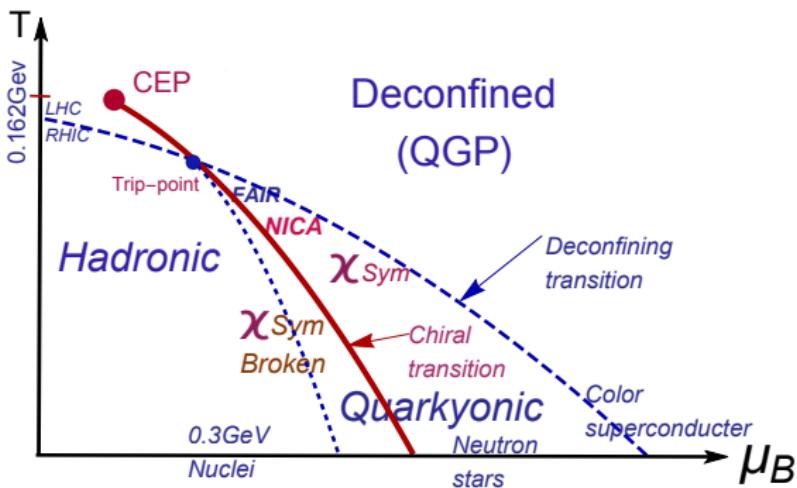
Outlook

- ① Our goal
- ② Our methods (Few remarks on holography)
- ③ The expected QCD phase diagram in (μ, T) -space
 - 1-st order phase transition vs crossover
 - Phase diagrams for heavy vs light quarks
- ④ Holographic QCD phase diagrams in (μ, T, B, ν) -space
 - Background phase transition
 - Phase diagrams for nonlocal observables
 - Phase diagrams for local observables
- ⑤ Conclusion and what next?

Few remarks on holography (AdS/CFT)

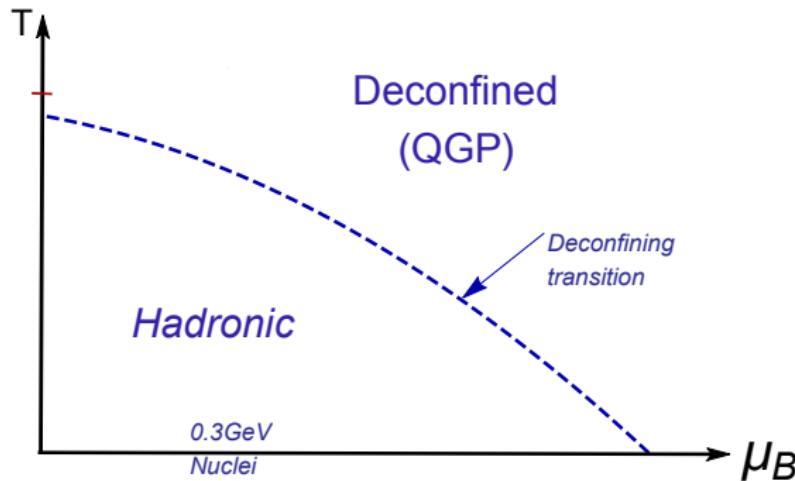
- Holography is nowadays one of the most effective tools to study quantum non-equilibrium physics of strongly interacting many body systems
- These systems include ultrarelativistic heavy-ion collisions, cold atom systems, quantum simulators, "ultrafast" techniques in condensed matter physics, etc.
- Holography translates the physics of quantum many body systems into a dual classical gravitational problem in a space-time with an extra dimension

The expected QCD phase diagram



QCD Phase Diagram: Early Conjecture

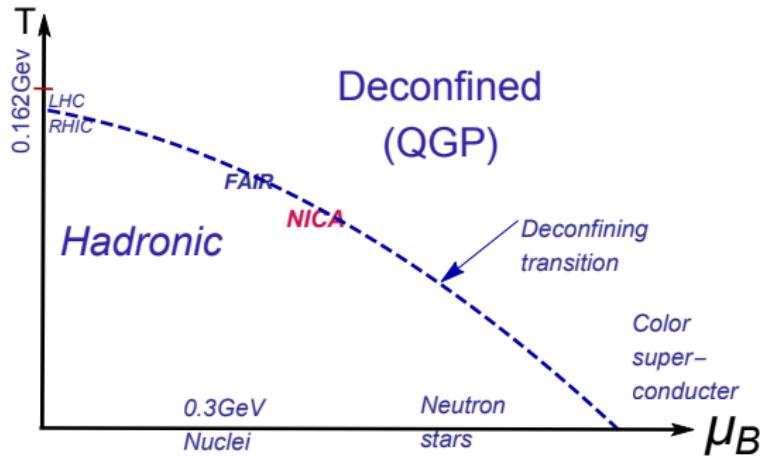
Cabibbo and Parisi, 1975



- μ a measure of the imbalance between quarks and antiquarks in the system

QCD Phase Diagram: Experiments

One of the main points at ...

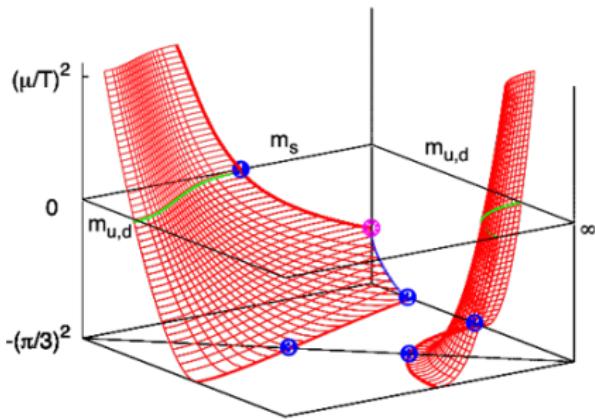
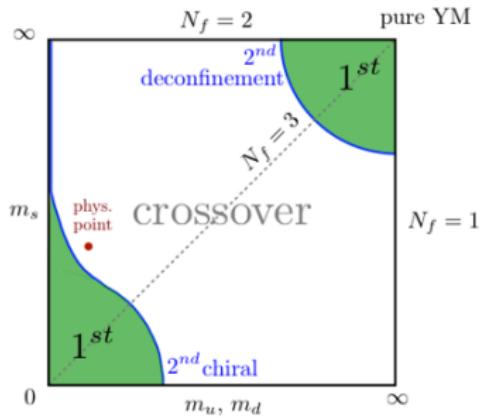


- μ a measure of the imbalance between quarks and antiquarks in the system

QCD Phase Diagram: Lattice

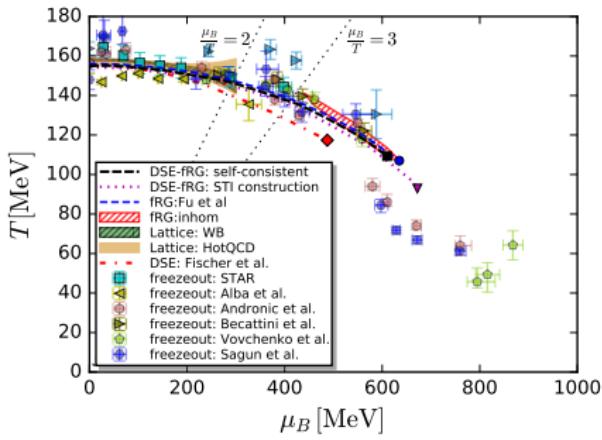
- Main problem with $\mu \neq 0$.

Imaginary chemical potential method.

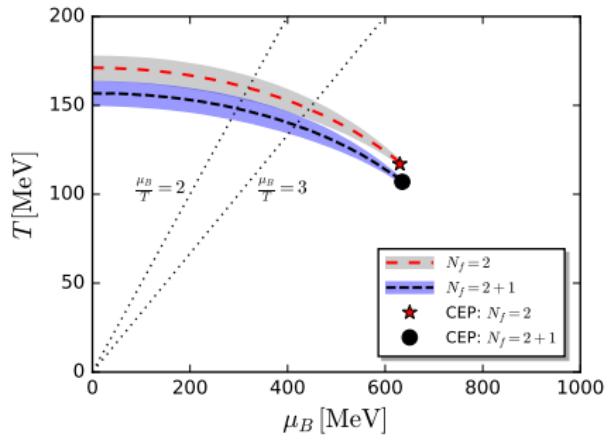


- A. Columbia plot. *F.R. Brown et al, Phys. Rev. Lett. (1990)*
- B. *O. Philipsen, C. Pinke, PRD (2016)*
- Does first order near the origine survive in the continuum?

QCD Phase Diagram: Lattice, phenomenological and freeze-out data



A



B

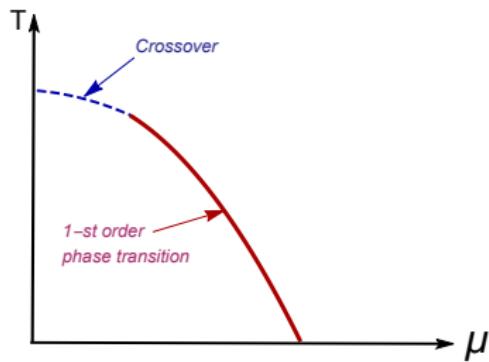
A. Phase diagram for $N_f = 2 + 1$ flavour QCD — different approaches and freeze-out data [Beam Energy Scan program]
The chiral crossover – the black dashed line

B. Phenomenological calculations from 1909.02991

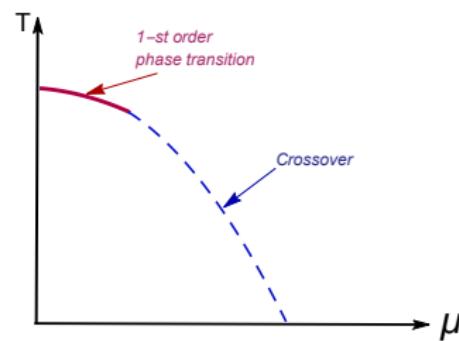
$$T_c(\mu_B)/T_c = 1 - \kappa (\mu_B/T_c)^2 + O(\mu_B^4)$$

“Heavy” and “light” quarks from Columbia plot

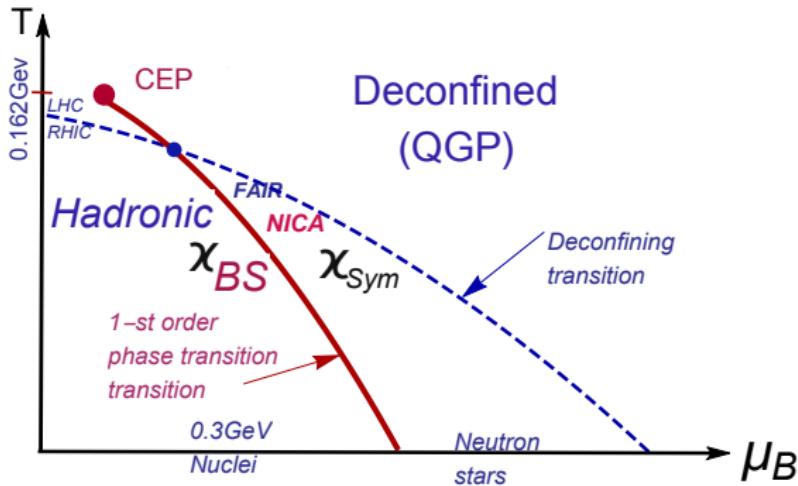
Light quarks



Heavy quarks

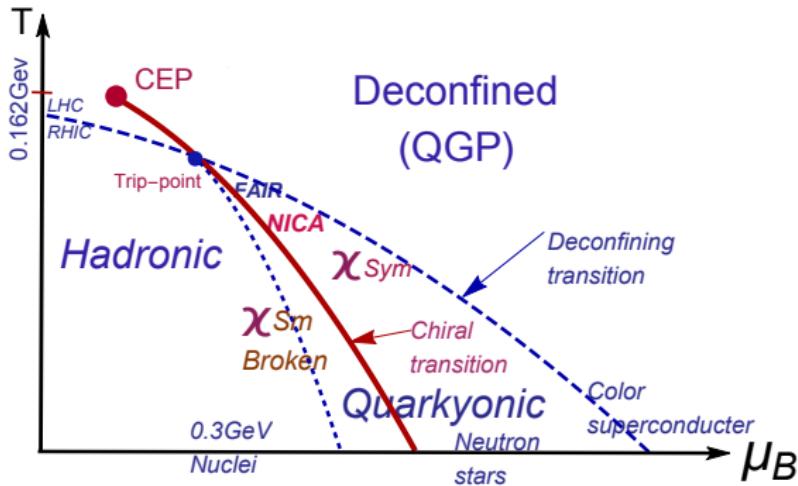


The expected QCD phase diagram



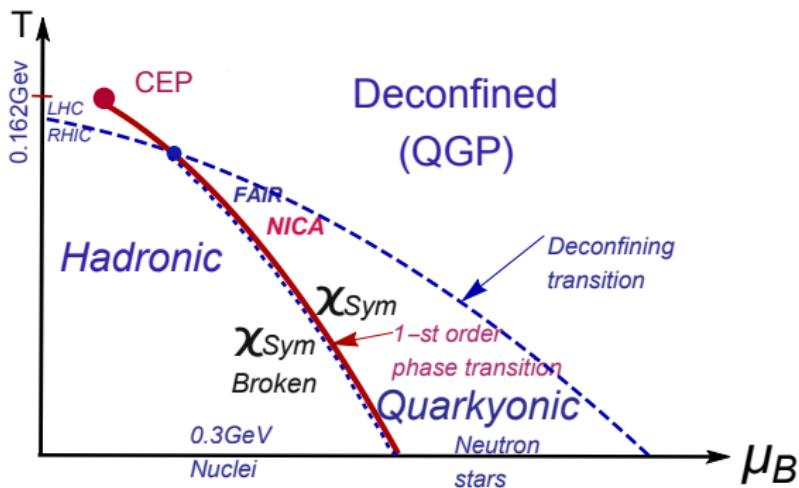
- Parameter of the chiral symmetry breaking $\langle \bar{\psi}\psi \rangle$
 - $\langle \bar{\psi}\psi \rangle = 0 \iff \chi\text{-symmetry}$
 - $\langle \bar{\psi}\psi \rangle \neq 0 \iff \text{broken } \chi\text{-symmetry}$

The expected QCD phase diagram



- Quarkyonic phase: baryon free \Rightarrow dense baryons *McLerran, Pisarski
0706.2191*
- Baryon density jumps

The expected QCD phase diagram



Anisotropy in QGP

- Origins of anisotropy

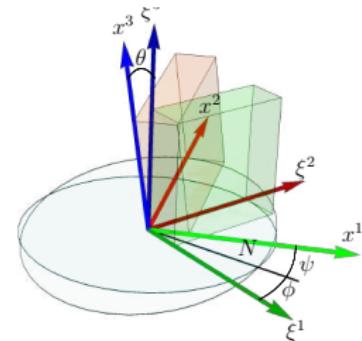
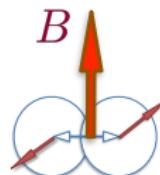
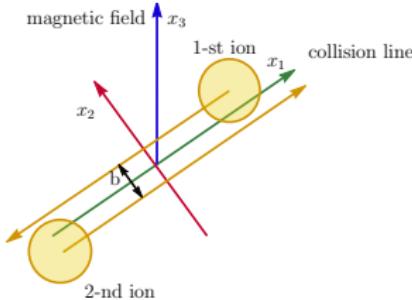
- Longitudinal and 2 transversal directions

- Multiplicity dependencies ALICE
 - I.A., Golubtsova, JHEP'14

$$\mathcal{M}(s) \sim s^{0.155(4)}$$

$$\mathcal{M} \sim s^{1/(\nu+2)} \Rightarrow \nu = 4.5$$

- Full anisotropy (in strong magnetic field, $eB \sim 5 - 10 m_\pi^2$, m_π – pion mass)



Peripheral HIC

Full Anisotropic Background

- Action

$$\int d^5x \sqrt{-g} \left[R - \frac{f_1(\phi)}{4} F_{(1)}^2 - \frac{f_2(\phi)}{4} F_{(2)}^2 - \frac{f_B(\phi)}{4} F_{(B)}^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]$$

- Kinetic functions: f_1 – fixed, f_2, f_B – to be find, as well as $V(\phi)$
- General anizotropic metric

$$ds^2 = \frac{\mathfrak{b}(z)}{z^2} \left[-g(z)dt^2 + \mathfrak{g}_1(z)dx_1^2 + \mathfrak{g}_2(z)dx_2^2 + \mathfrak{g}_3(z)dx_3^2 + \frac{dz^2}{g(z)} \right]$$

- In our model

$$ds^2 = \frac{\mathfrak{b}(z)}{z^2} \left[-\textcolor{red}{g(z)} dt^2 + dx^2 + z^{2-\frac{2}{\nu}} dy_1^2 + \textcolor{blue}{e^{c_B z^2}} z^{2-\frac{2}{\nu}} dy_2^2 + \frac{dz^2}{\textcolor{red}{g(z)}} \right]$$

Full Anisotropic Background

“Bottom-up” approach

- Wrap-factor $\mathfrak{b}(z) = e^{2\mathcal{A}(z)}$

Kinetic function $f_1(z)$

- heavy quarks

- $\mathcal{A}(z) = -cz^2/4 \rightarrow (\text{b, t})$
- $f_1 = z^{-2+\frac{2}{\nu}}$

Andreev, Zakharov (2006)

IA, Rannu, JHEP, 2018

- light quarks

- $\mathcal{A}(z) = -a \ln(bz^2 + 1) \rightarrow (\text{d, u})$
 - $f_1 = e^{-cz^2 - \mathcal{A}(z)} z^{-2+\frac{2}{\nu}}$
- $a = 4.046, b = 0.01613, c = 0.227$

Gursoy, Kiritsis, Nitti, JHEP'08

IA, Rannu, Slepov, JHEP, 2021

Li, Yang, Yuan (2017)

- hybrid (more realistic)

- $\mathcal{A}(z) = -a \ln(bz^2 + 1) + cz^2/2 + pz^4/4$
- $f_1 = e^{-cz^2 - \mathcal{A}(z)} z^{-2+\frac{2}{\nu}}$

IA, Rannu, Slepov in progress

- Maxwell's fields

$$A_\mu^{(1)} = A_t(z) \delta_\mu^0$$

$$F_{\mu\nu}^{(2)} = q \ dy^1 \wedge dy^2$$

$$F_{\mu\nu}^{(B)} = q_B \ dx \wedge dy^1$$

Full Anisotropic Background

- Boundary conditions

- Vector field

$$A_t(z) = \mu - \rho z^2 + \dots, \quad A_t(z_h) = 0$$

↓
the baryon density

- Scalar field

$$\phi(z_0) = 0 \rightarrow \sigma_{string}(T)$$

Anistropic solution for light quarks

$$ds^2 = \frac{\mathfrak{b}(z)}{z^2} \left[-g(z)dt^2 + dx^2 + z^{2-\frac{2}{\nu}}dy_1^2 + z^{2-\frac{2}{\nu}}dy_2^2 + \frac{dz^2}{g(z)} \right]$$

$$A_t = \mu \frac{e^{cz^2} - e^{cz_h^2}}{1 - e^{cz_h^2}}, \quad \rho = - \frac{\mu c}{1 - e^{cz_h^2}}$$

$$\begin{aligned} g = 1 & - \frac{\int_0^z (1 + b\xi^2)^{3a} \xi^{1+\frac{2}{\nu}} d\xi}{\int_0^{z_h} (1 + b\xi^2)^{3a} \xi^{1+\frac{2}{\nu}} d\xi} + \frac{2\mu^2 c}{1 - e^{cz_h^2}} \int_0^z e^{c\xi^2} (1 + b\xi^2)^{3a} \xi^{1+\frac{2}{\nu}} d\xi \times \\ & \times \left[1 - \frac{\int_0^z (1 + b\xi^2)^{3a} \xi^{1+\frac{2}{\nu}} d\xi}{\int_0^{z_h} (1 + b\xi^2)^{3a} \xi^{1+\frac{2}{\nu}} d\xi} \frac{\int_0^{z_h} e^{c\xi^2} (1 + b\xi^2)^{3a} \xi^{1+\frac{2}{\nu}} d\xi}{\int_0^z e^{c\xi^2} (1 + b\xi^2)^{3a} \xi^{1+\frac{2}{\nu}} d\xi} \right] \end{aligned}$$

Anistropic solution for heavy quarks

$$A_t = \mu \frac{e^{(c-2c_B)z^2/4} - e^{(c-2c_B)z_h^2/4}}{1 - e^{(c-2c_B)z_h^2/4}}, \quad \rho = -\frac{\mu(c-2c_B)}{4 \left(1 - e^{(c-2c_B)z_h^2/4}\right)}$$

$$f_B = -\frac{2c_B z^{1-\nu/2} g}{q_B^2 e^{cz^2/2}} \left(\frac{3cz}{2} + \frac{2}{\nu z} - c_B z - \frac{g'}{g} \right)$$

$$\begin{aligned} g = e^{c_B z^2} & \left\{ 1 - \frac{\Gamma\left(1 + \frac{1}{\nu}\right) - \Gamma\left(1 + \frac{1}{\nu}; \frac{3}{4}(2c_B - c)z^2\right)}{\Gamma\left(1 + \frac{1}{\nu}\right) - \Gamma\left(1 + \frac{1}{\nu}; \frac{3}{4}(2c_B - c)z_h^2\right)} - \right. \\ & - \frac{\mu^2 (2c_B - c)^{-\frac{1}{\nu}}}{4 \left(1 - e^{(c-2c_B)\frac{z_h^2}{4}}\right)^2} \left(\Gamma\left(1 + \frac{1}{\nu}\right) - \Gamma\left(1 + \frac{1}{\nu}; \frac{3}{4}(2c_B - c)z^2\right) \right) \times \\ & \times \left[1 - \frac{\Gamma\left(1 + \frac{1}{\nu}\right) - \Gamma\left(1 + \frac{1}{\nu}; \frac{3}{4}(2c_B - c)z^2\right)}{\Gamma\left(1 + \frac{1}{\nu}\right) - \Gamma\left(1 + \frac{1}{\nu}; \frac{3}{4}(2c_B - c)z_h^2\right)} \frac{\Gamma\left(1 + \frac{1}{\nu}\right) - \Gamma\left(1 + \frac{1}{\nu}; (2c_B - c)z_h^2\right)}{\Gamma\left(1 + \frac{1}{\nu}\right) - \Gamma\left(1 + \frac{1}{\nu}; (2c_B - c)z^2\right)} \right] \end{aligned}$$

$$\phi = \int_{z_0}^z \frac{1}{\nu \xi} \sqrt{4\nu - 4 + (4\nu c_B + 3(3c - 2c_B)\nu^2) \xi^2 + \left(\frac{3}{2} \nu^2 c^2 - 2c_B^2\right) \xi^4} d\xi, \quad z_0 \neq 0$$

Anistropic hybrid solution

$$A_t = \mu \frac{e^{(c - \frac{c_B}{2})z^2} - e^{(c - \frac{c_B}{2})z_h^2}}{1 - e^{(c - \frac{c_B}{2})z_h^2}}, \quad \rho = -\frac{\mu(c - c_B/2)}{1 - e^{(c - c_B/2)z_h^2}}$$

$$f_B = f_1, \quad g = 1 - \frac{I_3(z)}{I_3(z_h)} + \frac{\mu^2 (2c - c_B)}{\left(1 - e^{(c - \frac{c_B}{2})z_h^2}\right)^2} \left(I_7(z) - I_7(z_h) \frac{I_3(z)}{I_3(z_h)} \right) - \\ - \frac{q_B^2}{2} \left(c - \frac{c_B}{2} \right)^{1 - \frac{1}{\nu}} \left(I_G(z) - I_G(z_h) \frac{I_3(z)}{I_3(z_h)} \right)$$

$$I_3(z) = \int_0^z e^{\frac{(3c - 2c_B)\xi^2}{4}} (1 + b\xi^2)^{3a} \xi^{1 + \frac{2}{\nu}} d\xi, \quad I_7(z) = \int_0^z e^{\frac{(7c - 4c_B)\xi^2}{4}} (1 + b\xi^2)^{3a} \xi^{1 + \frac{2}{\nu}} d\xi$$

$$I_G(z) = \int_0^z \Gamma \left(-1 + \frac{1}{\nu}; (2c - c_B) \frac{\xi^2}{2} \right) e^{\frac{(3c - 2c_B)\xi^2}{4}} (1 + b\xi^2)^{3a} \xi^{1 + \frac{2}{\nu}} d\xi$$

$$f_2 = z^{1-4/\nu} \left(1 - \frac{1}{\nu} \right) \frac{e^{-(c-2c_B)\frac{z^2}{2}} g}{q^2(1+bz^2)^{2a}} \left[\frac{12abz}{1+bz^2} + \frac{4}{z} \left(1 + \frac{1}{\nu} \right) + (3c - 2c_B)z - \frac{2g'}{g} \right]$$

$$\phi = \int_{z_0}^z \sqrt{\left(\frac{3c^2}{2} - 2c_B^2 \right) \xi^2 + 3(3c - 2c_B) + \frac{4c_B}{\nu} + \frac{4(\nu - 1)}{\nu^2 \xi^2} + \frac{12ab(1 + c\xi^2)}{1 + b\xi^2} + \frac{24(1 + ab\xi^2)}{(1 + b\xi^2)^2}} d\xi$$

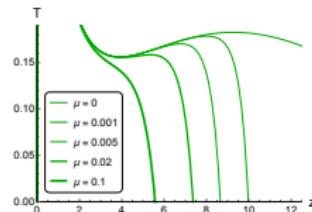
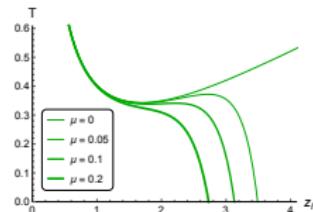
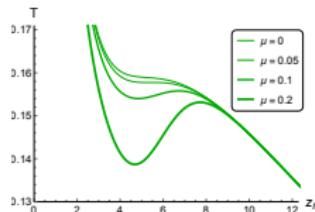
Anisotropic holographic models Refs

Holographic approach to the study of QGP is actively developed

- Isotropic models for light and heavy quarks
 - O. Andreev, V.I. Zakharov, PRD'06
 - Y. Yang, P.-H. Yuan, JHEP'15
 - M.-W. Li, Y. Yang, P.-H. Yuan, PRD'17
- Anisotropic models for light and heavy quarks
 - I.A., K. Rannu, JHEP'18
 - I.A., K. Rannu, P. Slepov, PLB'19
 - I.A., K. Rannu, P. Slepov, JHEP'21
- Magnetic catalysis/inverse catalysis
 - S. He, Y. Yang, P.H. Yuan 2004.01965
 - A. Ballon-Bayona, J.P. Shock, D. Zoakos, JHEP'20
 - H. Bohra, D. Dudal, A. Hajilou, S. Mahapatra, PRD'21
 - U. Gürsoy, M. Järvinen, G. Nijs, J.F. Pedraza, JHEP'21
 - I.A., K. Rannu, P. Slepov, 2011.07023
- Wilson loops in anisotropic model
 - D. Ageev, I.A., A. Golubtsova, E. Gourgoulhon, NPB'17
- Early papers + “Top-down” R.A. Janik, P. Witaszczyk'08; A. Rebhan, D. Steineder'08; D. Giataganas'12; D. Mateos, D. Trancanelli'11; ...

Non-monotonic dependence of T on the horizon

Isotropic models

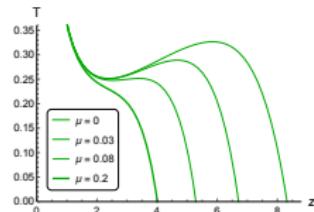
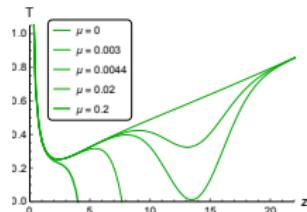


Model for:

light quarks

heavy quarks

heavy quarks + magnetic field



light + heavy quarks + magnetic field

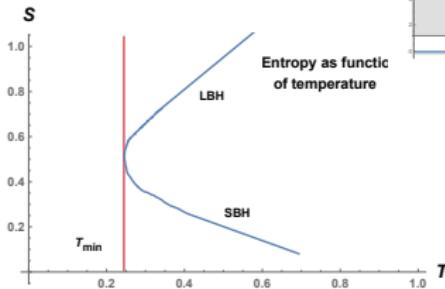
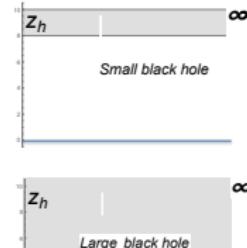
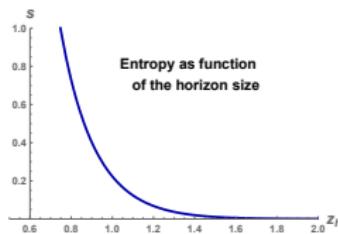
$$c_B = -0.1, q_B = 0.0001$$

$$c_B = -0.1, q_B = 0.02$$

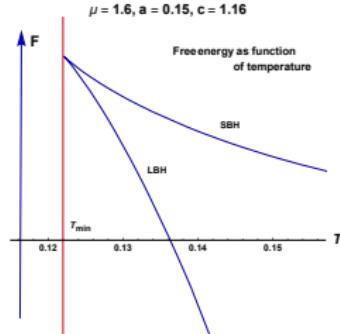
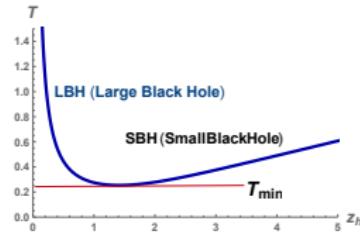
- We see: horizon dependence on temperature can be a multivalued function

Hawking-Page Phase Transitions

- Due to **non-monotonic** dependence of $T = T(z_h)$ the entropy $s = s(T)$ is **not monotonic**
- As a consequence the free energy $F = \int s dT$ undergoes the phase transition



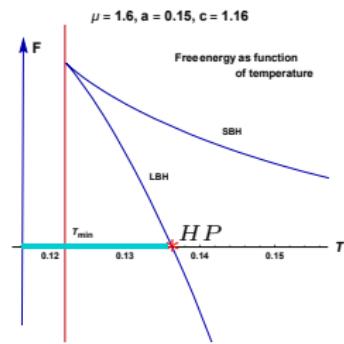
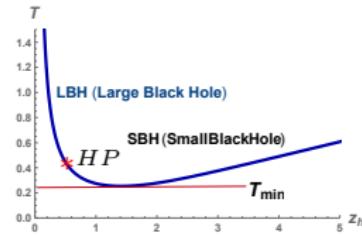
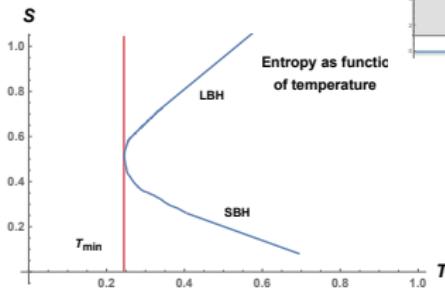
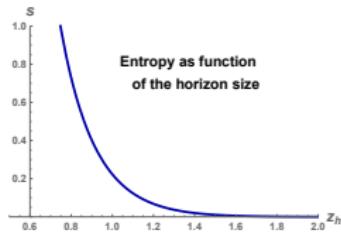
In all plots $\mu = 0$



Hawking-Page Phase Transitions

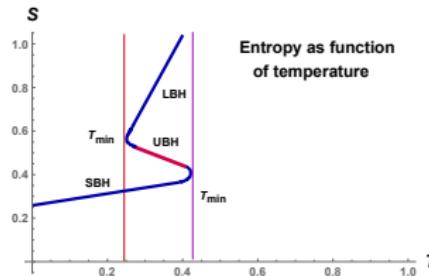
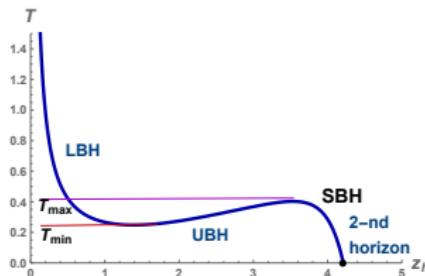
- Competition between BB and thermal AdS $\Rightarrow T_{HP} > T_{min}$

In all plots $\mu = 0$

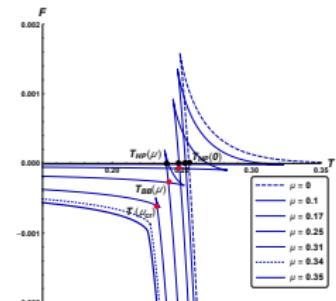


BB-phase Transitions

BB-PT describes transition from small black holes \rightarrow large black holes



Entropy as function
of temperature

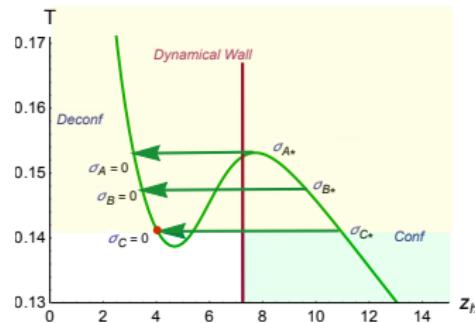
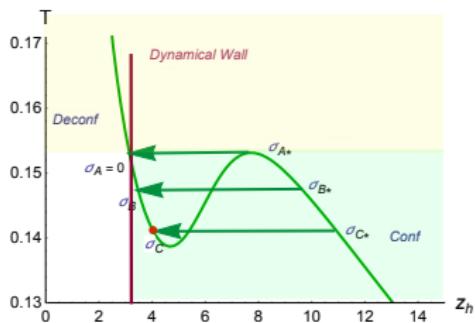


The swallow-tailed shape

- Physical quantities that probe backgrounds are smooth relative to z_h
 \Rightarrow their dependence on T should be taken from stable region
- BB-PT immediately provides the 1-st PT for corresponding characteristic of QCD

BB-phase Transitions

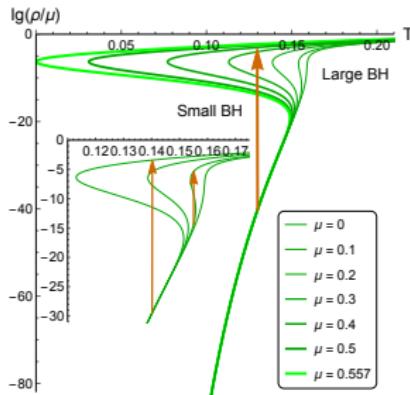
- Physical quantities that probe backgrounds are smooth relative to z_h
⇒ their dependence on T **should be taken from stable region**
- BB-PT immediately provides the 1-st PT for corresponding characteristic of QCD



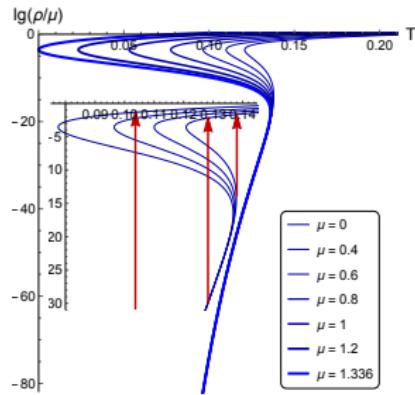
The arrows show transitions from the unstable phases to the stable ones

Change of Density across the 1-st Order Transition

$$A_t(z) = \mu - \rho z^2 + \dots$$



$\nu = 1$



$\nu = 4.5$

Density $\rho/\mu(T)$ in logarithmic scale for different μ

$$a = 4.046, b = 0.01613, c = 0.227$$

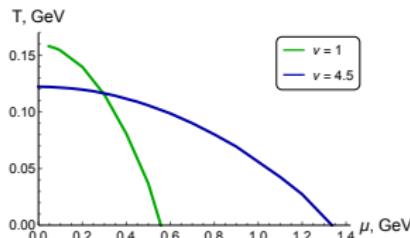
Inner plots show the fragments of main plots zoomed

- At a given T the value of the ratio ρ/μ increases with the transition from unstable to stable state. Quarkonic phase transition?

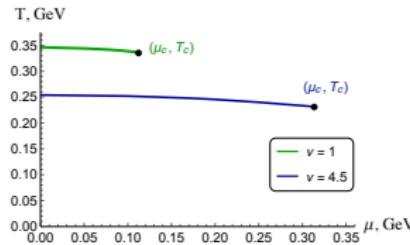
Anizotropic Phase Transitions

- One more phenomenological parameter — anisotropy parameter ν

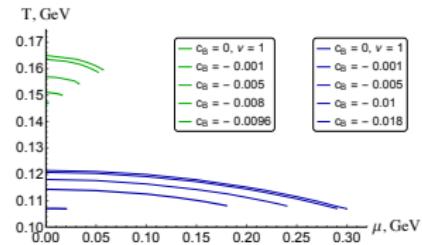
Location of isotropic and anisotropic 1-st order PTs
Blue lines for $\nu = 4.5$, the green lines for isotropic case



light quarks model



heavy quarks models
 $B = 0$

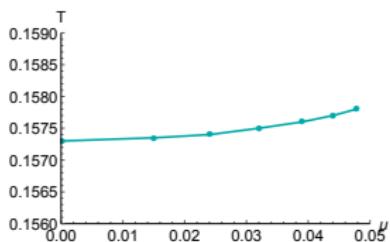


$B \neq 0$

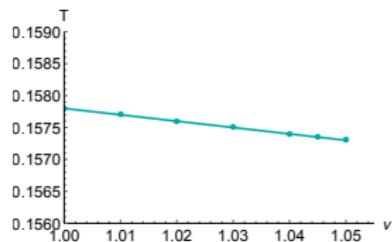
We see

- for the light quarks model the end of 1-st PT line (and CEP) moves to $\mu = 0$ with increasing ν
- the 1-st PT line becomes longer with increasing ν for heavy quarks model
- as full anisotropy increasing, i.e. increasing c_B (the magnetic field stronger), the whole transition line is shortened

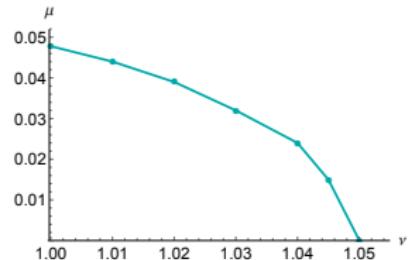
Position of CEP (Critical End Point)



A



B



C

- Position of CEP on $T - \mu$ (A), $T - \nu$ (B) and $\mu - \nu$ (C) planes

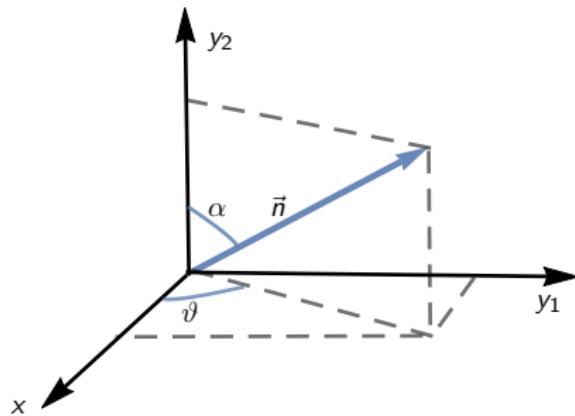
Phase Transitions for Various Probes

- Wilson loops through background PT
 - Temporal Wilson loop
 - string tension
 - confinement/deconfinrment PT
 - Spatial Wilson loop
 - Drag forces and energy lost
 - Light-light Wilson loop
- Extra local fields
 - Bosons (chiral symmetry breaking)
 - Extra Maxwell field (electric conductivity)
 - Maxwell field (Regge spectrum)

Temporal Wilson loops

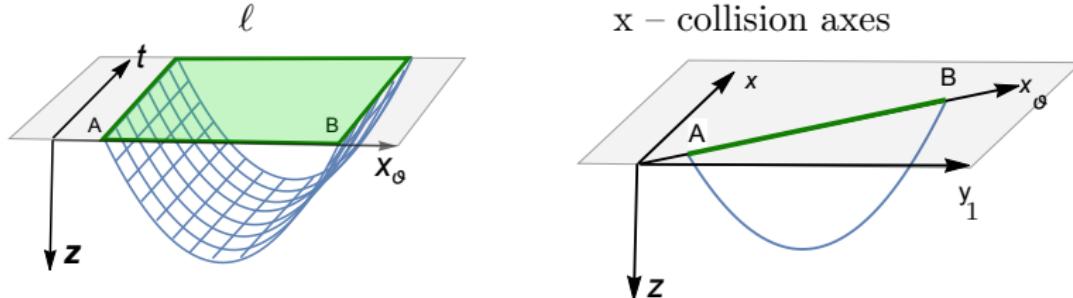
$$S = \frac{1}{2\pi\alpha'} \int d\xi^0 d\xi^1 \sqrt{-\det h_{\alpha\beta}}, \quad h_{\alpha\beta} = G_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu$$

$$\begin{aligned} X^0 &\equiv t = \xi^0, & X^1 &\equiv x = \xi^1 \cos \vartheta \sin \alpha, & X^2 &\equiv y_1 = \xi^1 \sin \vartheta \sin \alpha \\ X^3 &\equiv y_2 = \xi^1 \cos \alpha, & && X^4 &\equiv z = z(\xi^1) \end{aligned}$$



Temporal Wilson loop

$$W[C_\vartheta] = e^{-S_{\vartheta,t}} \quad \vec{n}: \quad n_x = \cos \vartheta, \quad n_{y_1} = \sin \vartheta, \quad n_{y_2} = 0$$



Two special cases:

- $\vartheta = 0$ WL (longitudinal)
- $\vartheta = \pi/2$ WT (transversal)

$$\ell \rightarrow \infty \quad S \sim \sigma_{DW} \ell$$

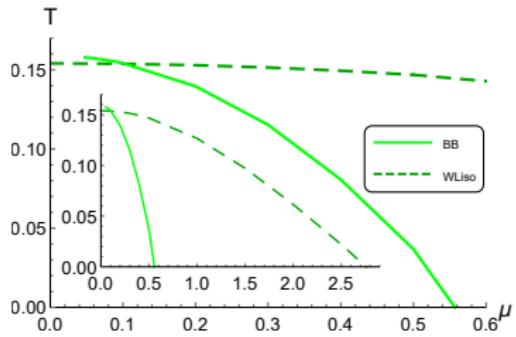


the string tension

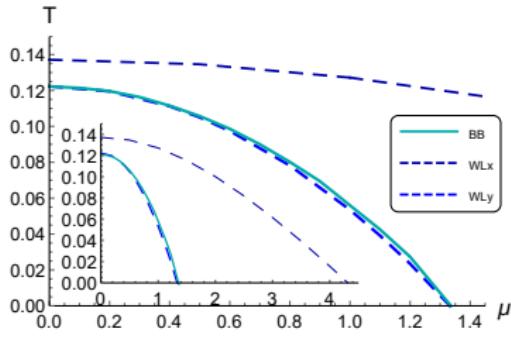
$$\sigma_{DW} = \frac{b(z) e^{\sqrt{\frac{2}{3}} \phi(z, z_0)}}{z^2} \sqrt{g(z) \left(z^{2-\frac{2}{\nu}} \sin^2(\vartheta) + \cos^2(\vartheta) \right)} \Big|_{z=z_{DW}}, \quad \left. \frac{\partial \sigma}{\partial z} \right|_{z=z_{DW}} = 0$$

I.A., K. Rannu, P. Slepov, PLB, 19

Phase diagram for light quarks ($\nu = 1, 4.5$)



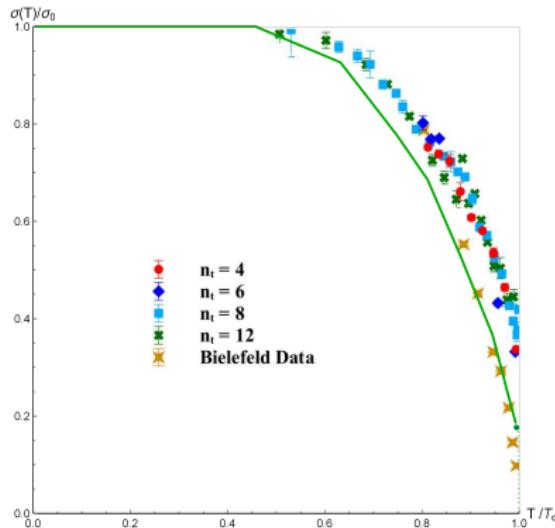
$\nu = 1$



$\nu = 4.5$

Our String tension vs Lattice

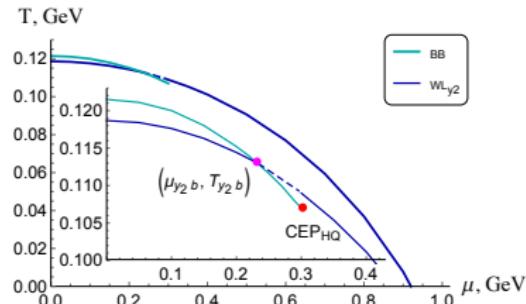
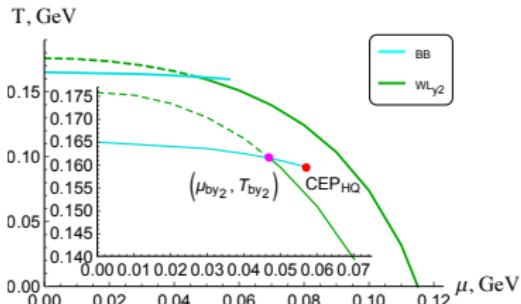
- We take $z_0 = 10 \exp(-z_h/4) + 0.1$ (#)



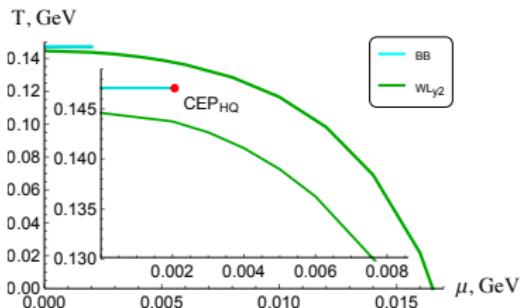
- The green curve shows the string tension as function of temperature for $z_0 = (\#)$, $\mu = 0$, $\nu = 1$, $a = 4.046$, $b = 0.01613$, $c = 0.227$
- The dots with different decorations Lattice *N.Cardoso, P.Bicudo, PRD'12*
- The thin green dotted line shows the WL phase transition

Phase transitions for heavy quarks

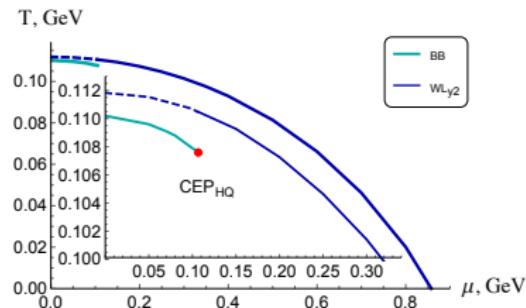
$$c_B = 0$$



$$c_B = -0.0096$$



$$c_B = -0.015$$



- We see the **inverse** magnetic catalysis

Spatial Wilson loops. Parametrization.

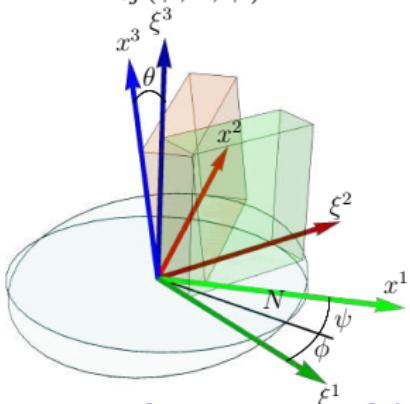
To describe the nesting of the 2-dimensional world sheet in 5-dimensional space time we use

$$X^0(\xi) = \text{const},$$

$$X^i(\xi) = \sum_{\alpha=1,2} a_{i\alpha}(\phi, \theta, \psi) \xi^\alpha, \quad i = 1, 2, 3, \quad \alpha = 1, 2,$$

$$X^4(\xi) = z(\xi^1),$$

where x^i are spatial coordinates and $a_{ij}(\phi, \theta, \psi)$ are entries of the rotation matrix



Brick's orientation is equivalent to rectangle's orientation

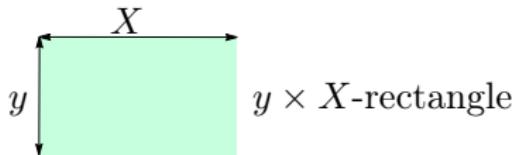
The Nambu-Goto action for SWL

$$\mathcal{S}_{SWL} = L \int d\xi \frac{\mathfrak{b}_s}{z^2} \sqrt{\mathfrak{g}_1 \mathfrak{g}_2 a_{33}^2 + \mathfrak{g}_1 \mathfrak{g}_3 a_{23}^2 + \mathfrak{g}_2 \mathfrak{g}_3 a_{13}^2 + \mathfrak{g}_3^2 a_{32}^2 (a_{31}^2 - a_{13}^2) + \frac{z'^2}{g} \bar{g}_{22}}$$

$$\mathcal{V}(z(\xi)) = \frac{\mathfrak{b}_s}{z^2} \sqrt{\mathfrak{g}_1 \mathfrak{g}_2 a_{33}^2 + \mathfrak{g}_1 \mathfrak{g}_3 a_{23}^2 + \mathfrak{g}_2 \mathfrak{g}_3 a_{13}^2 + \mathfrak{g}_3^2 a_{32}^2 (a_{31}^2 - a_{13}^2)}$$

- $g, \mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3$ are functions of z
- $\bar{g}_{22}, \bar{g}_{33}, \bar{g}_{23}$ are functions of z and the Euler angles

DW equations for SWL



- The equations for the DW for SWL in particular cases of orientation:

$$\mathcal{DW}_{xY_1} = \mathcal{DW}_{Xy_1} \equiv \left. \frac{2b'_s(z)}{b_s(z)} + \frac{\mathfrak{g}'_1(z)}{\mathfrak{g}_1(z)} + \frac{\mathfrak{g}'_2(z)}{\mathfrak{g}_2(z)} - \frac{4}{z} \right|_{z=z_{DW}} = 0$$

$$\mathcal{DW}_{y_2X} = \mathcal{DW}_{xY_2} \equiv \left. \frac{2b'_s(z)}{b_s(z)} + \frac{\mathfrak{g}'_1(z)}{\mathfrak{g}_1(z)} + \frac{\mathfrak{g}'_3(z)}{\mathfrak{g}_3(z)} - \frac{4}{z} \right|_{z=z_{DW}} = 0$$

$$\mathcal{DW}_{y_2Y_1} = \mathcal{DW}_{y_1Y_2} \equiv \left. \frac{2b'_s(z)}{b_s(z)} + \frac{\mathfrak{g}'_2(z)}{\mathfrak{g}_2(z)} + \frac{\mathfrak{g}'_3(z)}{\mathfrak{g}_3(z)} - \frac{4}{z} \right|_{z=z_{DW}} = 0$$

Spatial String Tensions

- String tension for different orientations

$$\begin{aligned}\sigma_{Xy_1} &= \frac{\mathfrak{b}_s}{z^2} \sqrt{\mathfrak{g}_1 \mathfrak{g}_2} \Big|_{z=z_*} & \sigma_{xY_2} &= \frac{\mathfrak{b}_s}{z^2} \sqrt{\mathfrak{g}_1 \mathfrak{g}_3} \Big|_{z=z_*} & \sigma_{y_1 Y_2} &= \frac{\mathfrak{b}_s}{z^2} \sqrt{\mathfrak{g}_2 \mathfrak{g}_3} \Big|_{z=z_*} \\ \sigma_{xY_1} &= \sigma_{Xy_1} = \sigma_1 & \sigma_{xY_2} &= \sigma_{Y_2 x} = \sigma_2 & \sigma_{y_1 Y_2} &= \sigma_{Y_1 Y_2} = \sigma_3\end{aligned}$$

$z_* = z_h$ or $= z_{DW}$ (if the DW exists)

- Or in term of vierbeins

$$\begin{aligned}\mathbf{e}^a &= e_\mu^a(z) dx^\mu, & e_\mu^a(z) &= \delta_\mu^a \sqrt{\frac{\mathfrak{b}_s}{z^2}} \mathfrak{g}_\mu, & a &= 0, \dots, 3, & \mu &= 0, \dots, 3, & \mathfrak{g}_0 &= g \\ g_{\mu\nu} &= \eta_{ab} e_\mu^a e_\nu^b, & \eta_{ij} &\text{ is the Minkowski metric}\end{aligned}$$

for Wilson loop extending in (a, b) -directions

$$\tau^{ab} = \mathbf{e}(z)^a \wedge \mathbf{e}(z)^b \Big|_{z=z_*} = \tau_{\mu\nu}^{ab} dx^\mu \wedge dx^\nu \quad \tau_{\mu\nu}^{ab} = \delta_\mu^a \delta_\nu^b \frac{\mathfrak{b}_s}{z^2} \sqrt{\mathfrak{g}_\mu \mathfrak{g}_\nu} \Big|_{z=z_*}$$

Spatial String Tensions and Drag Forces

The answers can be compared with drag forces:

Sin, Zahed, 2007
Andreev, 2018

For the drag forces in anizotropic models

I. A., Phys.Part.Nucl. 2020

$$f_i = g_{ii} \Big|_{z=z_*} v^i, \quad i = 1, 2, 3$$

here v^i is a constant velocity

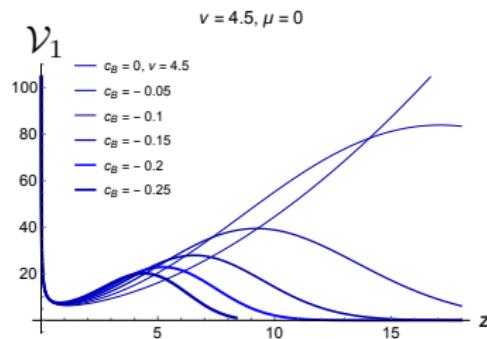
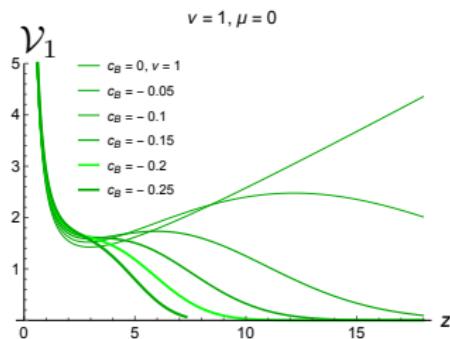
Next corrections

(cf. compare with isotropic case with Gursoy, Kiritsis,
Michalogiorgakis, Nitti, JHEP'09)

$$f_i = g_{ii} \Big|_{z=z_*} v^i + g'_{ii} \Big|_{z=z_*} v^{i\ 2} v^i + \dots$$

Spatial String Tension. Example.

- Effective potential $\mathcal{V}_1 = \frac{b_s}{z^2} \sqrt{g_1 g_2}$

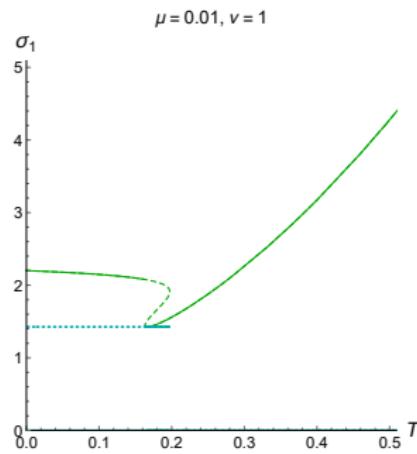
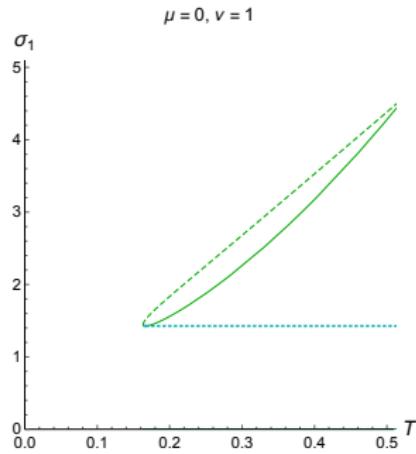


Spatial String Tension. Isotropic case and $c_B = 0$

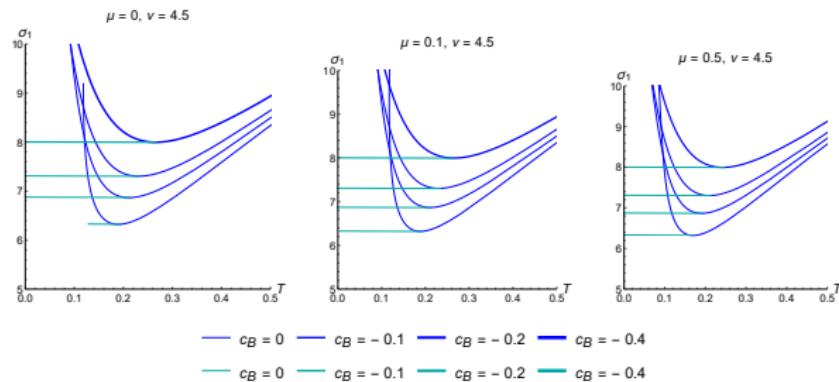
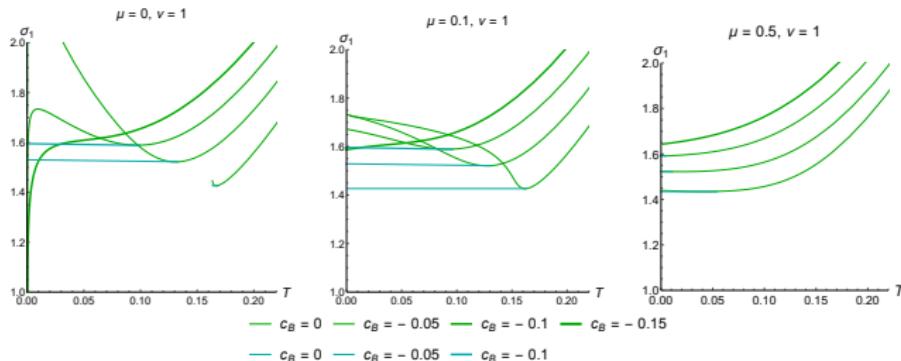
$$\sigma_1 \equiv \sigma_{Xy_1} = \frac{b_s}{z^2} \sqrt{g_1 g_2} \Big|_{z=z_*}$$

$z_* = z_{DW}$ or z_h

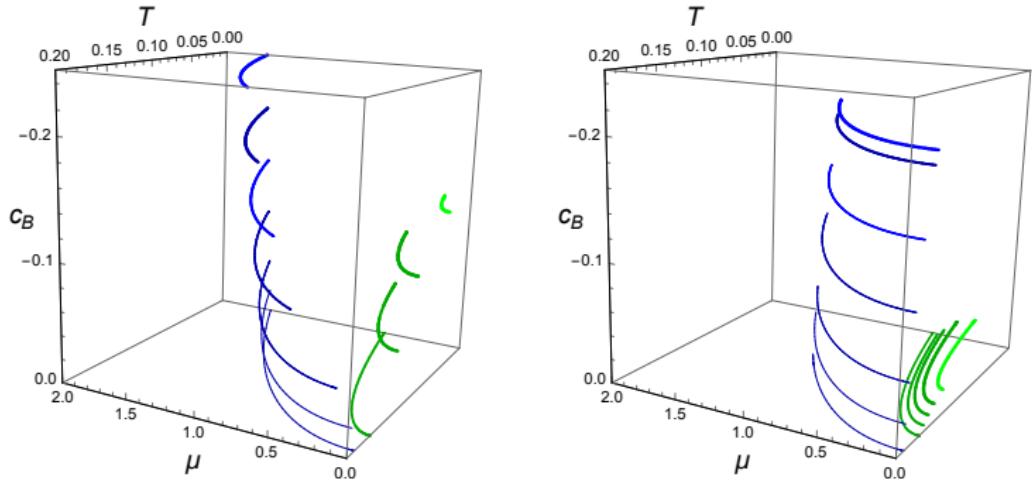
cyan (stable – solid, UNS – dashed) for DW
green (stable – solid, UNS – dashed) for z_h



Spatial String Tension σ_1 in isotropic and anisotropic cases ($\nu = 4.5$)



Drag Forces Phase Transitions



Phase transition surfaces are surfaces stretched on blue /green curves

- Under variation of (T , μ , magnetic field) the spatial string tension undergoes the phase transition I.A., K.Rannu, Slepov, 2012.05758
- The light green and blue lines represent to the end of the phase transitions
- Drag forces and energy losses undergo the phase transition as well

Direct photons emission rate and electric conductivity

- The number of photons emitted per unit time per unit volume Γ (photon emission rate) in thermal equilibrium is given by the light-like correlator

Caron-Huot, Kovtun, Moore, Starinets, Yaffe, JHEP' 06

$$d\Gamma = -\frac{d^3 k}{(2\pi)^3} \frac{e^2 n_b(|\mathbf{k}|)}{|\mathbf{k}|} \text{Im} [\text{tr} (\eta_{\mu\nu} G_R^{\mu\nu})] \Big|_{k^0=|\mathbf{k}|}$$

- $n_b(|\mathbf{k}|) = \frac{A}{e^{-k/T} - 1}$ is Bose-Einstein thermal distribution function
- Retarded Green's function is related to the electric conductivity through the Kubo relation

$$\sigma^{\mu\nu} = -\frac{G_R^{\mu\nu}}{iw}$$

Electric Conductivity

- To find the electric conductivity
 - perturb the background by adding Maxwell field with a dilaton kinetic term

$$S = S_{backgr} + S_{pert} \quad S_{pert} \sim \int d^5x \sqrt{-g} f_0(\phi) F^2$$

- Ansatz $A_M(t, x_3, z) = \psi_M(z) \exp(-i(wt - kx_3)) \quad M = 0, \dots, 4$

Iatrakis, Kiritis, Shen, Yang JHEP'16

- Conductivity

I.A., Ermakov, Slepov, 2104.14582

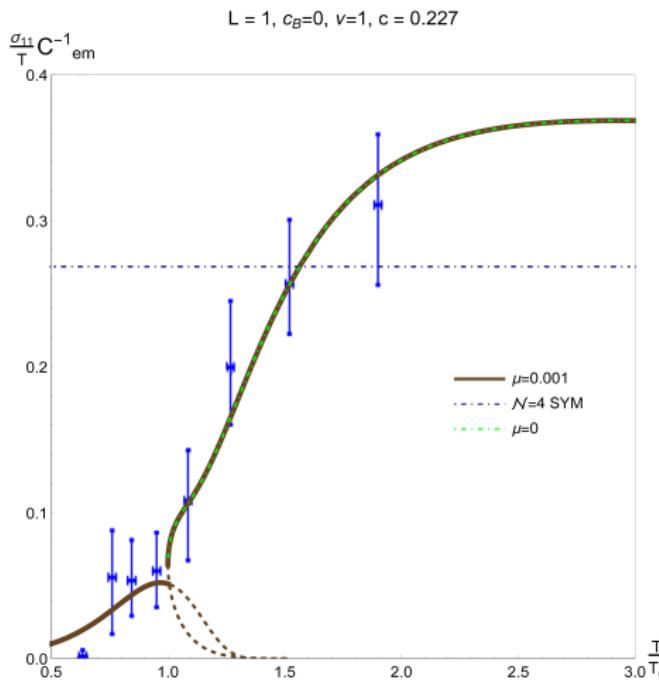
$$\sigma^{11} = \frac{f_0}{z} \sqrt{\frac{\mathfrak{b}_s \mathfrak{g}_3 \mathfrak{g}_2}{\mathfrak{g}_1}} \Big|_{z=z_h} \quad \sigma^{22} = \frac{f_0}{z} \sqrt{\frac{\mathfrak{b}_s \mathfrak{g}_3 \mathfrak{g}_1}{\mathfrak{g}_2}} \Big|_{z=z_h} \quad \sigma^{33} = \frac{f_0}{z} \sqrt{\frac{\mathfrak{b}_s \mathfrak{g}_1 \mathfrak{g}_2}{\mathfrak{g}_3}}$$

- Simple formula

$$\frac{\sigma^{ii}}{s} = f_0 g^{ii} \Big|_{z=z_h}$$

Electric conductivity

- The dependence of ratio σ^{11}/T on T



The solid brown line – stable phase $\mu = 0.001$

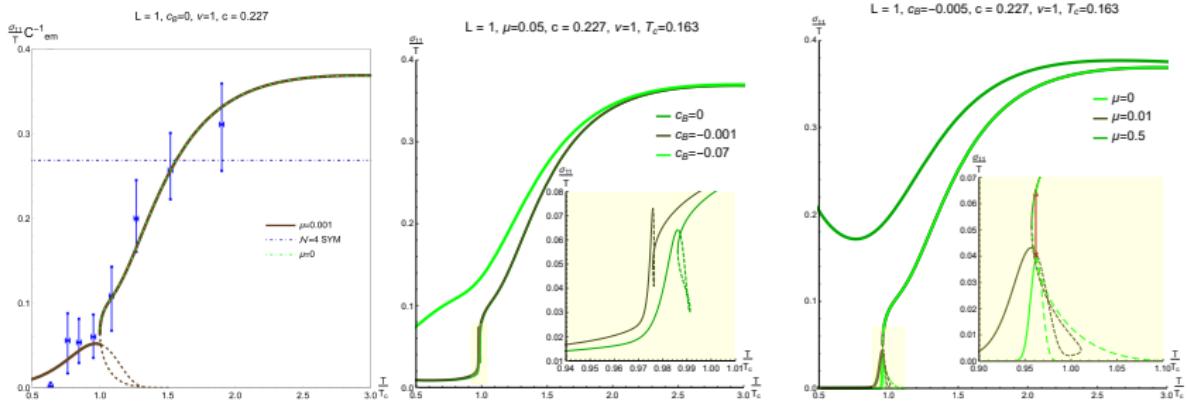
The green dash-dotted line – $\mu = 0$ case

The blue dash-dotted line – $N = 4 \text{ SYM}$

Blue dots Lattice **G. Aarts et al, JHEP'15**
for $N_c = N_f = 3$

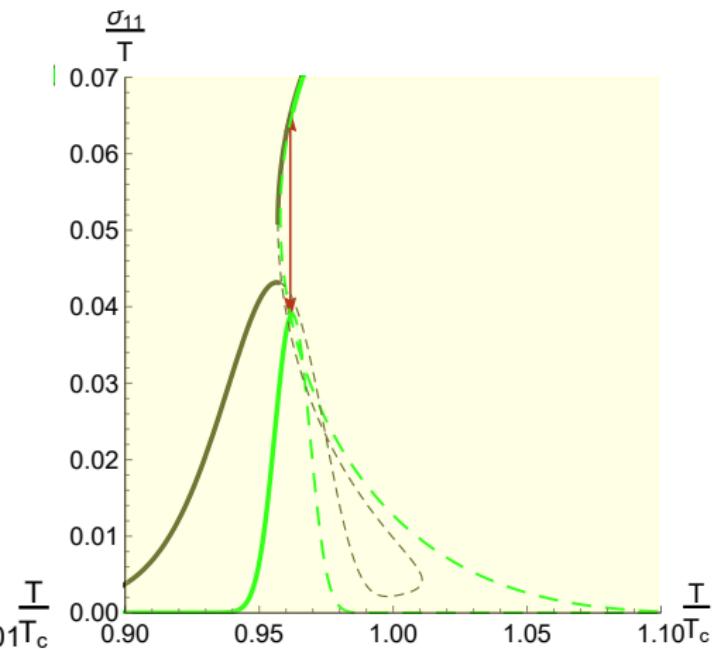
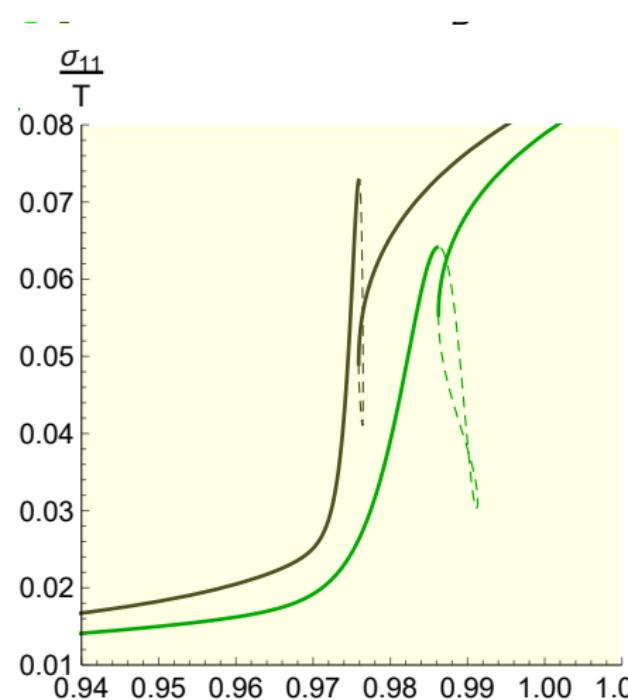
Electric conductivity

- The dependence of ratio σ^{11}/T on T for $\mu \neq 0$ and $c_B \neq 0$



Electric conductivity

- The dependence of ratio σ^{11}/T on T for $\mu \neq 0$ and $c_B \neq 0$



Conclusion

- The rich structure of phase transitions in (T, μ, B, ν) -space is found
- We have an interplay between two types of phase transitions
 - Crossover
 - 1-st order phase transition
- Our models have some parameters and one/two arbitrary functions
 - These functions have to be given from outside, say from lattice calculations at $\mu = 0, B = 0$ or other methods
 - The model provides the dependence of physical quantities on μ, B, ν
 - The analogy with RG (EOMs play the role of RG equations)
- What to do next
 - PT for “hybrid” model corresponding to realistic quark masses
 - Other characteristics (susceptibility, transport coefficients, eta/s, direct-photon spectra, jet quenching, thermalization time, etc)

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BACKUP. Born-Infeld type action

$$\mathcal{S} = \int_{-\ell/2}^{\ell/2} M(z(\xi)) \sqrt{\mathcal{F}(z(\xi)) + (z'(\xi))^2} d\xi, \quad V(z(\xi)) = M(z(\xi)) \sqrt{\mathcal{F}(z(\xi))} \quad (1)$$

We have two options to have $\ell \rightarrow \infty$

IA, EPJ Web Conf., 2018

- 1) The existence of a stationary point of $\mathcal{V}(z)$: $\mathcal{V}'\Big|_{z_{DW}} = 0$.

$$\ell \underset{z \rightarrow z_*}{\sim} \frac{1}{\sqrt{\mathcal{F}(z_{DW})}} \sqrt{\frac{\mathcal{V}(z_{DW})}{\mathcal{V}''(z_{DW})}} \log(z - z_*), \quad (2)$$

$$\mathcal{S} \underset{z \rightarrow z_*}{\sim} M(z_{DW}) \sqrt{\frac{\mathcal{V}(z_{DW})}{\mathcal{V}''(z_{DW})}} \log(z - z_*). \quad (3)$$

$$\mathcal{S} \sim M(z_{DW}) \cdot \sqrt{\mathcal{F}(z_{DW})} \cdot \ell, \quad (4)$$

$$\sigma_{DW} = M(z_{DW}) \sqrt{\mathcal{F}(z_{DW})}. \quad (5)$$

BACKUP. Born-Infeld type action

2) There is no stationary point of $\mathcal{V}(z)$ in the region $0 < z < z_h$, and we suppose it to be near horizon

$$F(z) = \mathfrak{F}(z_h)(z_h - z) + \mathcal{O}((z_h - z)^2), \quad (6)$$

if $M(z) \underset{z \rightarrow z_h}{\sim} \infty$ as

$$M(z) \underset{z \sim z_h}{\sim} \frac{\mathcal{M}(z_h)}{\sqrt{z - z_h}}, \quad (7)$$

$$\ell \underset{z \rightarrow z_h}{\sim} \frac{1}{\sqrt{\mathfrak{F}(z_h)}} \frac{1}{\sqrt{-\frac{2\mathcal{V}'(z_h)}{\mathcal{V}(z_h)}}} \log(z - z_h), \quad (8)$$

$$\mathcal{S} \underset{z \rightarrow z_*}{\sim} \mathcal{M}(z_h) \frac{1}{\sqrt{-\frac{2\mathcal{V}'(z_h)}{\mathcal{V}(z_h)}}} \log(z - z_h) \quad (9)$$

$$\sigma_h = \mathcal{M}(z_h) \mathfrak{F}^{1/2}(z_h). \quad (10)$$

BACKUP. Relations between 5-dim backgrounds and 4-dim models

- Relations between parameters of the 5-dim background (black hole) and thermodynamical parameters are the following:
 - $T_{BH} = T_{QCD}$, where T_{BH} is the temperature of the 5-dim black hole;
 - $A_0(z) = \mu_B - \rho_B z^2 + \mathcal{O}(z)$, where $A_0(z)$ is the 0-component of the electromagnetic field $A_\mu(z)$, μ_B is the baryonic chemical potential, ρ_B is the density and z is the 5-dimentional coordinate;
 - $S_{BH} = s$, where S_{BH} is the entropy of the black hole, which as usual is defined by the square of the black hole horizon, s is the thermodynamical entropy;
 - $F_{BH} = -p$, where F_{BH} is the free energy of the black hole, p is the thermodynamical pressure.

BACKUP. RG vs HRG

Relations between HRG & QCD flows are the following:

- 5-dim background

$$ds_5^2 = G_{MN} dx^M dx^N = w(z)(ds_4^2 + dz^2)$$

- 5-th coordinate z takes a role of an energy scale,
- the dilaton field $\phi = \phi(z)$ defines the running coupling $\lambda = \exp \phi(z)$ in 4-dim
- holographic $\beta = \frac{d\lambda(z)}{d \log w(z)}$.
- The field $X(z) = \beta/(3\lambda)$ satisfies $\frac{dX}{d\phi} = \mathcal{X}(\phi, X)$ (follows from the Einstein EOM for the background).
- We know the QCD RG flow of the running coupling

$$\beta(E) = \frac{d\lambda(E)}{d \log E}$$

perturbatively in UV and expect its form in IR from lattice calculations. Note that the form of the renormgroup flow depends on the mass of quarks, number of flavours, temperature, chemical potential, etc.