String size black holes

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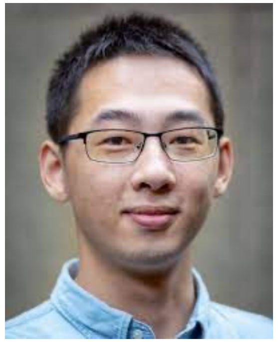
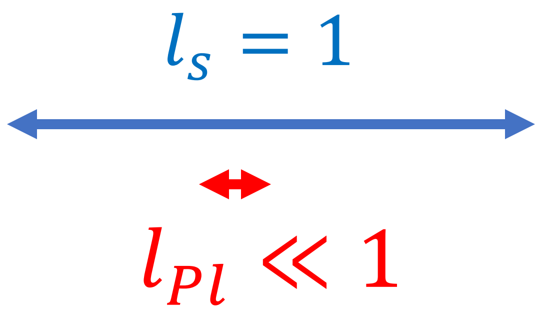
Outline

- Review
  - String size black holes.
  - Highly excited strings.
  - Strings in D=4 and the Horowitz-Polchinski solution.
  - A D=2 black hole: an exact string background based on SL(2)/U(1).
  - 2d black hole from large D black holes.

- Work in progress with Yiming Chen
  - Using the SL(2)/U(1) + large D approximation to describe stringy black hole.
  - Appearance of a large stringy thermal atmosphere as we tune to a critical mass/temperature.
  - Discussion on the chaos exponent.



- We will be considering a weakly coupled string theory, $g \ll 1$.
- Set ℓ_s =1= $\sqrt{\alpha'}$ and $G_N \sim g^2$
- We will discuss the bosonic string. Similar statements hold for the type II and heterotic superstrings.

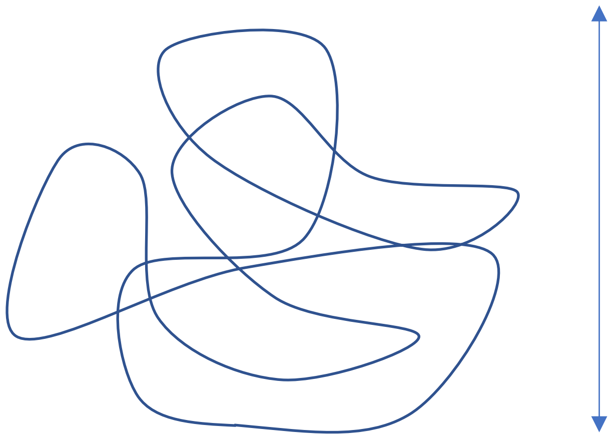


Highly excited free string

$M \sim 2\sqrt{N}$, $S = 4\pi\sqrt{\frac{c}{6}N}$

c= 24 for the bosonic string.

$$\beta = \frac{dS}{dM} = 4\,\pi \equiv \beta_H$$



Size $\sim N^{\frac{1}{4}}$

Hagedorn (inverse) temperature.

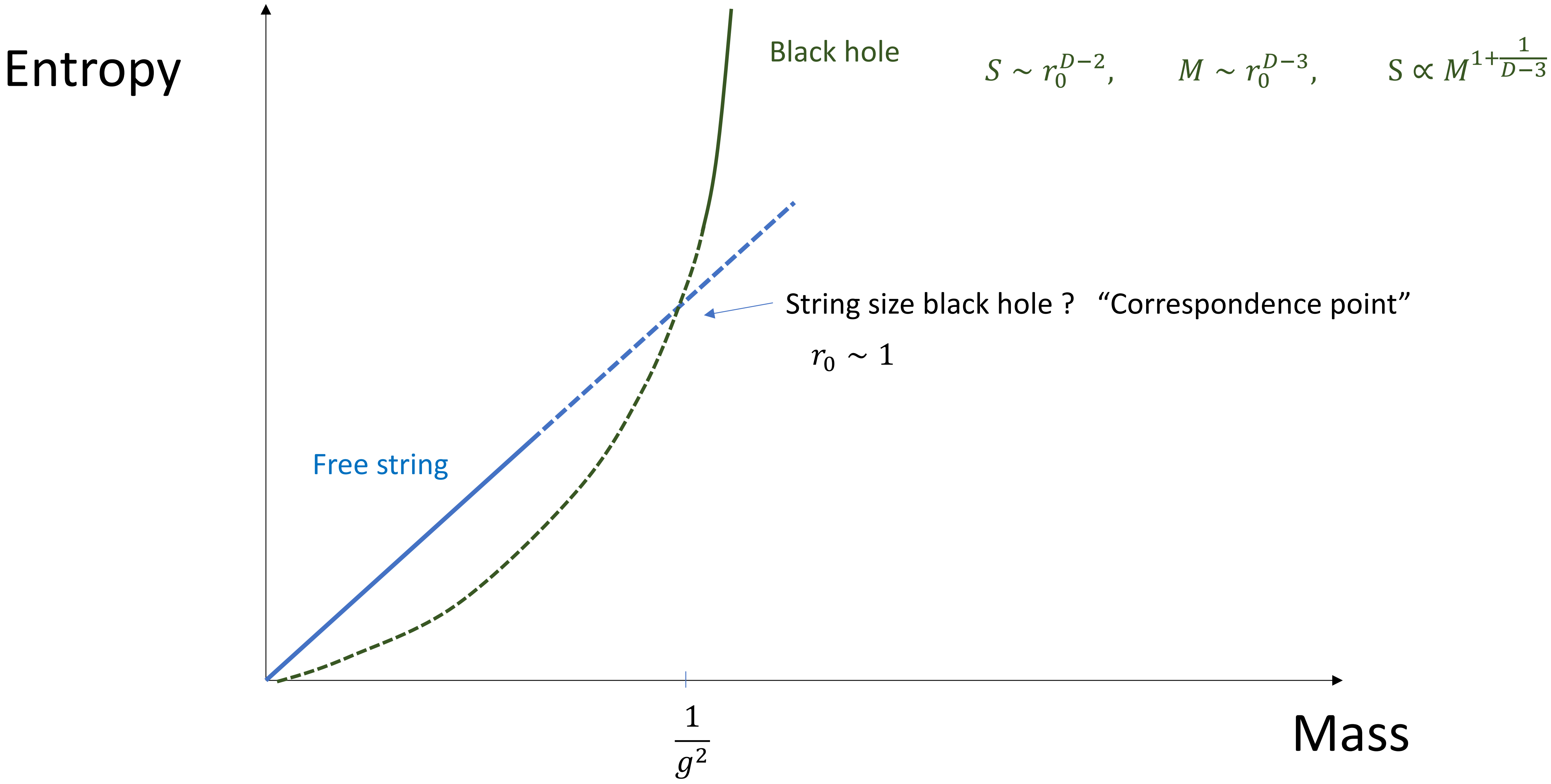
Thermal ensemble is well defined only for $\beta > \beta_H$

$$Z \sim \int d\ M\ (\ldots)e^{\,(\beta_H - \beta)M}$$

Black holes

- Well defined if $r_s \gg 1$
- Leading order α' corrections were computed.
- What happens as they approach the string size?.

Callan, Myers, Perry
Myers (type II).



Bowick, Smolin, Wijewardhana; Susskind; Horowitz, Polchinski

Corrections are important before we can reach the correspondence point.

Is there a smooth transition between black holes and highly excited strings ?

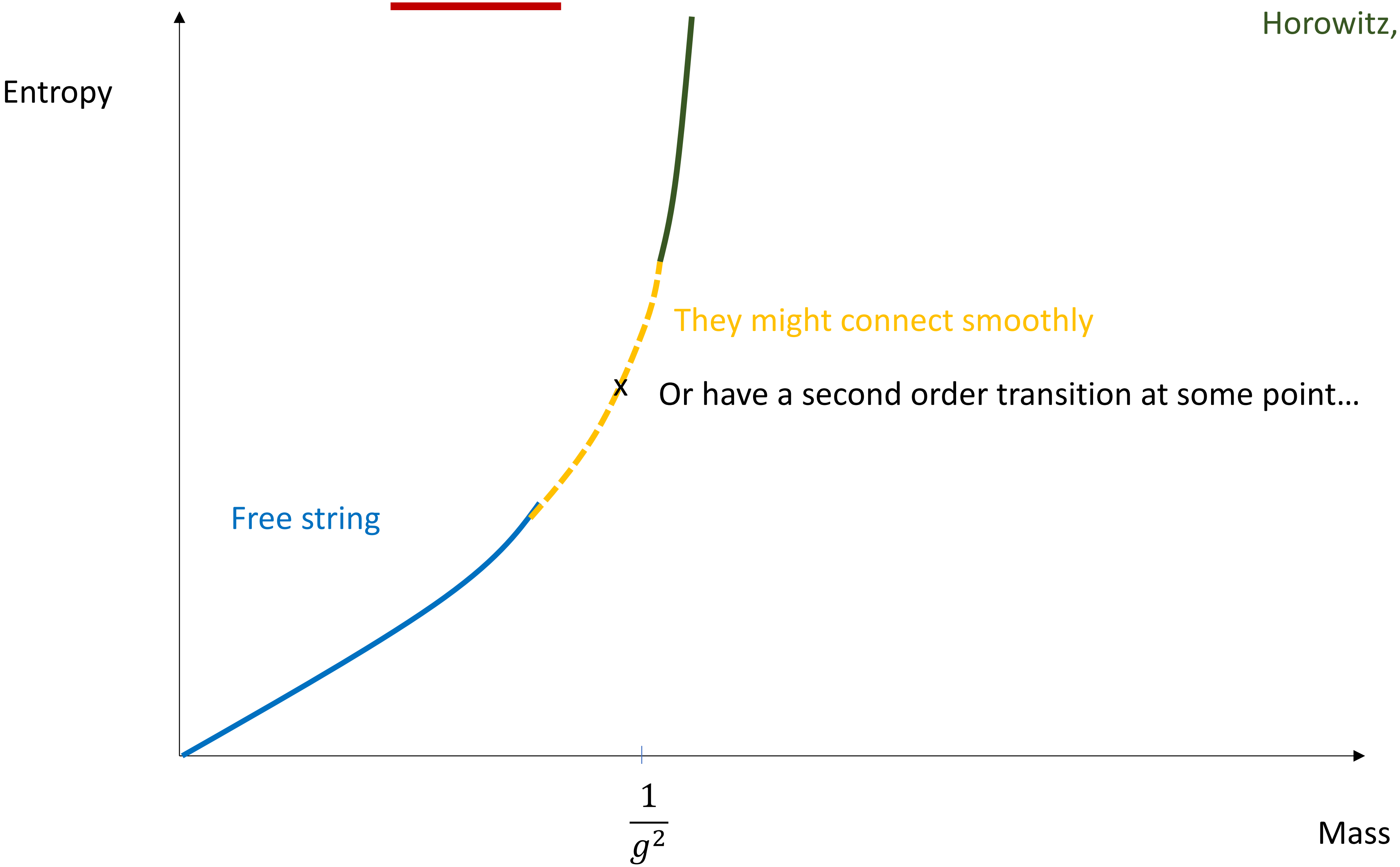
Motivation:

- String picture: Microstates are explicit, but no interior.
- Black holes: there is an interior, but not obvious microstates.

Review

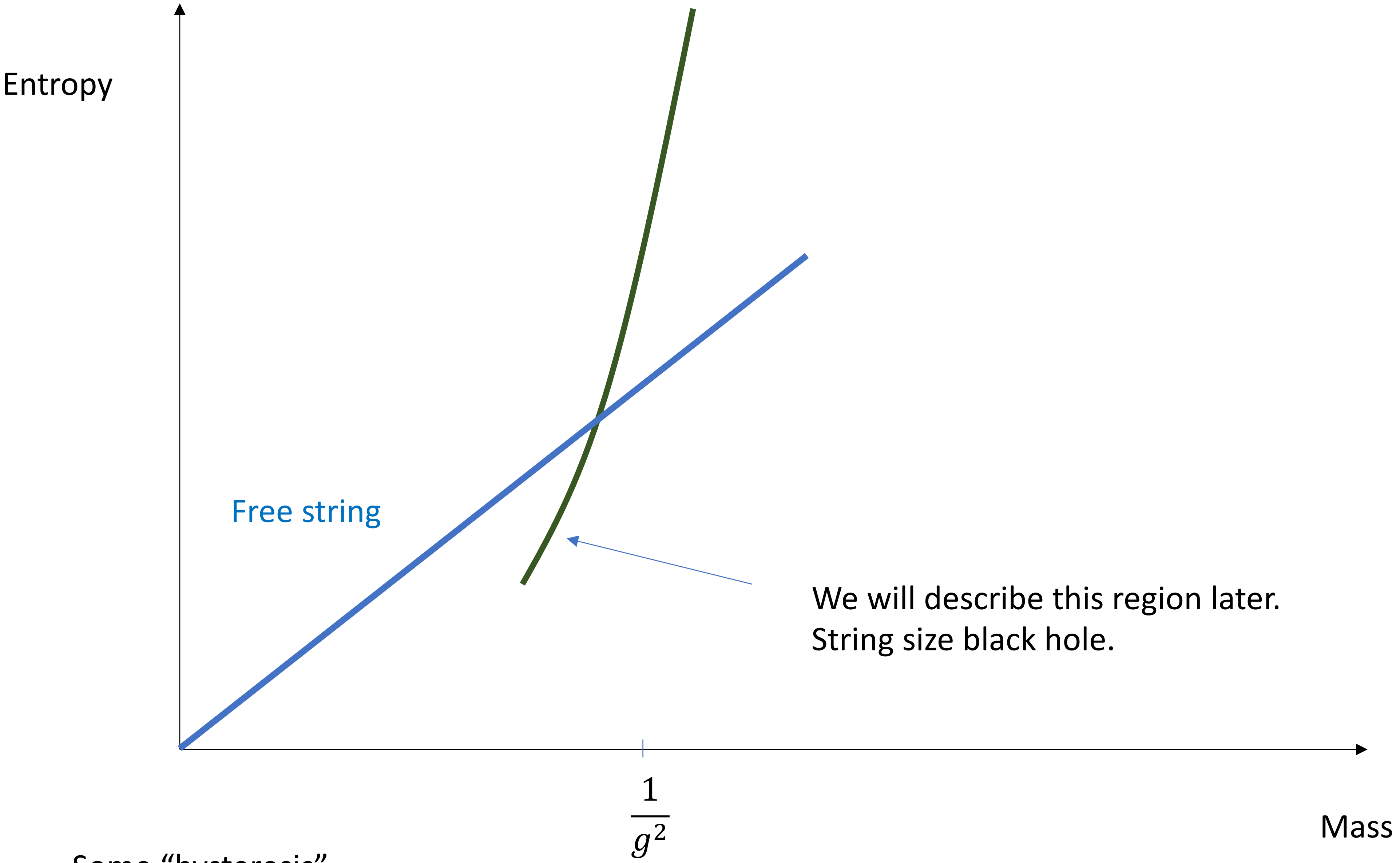
D=4

Horowitz, Polchinski



Horowitz, Polchinski

D>>4



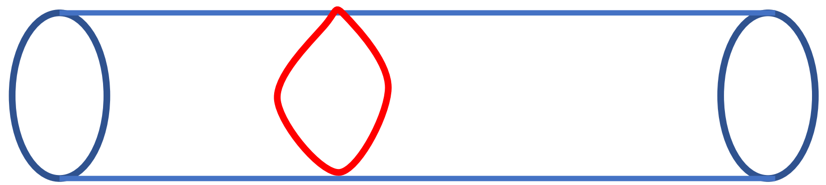
Some “hysteresis”

Some comments on strings at finite temperature

Sathiapalan; Kogan; Atick, Witten

Winding mode formalism

- Finite temperature → compactify the Euclidean time direction



Winding mode.

$$S = \frac{1}{g^2}\int |\nabla\chi|^2 + m^2(\beta)|\chi|^2,$$

$$\mathrm{m}^2(\beta) \propto (\beta^2 - \beta_H^2)$$

Becomes light as $\beta \to \beta_H$

Self interactions ? The most important one is gravity

$$S = \frac{1}{g^2}\int \sqrt{g}R + \frac{1}{g^2}\int |\nabla\chi|^2 + m^2(\beta)|\chi|^2,$$

For $\beta \sim \beta_H$ → winding mode is light and the field theory approximation is good.

Simple action for the thermodynamics of strings

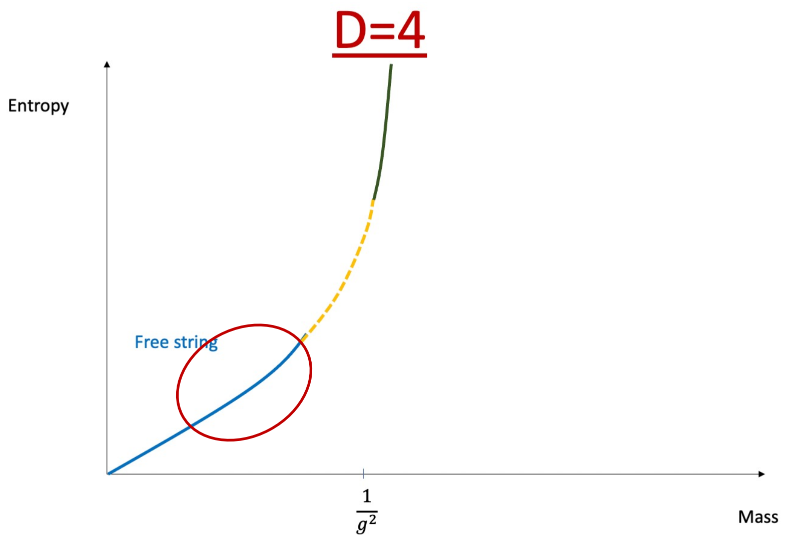
$$S = \frac{1}{g^2}\int \sqrt{g}R + \frac{1}{g^2}\int |\nabla\chi|^2 + m^2(\beta)|\chi|^2$$

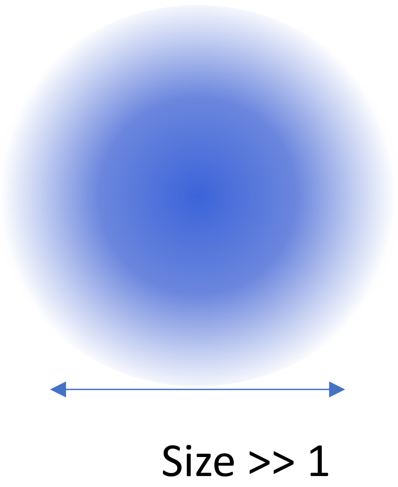
This leads to an interesting solution in D=4.

Self gravitating string

Horowitz-Polchinski

- Localized solution in 3 spatial dimensions. (D=4).
- Localized profile for the winding mode.
- Describes a self gravitating string in thermodynamic equilibrium.
- Size decreases as mass increases. Size $\sim \frac{1}{g^2 M}$. Should be larger than 1 to trust the gravity approximation. Breaks down before the correspondence point.
- Temperature decreases as mass increases. (negative specific heat)





Entropy of the self gravitating string

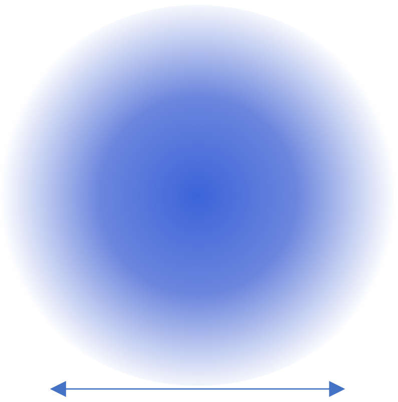
- We can compute the entropy from the classical action.
- Entropy of order $\frac{1}{g^2}$.

$$S = \left(1 - \beta\partial_\beta\right)(-I) = \frac{\beta}{g^2}\int d^{D-1}x\, [\beta\, \partial_\beta m^2(\beta)]\, |\chi|^2 = 2\, \frac{\beta}{g^2}\int d^{D-1}x\, \left(\frac{\beta^2}{4\pi^2}\right) |\chi|^2$$

Only a contribution from the explicit dependence on β.

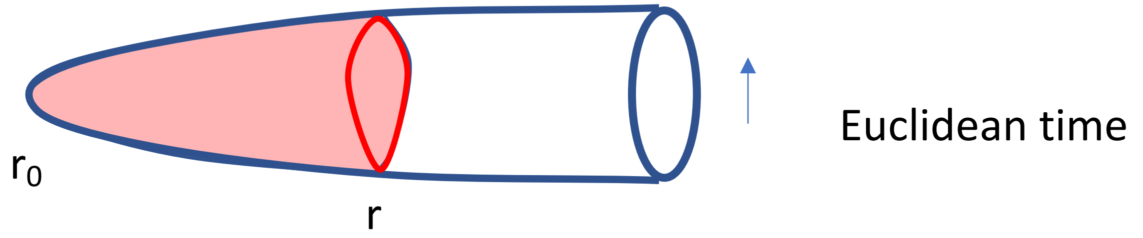
To leading order this gives $S = \beta_H M + \#\, g^4 M^3 + \cdots$

We computed correction using the solution.



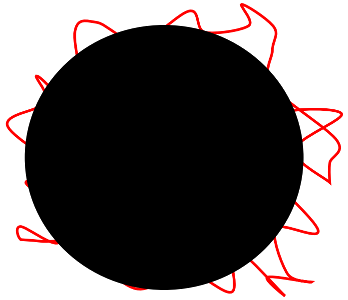
size

Black holes and winding condensates



Winding one function → computed by a worldsheet wrapping the cigar.

$$\langle \chi(r) \rangle \propto e^{-T\,A} \sim e^{-\beta(r-r_0)}, \qquad \beta \gg \beta_H$$

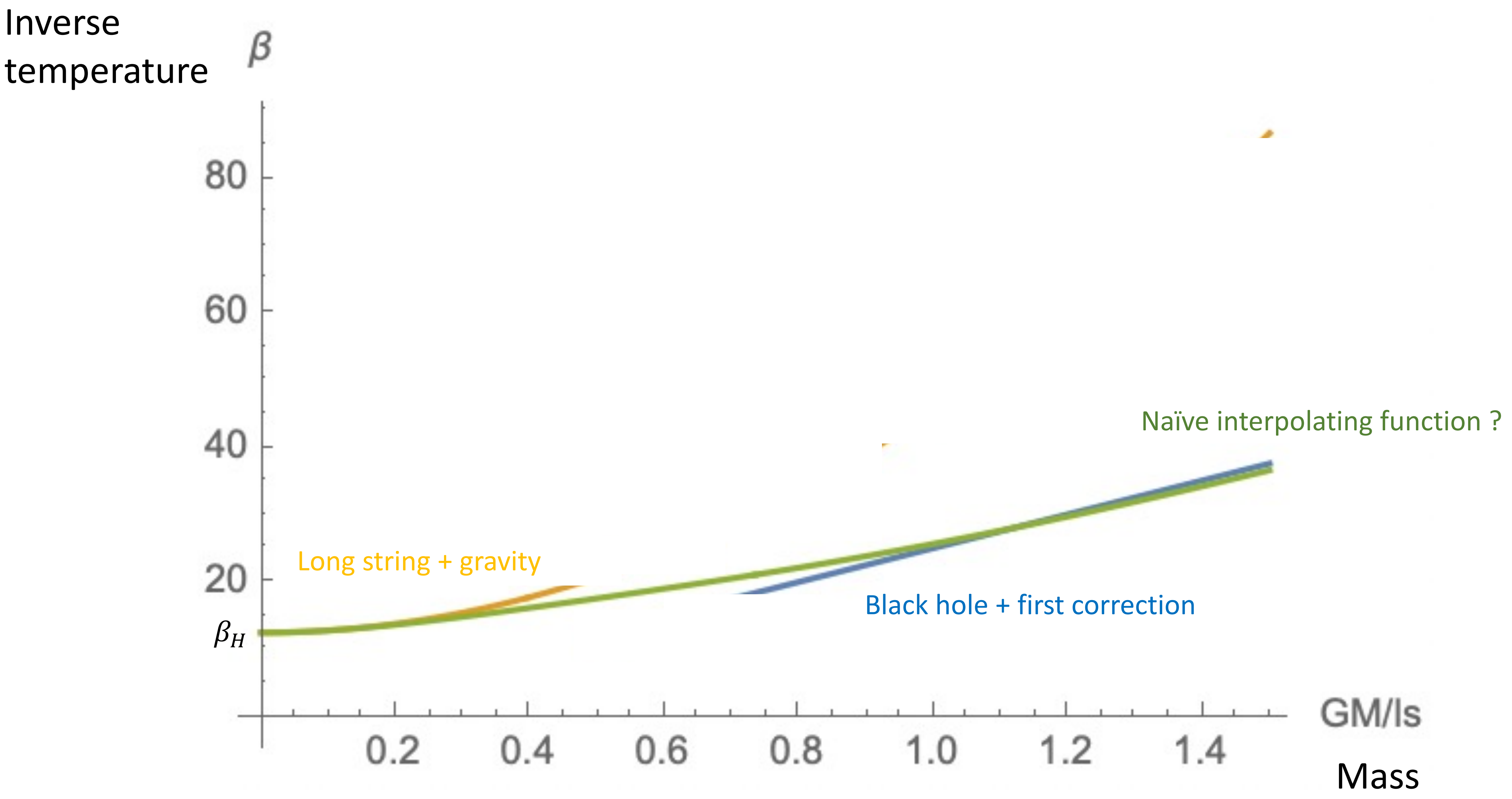


Is present for any black hole, but it is small.

We can view it as a thermal atmosphere of strings.

It is a classical contribution to the entropy, formally of order $\frac{1}{g^2}$, but not calculable (to my knowledge), since it is concentrated near the horizon.

Susskind, Uglum ; Dabholkar



Inverse temperature
β
80
60
40
20
β_H
Long string + gravity
Naïve interpolating function ?
Black hole + first correction
0.2
0.4
0.6
0.8
1.0
1.2
1.4
GM/ls
Mass

Conclusion

- In D=4 there might be an interpolating worldsheet conformal field theory that connects the black hole with the string at finite temperature.

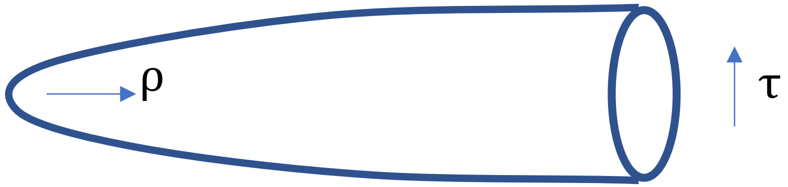
We now review a particular black hole solution, which is exactly solvable in string theory (as a worldsheet CFT).

The cigar, or SL(2)/U(1) black hole

- This is a two dimensional black hole.
- It arises as a gauged WZW model.
- It can be analyzed with the tools of Kac-Moody current algebras.

Witten
Mandal, Sengupta, Wadia
Dijkgraaf, Verlinde, Verlinde,
Becker, Becker,
…
Kazakov, Kostov, Kutasov,
Giveon, Kutasov,
….

Dilaton goes to weak coupling at infinity.



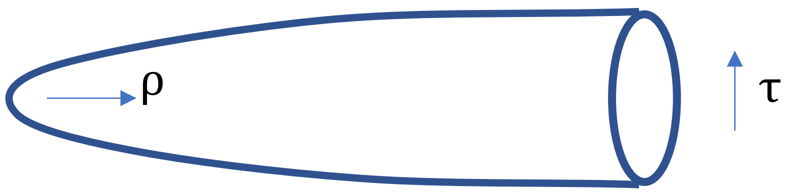
$$ds^2 = k\,(d\rho^2 + \tanh^2\!\rho\, d\tau^2)\,, \qquad e^{-2\phi} = e^{-2\phi_h}\cosh^2\!\rho$$

← String coupling is space dependent

k = parameter setting the radius. This metric description is good for k>>1.
The algebraic description is good for any k → stringy curvatures.

ϕ_h is a constant that sets the dilaton at the tip → gives the entropy (for k>>1).

The cigar, or SL(2)/U(1) black hole



Witten
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….

k sets the gradient of the dilaton far away and the central charge of the wordsheet CFT.

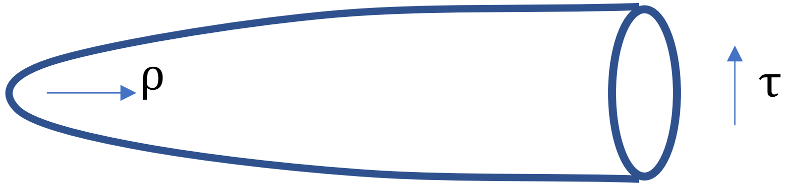
$$(\nabla\phi)^2 = \frac{1}{\mathrm{k}-2}$$

$$c = 2 + \frac{6}{k-2}$$

Radius of the circle at infinity, $\beta = 2\pi\sqrt{k}$

Temperature is fixed for fixed theory (i.e. fixed k)

Winding condensate



Fateev, Zamolodchikov, Zamolodchikov
Kazakov, Kostov, Kutasov.

The winding condensate can be computed explicitly. We find

$$\chi = e^{-(k-2)\rho}, \qquad e^{-\phi}\chi = \hat{\chi} = e^{-\phi_h} e^{-(k-3)\rho} \qquad S = \int e^{-2\phi}(|\nabla\chi|^2 + \cdots), \to \hat{\chi} = e^{-\phi}\chi$$

Decreases only for k>3.

Canonically normalized field

The solution changes qualitatively at k=3. The winding condensate changes from being localized near the tip to being sourced at infinity. It changes from being normalizable to non-normalizable.

Karczmarek, JM, Strominger

The contribution to the entropy from the winding condensate becomes infinite at k=3.

Some conclusions:

- The black hole picture and intuition good for k>3 (or $\beta > \sqrt{3}\, 2\pi$)
- For k→ 3 a large winding condensate emerges → atmosphere of strings extends to infinity.

- It was suggested that we should interpret the solution for k<3 as a condensate of winding strings only, with no black hole.

Giveon, Kutasov, Rabinovici, Sever

…

- Maybe in all cases we have two alternative pictures ?

Jafferis, Schneider

Let us now review one more idea…

Large D Schwarzschild black hole →

Sphere x (2 d black hole)

Emparan, Grumiller, Tanabe

From D to 2 dimensional black holes

Emparan, Grumiller, Tanabe

- A D-dimensional Schwarzschild black hole can be approximated by the two dimensional one when $D \gg 1$.

$$ds^2 = f\,dt^2 + \frac{dr^2}{f} + r^2 d\Omega_{D-2}^2\,, \qquad f = 1 - \frac{r_0^{D-3}}{r^{D-3}}$$

Define

$$r^{D-3} = r_0^{D-3}\cosh^2\rho$$

The r and t components of the metric become $\quad ds^2 = k\,(\,d\rho^2 + \tanh^2\rho\, d\tau^2\,)$

with: $\quad k = \frac{2\,r_0}{D}, \qquad \mathrm{k} \gg 1$

Same as two dimensional black hole

This can be viewed as giving $e^{-2\phi} \sim \sqrt{g} \sim r^{D-2} \propto \cosh^2\rho$

Now we come to our work in progress

Yiming Chen, JM

We want to extend the large D observation to small k, or stringy curvatures. (the curvature is high in two of the dimensions)

And use it to make a more precise statement for Schwarzschild black holes in 26 or 10 dimensions, in the approximation that $26, 10 \gg 1$.

Matching the central charges to determine k

Sphere part of the sigma model can be approximated as an almost CFT with a slowly varying radius.

Its central charge can be computed using large radius formulas. Tseytlin

$$c_{Sphere} = D - 2 - \frac{6}{4} R \sim D - 2 - 6\left(\frac{D}{2r}\right)^2, \qquad \rightarrow \qquad 6\left(\frac{D}{2r_0}\right)^2 = \frac{6}{k-2}$$

Match the central charge deficit on the sphere with the excess in the cigar

$$\frac{\beta}{2\pi} = \sqrt{k}$$

Exact temperature to r_0 relation. (at large D)

Check:

The leading correction can be matched against the large D limit of the α' corrected black holes.

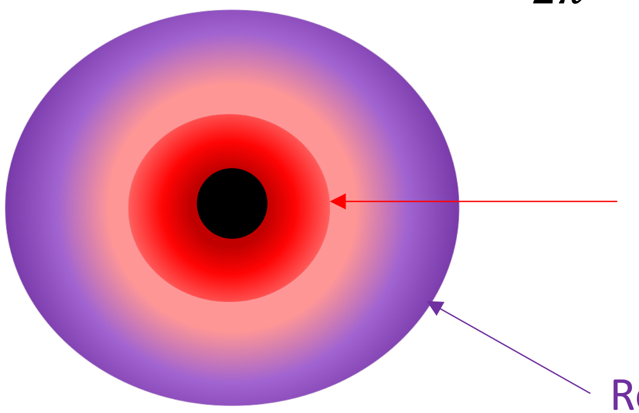
Callan, Myers, Perry (bosonic, heterotic)

Myers (type II) (needs correction)

- As we vary the size of the horizon, we are varying k, the parameter of the cigar theory.

Expansion of the stringy thermal atmosphere

Now consider the limit k → 3^+ $\frac{\beta}{2\pi} = \sqrt{k} \to \sqrt{3},$ $\frac{\beta_H}{2\pi} = 2$



Region where the SL(2)/U(1) black hole approximation is good

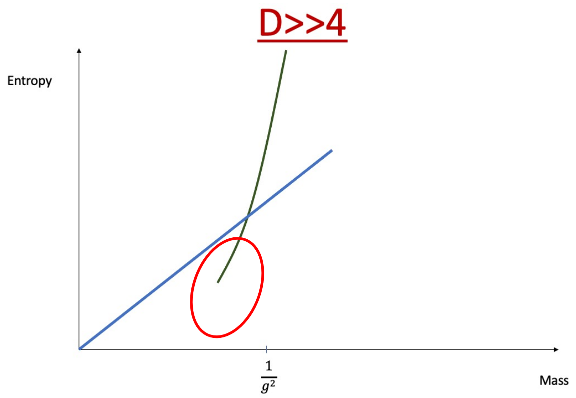
Region where the winding mode is tachyonic

The winding condensate expands far from the horizon → makes a large contribution the entropy and the mass.

Black hole becomes dominated by the highly excited string.

Our method ceases to be valid at $k - 3 \sim \frac{1}{D}$.

We do not know what happens for lower masses.

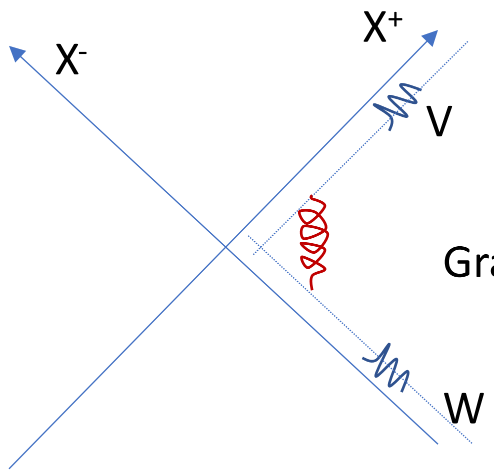


Let us now discuss another computation we can do with this description

Chaos exponent

Shenker, Stanford; Kitaev

- We expect that the dynamics of black holes is chaotic.
- This can be made very concrete by looking at special correlation functions: Out of time order correlators.
- These receive an interesting contribution from a scattering process near the horizon.



Graviton exchange → spin = 2 → $\lambda = \frac{2\pi}{\beta}$

$$\frac{\langle V(t)W(0)V(t)W(0)\rangle}{\langle W\,W\rangle\langle VV\rangle} = 1 - \mathrm{G_N}\mathrm{e}^{\lambda t}$$

Chaos exponent from the spin of the exchanged state

Conjectured candidate exchanged state $$|\Psi\rangle = (J^{+}_{-1}\bar{J}^{-}_{-1})^{\frac{s}{2}}\left|j,-\frac{1}{2},\frac{1}{2}\right\rangle, \qquad \text{for } j\to\frac{1}{2}$$

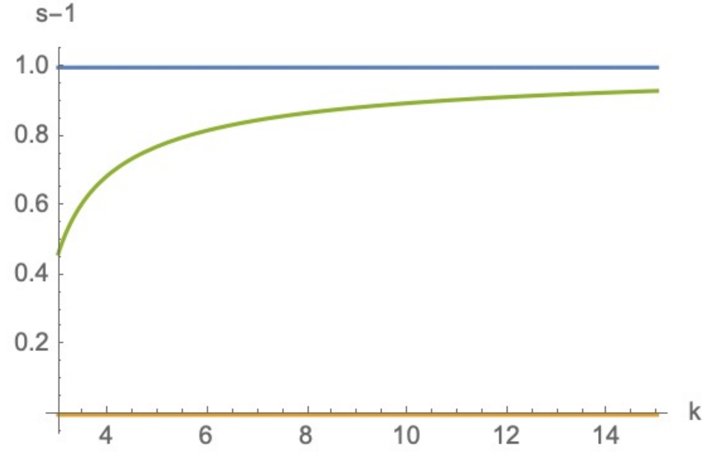
We expect that the type of state we exchange has an ``orbital'' wavefunction involving $\delta(X^{+})$ whose spin is minus one.

Metric shock wave: $\delta g_{++} \sim \delta(X^{+})$

$$L_0 = N - \frac{j(j-1)}{k-2} + \frac{(J_0^3)^2}{k}$$

$j=\frac{1}{2}$ $\quad J_0^3 = \frac{s-1}{2}$

$$1 = \frac{s}{2} + \frac{1}{4\,(k-2)} + \frac{(s-1)^2}{4\,k}$$



$$s-1 = 1 - \frac{1}{k} + \cdots \ \text{for large } k \ \rightarrow$$ Matches the first correction of Shenker and Stanford.

We get a non-zero value at k=3.

Conclusions

- We discussed how the large D approximation for Schwarzschild black holes leads to the cigar geometry.
- The cigar geometry can have a string scale curvature and we can still solve it.
- Used it to explore the geometry of a black hole as we approach (and surpass) the Hagedorn temperature.
- At a critical size/temperature the black hole develops a large stringy “halo” or atmosphere.
- We do not think that the black hole makes sense for lower masses.
- We computed the chaos exponent and found it is non-zero at the transition point.

Questions

- Can similar large D approximations be used for other string scale black holes?
- What can we say about the gravitational picture for the microstates and the black hole interior?