Periods of multiple BHK mirrors

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Plan

- Introduction
- Calabi-Yau orbiflds
- Berglund–Hübsch–Krawitz mirror construction
- Coincidence the periods of BHK multiple mirrors

Abstract

We consider the multiple Calaby-Yau (CY) mirror phenomenon which appears in Berglund-Hübsch-Krawitz (BHK) mirror symmetry. We show that for any pair of Calabi–Yau orbifolds that are BHK mirrors of a loop–chain type pair of Calabi–Yau manifolds in the same weighted projective space the periods of the holomorphic nonvanishing form coincide.

Introduction

Calabi-Yau manifolds (CY) arise in the context of spacetime supersymmetric compactifications of string theories.

An important property of CY manifolds is mirror symmetry , which reflects geometric relation between pairs of CY manifolds.

Namely, for a pair of n-dimensional Calabi-Yau manifolds X and Y, cohomologies are isomorphic, $H^{p,q}(X,\mathbb{C}) = H^{n-p,q}(Y,\mathbb{C})$. Calabi-Yau manifolds possess the structure of complex and Kähler manifold which admit deformations , giving rise to the moduli spaces $M_C(X)$ and $M_K(X)$. Mirror symmetry can be considered as matching of the Special geometries on the moduli spaces

$$M_C(X) \simeq M_K(Y)\,, \quad M_K(X) \simeq M_C(Y)\,. \tag{1}$$

Calabi-Yau manifolds

Here we consider the class of CY manifolds, defined as hypersurfaces or orbifolds in weighted projective spaces $\mathbb{P}^4_{(k_1,k_2,k_3,k_4,k_5)}$ cut out by non-degenerate quasihomogeneous invertible polynomial of degree $d = k_1 + k_2 + k_3 + k_4 + k_5$, which consists of five monomials

$$W^0_M(x) = \sum_{i=1}^{5} \prod_{j=1}^{5} x_j^{M_{ij}} \tag{2}$$

The nondegeneracy requires that these polynomials are sum of the polynomials of three basic types, Fermat, Chain or Loop:

$$x_1^{A_1} + x_2^{A_2} + ... + x_n^{A_n} - \text{ Fermat,} \tag{3}$$

$$x_1^{A_1} x_2 + x_2^{A_2} x_3 + ... + x_n^{A_n} - \text{ Chain ,} \tag{4}$$

$$x_1^{A_1} x_2 + x_2^{A_2} x_3 + ... + x_n^{A_n} x_1 - \text{ Loop .} \tag{5}$$

Multiple mirrors

The multiple mirror phenomenon occurs when a given CY threefold possesses more than one mirror in different weighted projective spaces.

More specifically the phenomenon is the following. Some weighted projective spaces allow the existence of a few CY manifolds defined by the polynomials that belong to different types of Kreuzer-Skarke class. In such cases the Berglund-Hübsch-Krawitz (BHK) mirrors of these two CY manifolds generally appear in two different weighted projective spaces.

We obtained examples of such phenomenon, which shows all the cases when the weighted projective space allow the simultaneous occurrence of CY-manifolds of loop and chain types of Kreuzer-Skarke class CY manifolds.

Deformations of complex structure

X_M^0 admits the deformation of complex structure. The full family of X_M is given by zero locus of

$$W(x,\varphi) = \sum_{i=1}^{5} \prod_{j=1}^{5} x_j^{M_{ij}} + \sum_{l=1}^{h} \varphi_l e_l(x). \tag{6}$$

Monomials e_l form the basis in the space of deformations of complex structure.
BH proposed that the mirror for X_M is related to the hypersurface X_{M^T} cut out by

$$W_{M^T}(z) = \sum_{i=1}^{5} \prod_{j=1}^{5} x_j^{(M^T)_{ij}} \tag{7}$$

in another space $\mathbb{P}^4_{(\bar{k}_1,\bar{k}_2,\bar{k}_3,\bar{k}_4,\bar{k}_5)}$, realized as a quotient of X_{M^T} by some subgroup of the phase symmetries of the W_{M^T}.

Phase symmetry of polynomial W_M^0

Let Calabi-Yau X_M be defined in $\mathbb{P}^4_{(k_1,k_2,k_3,k_4,k_5)}$ by zero locus of

$$W_M(x) = \sum_{i=1}^{5} \prod_{j=1}^{5} x_j^{M_{ij}}. \tag{8}$$

In view of the quasi-homogeneity of W_M it is invariant under the action of the group J_M generated by the action

$$x_i \mapsto \omega^{k_i} x_i, \quad \omega^d = 1. \tag{9}$$

W_M a can have a larger group of diagonal automorphisms and $J_M \subseteq \mathrm{Aut}(M)$. The group $\mathrm{Aut}(M)$ is generated by $q_i(M)$

$$q_i(M) : x_j \mapsto e^{2\pi i B_{ji}} x_j, \tag{10}$$

where the matrix $B = M^{-1}$.

The allowable symmetries

Group $\mathrm{Aut}(M)$ acts on each term of W_M as

$$q_l(M) \cdot \prod_{j=1}^{5} x_j^{M_{ij}} = e^{2\pi i B_{ij} M_{jl}} \prod_{l=1}^{5} x_l^{M_{il}} = e^{2\pi i \delta_{il}} \prod_{l=1}^{5} x_l^{M_{il}} = \prod_{l=1}^{5} x_l^{M_{il}}. \qquad (11)$$

C-Y X_M admits the holomorphic, nowhere vanishing 3-form Ω. Subgroups of the phase group preserving the form Ω, or $\prod_i x_i$ are called allowable. Let $SL(M)$ be the maximal allowable group with generators $p_s(M)$,

$$SL(M) := \left\{ p_s(M) \in \mathrm{Aut}(M) \mid p_s(M) \cdot \prod_{j=1}^{5} x_j = \prod_{j=1}^{5} x_j \right\}. \qquad (12)$$

$J_M \subseteq SL(M)$. Let G_0 is an subgroup, $J_M \subseteq G_0 \subseteq SL(M)$ and $G := G_0/J_M$. Then $Z(M,G) := X_M/G$ is a Calabi-Yau orbifold X.

Calabi-Yau orbifolds

The full family X is cut out by $\{W^X = 0\}$

$$W^X(x,\varphi) = \sum_{i=1}^{5}\prod_{j=1}^{5} x_j^{M_{ij}} + \sum_{l=1}^{h}\varphi_l \prod_{j=1}^{5} x_j^{S_{lj}}. \tag{13}$$

φ_l are moduli of complex structure deformations,

$$e_l := \prod_{j=1}^{5} x_j^{S_{lj}}. \tag{14}$$

e_l are quasi-homogeneous and invariant under the group G. They belong to G-invariant subring of the chiral ring $\mathbb{C}[x_1,\ldots,x_5]/\langle \frac{\partial W_M}{\partial x_j}\rangle$. We denote

$$e_{h_X} = \prod_{i=1}^{5} x_i. \tag{15}$$

Actually, the monomials e_l are the subset in the basis of deformations of complex structure e_s, $s = 1,\ldots,h$ of the original CY family X_M.

Berglund–Hübsch–Krawitz mirror construction

The polynomial

$$W_{M^T}(z) = \sum_{i=1}^{5} \prod_{j=1}^{5} z_j^{M_{ji}} \tag{16}$$

of the degree $\bar{d} = \sum_j \bar{k}_j M_{ji}$ defines the Calabi-Yau hypersurface X_{M^T} in another projective space $\mathbb{P}^4_{(\bar{k}_1,\bar{k}_2,\bar{k}_3,\bar{k}_4,\bar{k}_5)}$. We can define a similar set of groups for the transposed matrix: $J_{M^T} \subseteq G_0^T \subseteq SL(M^T) \subseteq \text{Aut}(M^T)$. Taking the quotient $G^T := G_0^T / J_{M^T}$, we define the Calabi-Yau orbifold as

$$Z(M^T, G^T) := X_{M^T}/G^T. \tag{17}$$

The groups G and G^T can be choosen in different ways.
Is it possible to choose a group G^T for a given group G, and if so, how to do this so that the manifolds $Z(M, G)$ and $Z(M^T, G^T)$ form a mirror pair?

Berglund–Hübsch–Krawitz mirror construction

Krawitz's construction defines the generators of the group G_0^T. Namely, they are constructed using the exponents S_{li} of the invariant monomials as follows

$$\rho_l^T := \prod_{i=1}^{5} q_i(M^T)^{S_{li}}, \tag{18}$$

where $q_i(M^T)$ are generators of the $\mathrm{Aut}(M^T)$ acting on each coordinate y_j in $\mathbb{P}^4_{(\bar{k}_1,\bar{k}_2,\bar{k}_3,\bar{k}_4,\bar{k}_5)}$ as

$$q_i(M^T) : y_j \mapsto e^{2\pi i B_{ij}} y_j. \tag{19}$$

It follows that the group G_0^T acts on the coordinates y_j as

$$\rho_l^T : y_j \mapsto e^{2\pi i \sum_i S_{li} B_{ij}} y_j. \tag{20}$$

Berglund–Hübsch–Krawitz mirror construction

The full family of the mirror Calabi-Yau orbifold $Z(M^T, G^T)$ is given by zero locus of

$$W^{Z(M^T,G^T)}(y,\psi) = \sum_{i=1}^{5} \prod_{j=1}^{5} y_j^{M_{ji}} + \sum_{m=1}^{h_Y} \psi_m \prod_{j=1}^{5} y_j^{R_{mj}}, \tag{21}$$

where ψ_m are moduli of complex structure of $Z(M^T, G^T)$. The monomials $\bar{e}_m := \prod_{j=1}^{5} y_j^{R_{mj}}$ are invariant under the G_0^T action:

$$\rho_l^T \cdot \bar{e}_m = e^{2\pi i \sum_{ij} B_{ij} S_{li} R_{mj}} \prod_{j=1}^{5} y_j^{R_{mj}} = \prod_{j=1}^{5} y_j^{R_{mj}}. \tag{22}$$

S_{li}, and R_{mj} are components of vectors $(\vec{S}_l)_j = S_{lj}$, and $(\vec{R}_m)_i = R_{mi}$. The invariance condition can be rewritten in terms ofmatrix B as

$$(\vec{S}_l, \vec{R}_m) = \sum_{i,j=1}^{5} B_{ij} S_{li} R_{mj} \in \mathbb{Z}. \tag{23}$$

Berglund–Hübsch–Krawitz mirror construction

Last relation is is a strong constraint because S and R are integers, although the elements in B are rational. Taking into account also the condition of quasi-homogeneity

$$\sum_{i=1}^{5} R_{mi}\bar{k}_i = \bar{d}, \tag{24}$$

we conclude that the equations above have finite non-negative number of solutions. This number is h_Y.

The Chiodo-Ruan theorem states that orbifolds $Z(M,G)$ and $Z(M^T,G^T)$ form a mirror pair on the level of cohomology

$$H^{p,q}(Z(M,G),\mathbb{C}) = H^{3-p,q}(Z(M^T,G^T),\mathbb{C}). \tag{25}$$

Thus the Berglund–Hübsch–Krawitz construction allows one to determine the polynomial $W^{Z(M^T,G^T)}$ which defines the full family of $Z(M^T,G^T)$.

Periods BHK mirrors

Let $X(M)$ be the CY family defined in $\mathbb{P}_{\vec{k}}$ by zeros of

$$W_M(x) = \sum_{i=1}^{5} \prod_{j=1}^{5} x_j^{M_{ij}} + \sum_{l=1}^{h} \phi_l \prod_{j=1}^{5} x_j^{S_{lj}}. \tag{26}$$

The mirror family $Y(\bar{M})$ is defined in $\mathbb{P}_{\vec{\bar{k}}} = \mathbb{P}^4_{(\bar{k}_1,\bar{k}_2,\bar{k}_3,\bar{k}_4,\bar{k}_5)}$ by zeros of the polynomial

$$W_{\bar{M}}(x) = \sum_{i=1}^{5} \prod_{j=1}^{5} x_j^{\bar{M}_{ij}} + \sum_{r=1}^{\bar{h}} \psi_r \prod_{j=1}^{5} x_j^{R_{rj}} \ , \tag{27}$$

where $\bar{M} := M^T$ and there holds $\sum_{j=1}^{5} R_{rj} \bar{k}_j = \bar{d}$ and $\sum_{ij} B_{ij} S_{li} R_{rj} \in \mathbb{Z}$. Periods of the form Ω on $X(M)$ can be presented by contour integrals

$$\sigma_{\vec{\mu}}(\phi) = \oint_{\Gamma_{\vec{\mu}}} e^{-W_M(x,\phi)} d^5 x \ . \tag{28}$$

Chiral ring

The number of the contours $\Gamma_{\vec{\mu}}$ is equal to $\dim \mathbb{R}(M) = 2h + 2$, where $\mathbb{R}(M)$ is the chiral ring of polynomials of $x_1, ..., x_5$ modulo $\{\frac{\partial W_M^{\mathbf{0}}}{\partial x_i}\}$. The chiral ring $\mathbb{R}(M)$ has a basis

$$e_{\vec{\mu}}(x) = \prod_{i=1}^{5} x_i^{\mu_i} \ , \text{ where } \sum_{i=1}^{5} k_i \mu_i = 0, \ d, \ 2d, \ 3d. \tag{29}$$

Let $A_i := M_{ii}$, then non-negative integers μ_i are subject to constraints: In the Fermat case

$$0 \leq \mu_i \leq A_i - 2 \ . \tag{30}$$

In the Loop case

$$0 \leq \mu_i \leq A_i - 1 \ . \tag{31}$$

In Chain case covenient bases are given by Kreuzer and Krawitz. The ring $\mathbb{R}(M)$ is graded,

$$\mathbb{R}(M) = \mathbb{R}^0(M) \oplus \mathbb{R}^d(M) \oplus \mathbb{R}^{2d}(M) \oplus \mathbb{R}^{3d}(M). \tag{32}$$

Similar statements exist for the mirror $Y(\bar{M})$.

Periods

$$W_M(x,\phi) = W_M^0(x) + \sum_{l=1}^{h} \phi_l e_l(x), \tag{33}$$

so, expanding the expression for the period to the series, we obtain

$$\sigma_{\vec{\mu}}(\phi_1, ..., \phi_h) = \int_{\Gamma_{\vec{\mu}}} d^5 x e^{-W_M^0(x)} e^{-\sum_{l=1}^{h} \phi_l e_l(x)}$$

$$= \sum_{\{m_l\}} \prod_{l=1}^{h} \frac{\phi_l^{m_l}}{m_l!} \int_{\Gamma_{\vec{\mu}}} d^5 x e^{-W_M^0(x)} \prod_{j=1}^{5} x_j^{\sum_{l=1}^{h} m_l S_{lj}}.$$

Using the technique (KAAB), we get

$$\sigma_{\vec{\mu}}(\phi_1, ..., \phi_h) \sim \sum_{\{m_l\}\in\Sigma_{\vec{\mu}}} C(\{m_l\}) \prod_{l=1}^{h} \frac{\phi_l^{m_l}}{m_l!}\ , \tag{34}$$

$$C(\{m_l\}) = \prod_{i=1}^{5} \Gamma\left(\sum_{j} (m_l S_{lj} + 1) B_{ji}\right)\ . \tag{35}$$

Periods

The canonical choice of $\Gamma_{\vec{\mu}}$ defines it as a cycle dual, to the the element of the base of the chiral ring $e_{\vec{\mu}}$

$$\int_{\Gamma_{\vec{\mu}}} e_{\vec{\nu}}(x) e^{-W_0(x)} d^5 x = \delta_{\vec{\mu},\vec{\nu}} \ . \tag{36}$$

It leads to the following definition $\Sigma_{\vec{\mu}}$. The set of non-negative integers m_l, where $l = 1, ..., h$, belongs to $\Sigma_{\vec{\mu}}$, i.e., $\{m_l\} \subset \Sigma_{\vec{\mu}}$, if the following combination of the vectors S_{li} and the five-component vector $\vec{\mu} = \mu_1, ..., \mu_5$ can be expressed in terms of five non-negative integers $n_j, j = 1, ..., 5$ and the matrix M as

$$m_l S_{li} = (\vec{\mu})_i + n_j M_{ji} \ . \tag{37}$$

From the formula for periods we see that they are determined by the sets of h the products $S_{li} B_{ij}$ and by the sets $\{m_l\}$, $l = 1, ..., h$, where $l = 1, .., h$ and $j = 1, ..., 5$.

Periods

Now we prove that the periods of two CY mirror orbifolds $Y(\bar{M}(1))$ and $Y(\bar{M}(2))$, defined in two different spaces $\mathbb{P}_{\vec{k}(1)}$ and $\mathbb{P}_{\vec{k}(2)}$, are the same, if they are BHK mirrors of two CY manifolds $X(M(1))$ and $X(M(2))$, defined in the same space $\mathbb{P}_{\vec{k}}$.
Here initial loop matrix is $M(1)$ and initial chain matrix is $M(2)$. Let $S_{lj}(1) \in \mathbb{Z}_{\geq 0}$ and $S_{lj}(2) \in \mathbb{Z}_{\geq 0}$ are vectors of the exponents in

$$e_l(1) = \prod_{j=1}^{5} x_j^{S_{lj}(1)} , \quad e_l(2) = \prod_{j=1}^{5} x_j^{S_{lj}(2)} \tag{38}$$

$$\sum_{j=1}^{5} S_{lj}(1) k_j = \sum_{j=1}^{5} S_{lj}(2) k_j = d . \tag{39}$$

The sets $\vec{S}_l(1)$ and $\vec{S}_l(2)$ are really different.

However, from Table 1 we come to the following important observation: *When a loop polynomials $W_{M(1)}$ and a chain polynomials $W_{M(2)}$ appear in the same weighted projective space $\mathbb{P}(k_1, \ldots, k_5)$ and are connected by Kreuzer-Skarke cleaves, in all these 111 cases listed in Table the weight $k_5 = 1$.*

From this fact we obtain that the sets of exponents of monomials of their chiral rings shifted by the vector $(1, \ldots, 1)$ not only belong to the same 4-dimensial lattice, but also both contain the integral basis of the lattice, which consists of the four vectors $\vec{V}_a, a = 1, 2, 3, 4$

$$\begin{array}{l}(-1, 0, 0, 0, k_1),\\ (0, -1, 0, 0, k_2),\\ (0, 0, -1, 0, k_3),\\ (0, 0, 0, -1, k_4).\end{array}$$

We use this fact to prove Main statement.

$Y(1)$ and $Y(2)$ are CY orbifolds in $P_{\vec{k}(1)}$ and $P_{\vec{k}(2)}$.
The weights $\vec{k}(1)$ and $\vec{k}(2)$ satisfy

$$\sum_{j=1}^{5}(\bar{M}(\alpha))_{ij}\bar{k}_j(\alpha)=\bar{d}(\alpha)=\sum_{j=1}^{5}\bar{k}_j(\alpha)\ , \quad \alpha=1,2\ . \tag{40}$$

and two sets of the vectors $R_{mj}(1)$ and $R_{mj}(2)$, satisfy

$$\sum_{j=1}^{5}(\bar{R}(\alpha))_{rj}\bar{k}_j(\alpha)=\bar{d}(\alpha)\ , \quad \alpha=1,2\ . \tag{41}$$

From the above BHK mirror conditions

$$(\vec{S}(\alpha)_l,\vec{R}(\alpha)_r)=\sum_{i,j=1}^{5}B(\alpha)_{ij}S(\alpha)_{li}R(\alpha)_{rj}\in\mathbb{Z}\ ,, \tag{42}$$

where $B(\alpha)$ is the inverse for $M(\alpha)$ and from the fact that two sets $\vec{S}_l(1)$ and $\vec{S}_l(2)$ contain the same basis , one finds

$$R_{ri}(1)\bar{B}_{ij}(1)=R_{ri}(2)\bar{B}_{ij}(2)\ . \tag{43}$$

The sets of m_r in the expressions for both mirrors also coincide, what is verified multiplying both equations by $B(1)_{ji}$.
Then we obtain the linear system

$$m_r \sum_{j=1} R(1)_{rj} B(1)_{ji} = \sum_{j=1} (\vec{\mu}(1))_j B(1)_{ji} + n_k \sum_{j=1} \bar{M}(1)_{kj} B(1)_{ji}, \qquad (44)$$

$$m_r \sum_{j=1} R(1)_{rj} B(1)_{ji} = \sum_{j=1} (\vec{\mu}(1))_j B(1)_{ji} + n_i(1). \qquad (45)$$

Similarly, for the second BHK mirror we get

$$m_r \sum_{j=1} R(2)_{rj} B(2)_{ji} = \sum_{j=1} (\vec{\mu}(2))_j B(2)_{ji} + n_i(2). \qquad (46)$$

Taking into account that $\sum_{j=1} R_{rj}(1)\bar{B}_{ji}(1) = \sum_{j=1} R_{rj}(2)\bar{B}_{ji}(2)$ and $\sum_{j=1} (\vec{\mu}(1))_j B(1)_{ji} = \sum_{j=1} (\vec{\mu}(2))_j B(2)_{ji}$, we see that coefficients in both eqs are the same. It follows that $n_i(1) = n_i(2)$.

This completes the proof of our Main statement that the periods of the multiple BHK mirrors coincide.

N	$\{k_i(2)\}$	$\{A_i\}^C \& \{A_i\}^{C(mir)}$	$\{k_i\}$	$\{A_i\}^L \& \{A_i\}^{L(mir)}$	$\{k_i(1)\}$
1	{126,42,70,13,1}	{2,3,3,14,239}	{92,55,74,17,1}	{2,3,3,14,147}	{77,26,43,8,1}
2	{270,90,150,13,17}	{2,3,3,30,31}	{12,7,10,1,1}	{2,3,3,30,19}	{157,58,91,8,17}
3	{6,2,2,1,1}	{2,3,5,10,11}	{4,3,2,1,1}	{2,3,5,10,7}	{83,36,31,16,25}
4	{216,36,66,61,53}	{2,6,6,6,7}	{3,1,1,1,1}	{2,6,6,6,4}	{97,25,37,35,53}
5	{64,48,52,51,41}	{4,4,4,4,5}	{1,1,1,1,1}	{4,4,4,4,4}	{1,1,1,1,1}
6	{96,32,20,43,1}	{2,3,8,4,149}	{52,45,14,37,1}	{2,3,8,4,97}	{62,21,13,28,1}
7	{132,44,20,61,7}	{2,3,11,4,29}	{10,9,2,7,1}	{2,3,11,4,19}	{83,30,13,40,7}
8	{160,40,28,73,19}	{2,4,10,4,13}	{5,3,1,3,1}	{2,4,10,4,8}	{89,27,17,45,19}
9	{108,27,63,17,1}	{2,4,3,9,199}	{82,35,59,22,1}	{2,4,3,9,117}	{63,16,37,10,1}
10	{168,42,98,17,11}	{2,4,3,14,29}	{12,5,9,2,1}	{2,4,3,14,17}	{93,26,57,10,11}
11	{324,36,204,37,47}	{2,9,3,12,13}	{6,1,4,1,1}	{2,9,3,12,7}	{151,22,109,20,47}
12	{120,24,54,31,11}	{2,5,4,6,19}	{8,3,4,3,1}	{2,5,4,6,11}	{64,15,31,18,11}
13	{224,32,104,43,45}	{2,7,4,8,9}	{4,1,2,1,1}	{2,7,4,8,5}	{34,7,19,8,15}
14	{180,15,115,49,1}	{2,12,3,5,311}	{146,19,83,62,1}	{2,12,3,5,165}	{95,8,61,26,1}
15	{450,15,295,121,19}	{2,30,3,5,41}	{20,1,11,8,1}	{2,30,3,5,21}	{221,8,151,62,19}
16	{126,18,78,29,1}	{2,7,3,6,223}	{100,23,62,37,1}	{2,7,3,6,123}	{69,10,43,16,1}
17	{324,18,210,73,23}	{2,18,3,6,25}	{12,1,7,4,1}	{2,18,3,6,13}	{157,10,109,38,23}
18	{288,24,184,49,31}	{2,12,3,8,17}	{8,1,5,2,1}	{2,12,3,8,9}	{137,14,97,26,31}
19	{112,16,52,43,1}	{2,7,4,4,181}	{80,21,34,45,1}	{2,7,4,4,101}	{62,9,29,24,1}
20	{128,16,60,49,3}	{2,8,4,4,69}	{31,7,13,17,1}	{2,8,4,4,38}	{23,3,11,9,1}
21	{160,16,76,61,7}	{2,10,4,4,37}	{17,3,7,9,1}	{2,10,4,4,20}	{83,9,41,33,7}
22	{272,16,132,103,21}	{2,17,4,4,21}	{10,1,4,5,1}	{2,17,4,4,11}	{44,3,23,18,7}
23	{180,45,21,113,1}	{2,4,15,3,247}	{94,59,11,82,1}	{2,4,15,3,153}	{111,28,13,70,1}
24	{300,75,21,193,11}	{2,4,25,3,37}	{14,9,1,12,1}	{2,4,25,3,23}	{181,48,13,120,11}
25	{234,39,33,145,17}	{2,6,13,3,19}	{8,3,1,6,1}	{2,6,13,3,11}	{127,24,19,84,17}
26	{100,20,36,41,3}	{2,5,5,4,53}	{22,9,8,13,1}	{2,5,5,4,31}	{19,4,7,8,1}
27	{210,15,81,113,1}	{2,14,5,3,307}	{144,19,41,102,1}	{2,14,5,3,163}	{111,8,43,60,1}
28	{480,15,189,257,19}	{2,32,5,3,37}	{18,1,5,12,1}	{2,32,5,3,19}	{237,8,97,132,19}
29	{324,27,69,193,35}	{2,12,9,3,13}	{6,1,1,4,1}	{2,12,9,3,7}	{157,16,37,104,35}

30	{220,20,84,89,27}	{2,11,5,4,13}	{6,1,2,3,1}	{2,11,5,4,7}	{35,4,15,16,9}
31	{224,28,60,97,39}	{2,8,7,4,9}	{4,1,1,2,1}	{2,8,7,4,5}	{35,6,11,18,13}
32	{24,24,16,7,1}	{3,2,3,8,65}	{14,23,19,8,1}	{3,2,3,8,51}	{37,38,25,11,2}
33	{27,27,18,7,2}	{3,2,3,9,37}	{8,13,11,4,1}	{3,2,3,9,29}	{41,43,28,11,4}
34	{33,33,22,7,4}	{3,2,3,11,23}	{5,8,7,2,1}	{3,2,3,11,18}	{49,53,34,11,8}
35	{25,25,10,13,2}	{3,2,5,5,31}	{6,13,5,6,1}	{3,2,5,5,25}	{39,41,16,21,4}
36	{32,48,20,27,1}	{4,2,4,4,101}	{15,41,19,25,1}	{4,2,4,4,86}	{27,41,17,23,1}
37	{72,108,30,43,35}	{4,2,6,6,7}	{1,3,1,1,1}	{4,2,6,6,6}	{53,97,25,37,35}
38	{28,28,8,19,1}	{3,2,7,4,65}	{12,29,7,16,1}	{3,2,7,4,53}	{45,46,13,31,2}
39	{52,52,8,37,7}	{3,2,13,4,17}	{3,8,1,4,1}	{3,2,13,4,14}	{81,88,13,61,14}
40	{9,9,2,5,2}	{3,2,9,5,11}	{2,5,1,2,1}	{3,2,9,5,9}	{67,77,16,41,20}
41	{30,45,25,19,1}	{4,2,3,5,101}	{16,37,27,20,1}	{4,2,3,5,85}	{25,38,21,16,1}
42	{36,54,30,19,5}	{4,2,3,6,25}	{4,9,7,4,1}	{4,2,3,6,21}	{29,46,25,16,5}
43	{18,99,39,59,1}	{12,2,3,3,157}	{8,61,35,52,1}	{12,2,3,3,149}	{17,94,37,56,1}
44	{9,54,21,32,1}	{13,2,3,3,85}	{4,33,19,28,1}	{13,2,3,3,81}	{17,103,40,61,2}
45	{9,63,24,37,2}	{15,2,3,3,49}	{2,19,11,16,1}	{15,2,3,3,47}	{17,121,46,71,4}
46	{9,81,30,47,4}	{19,2,3,3,31}	{1,12,7,10,1}	{19,2,3,3,30}	{17,157,58,91,8}
47	{15,45,20,17,8}	{7,2,3,5,11}	{1,4,3,2,1}	{7,2,3,5,10}	{25,83,36,31,16}
48	{66,99,15,83,1}	{4,2,11,3,181}	{24,85,11,60,1}	{4,2,11,3,157}	{57,86,13,72,1}
49	{126,189,15,163,11}	{4,2,21,3,31}	{4,15,1,10,1}	{4,2,21,3,27}	{107,166,13,142,11}
50	{21,42,9,32,1}	{5,2,7,3,73}	{8,33,7,24,1}	{5,2,7,3,65}	{37,75,16,57,2}
51	{39,78,9,62,7}	{5,2,13,3,19}	{2,9,1,6,1}	{5,2,13,3,17}	{67,141,16,111,14}
52	{27,81,12,59,10}	{7,2,9,3,13}	{1,6,1,4,1}	{7,2,9,3,12}	{47,151,22,109,20}
53	{7,14,3,8,3}	{5,2,7,4,9}	{1,4,1,2,1}	{5,2,7,4,8}	{15,34,7,19,8}
54	{30,75,21,53,1}	{6,2,5,3,127}	{12,55,17,42,1}	{6,2,5,3,115}	{27,68,19,48,1}
55	{15,45,12,31,2}	{7,2,5,3,37}	{3,16,5,12,1}	{7,2,5,3,34}	{27,83,22,57,4}
56	{72,48,84,11,1}	{3,3,2,12,205}	{56,37,94,17,1}	{3,3,2,12,149}	{52,35,61,8,1}
57	{168,112,196,11,17}	{3,3,2,28,29}	{8,5,14,1,1}	{3,3,2,28,21}	{116,83,141,8,17}
58	{60,24,78,17,1}	{3,5,2,6,163}	{48,19,68,27,1}	{3,5,2,6,115}	{42,17,55,12,1}
59	{168,48,228,23,37}	{3,7,2,12,13}	{4,1,6,1,1}	{3,7,2,12,9}	{104,35,157,16,37}
60	{32,24,52,19,1}	{4,4,2,4,109}	{23,17,41,27,1}	{4,4,2,4,86}	{25,19,41,15,1}

N	$\{k_i(2)\}$	$\{A_i\}^C \& \{A_i\}^{C(mir)}$	$\{k_i\}$	$\{A_i\}^L \& \{A_i\}^{L(mir)}$	$\{k_i(1)\}$
61	{80,48,136,23,33}	{4,5,2,8,9}	{2,1,4,1,1}	{4,5,2,8,7}	{18,13,35,6,11}
62	{88,16,124,35,1}	{3,11,2,4,229}	{72,13,86,57,1}	{3,11,2,4,157}	{60,11,85,24,1}
63	{184,16,268,71,13}	{3,23,2,4,37}	{12,1,14,9,1}	{3,23,2,4,25}	{120,11,181,48,13}
64	{132,24,186,35,19}	{3,11,2,6,19}	{6,1,8,3,1}	{3,11,2,6,13}	{84,17,127,24,19}
65	{64,24,116,35,17}	{4,8,2,4,13}	{3,1,5,3,1}	{4,8,2,4,10}	{45,19,89,27,17}
66	{48,60,114,29,37}	{6,4,2,6,7}	{1,1,3,1,1}	{6,4,2,6,6}	{35,53,97,25,37}
67	{24,32,44,19,1}	{5,3,2,4,101}	{16,21,38,25,1}	{5,3,2,4,85}	{20,27,37,16,1}
68	{140,56,26,197,1}	{3,5,14,2,223}	{60,43,8,111,1}	{3,5,14,2,163}	{102,41,19,144,1}
69	{75,30,13,106,1}	{3,5,15,2,119}	{32,23,4,59,1}	{3,5,15,2,87}	{109,44,19,155,2}
70	{85,34,13,121,2}	{3,5,17,2,67}	{18,13,2,33,1}	{3,5,17,2,49}	{123,50,19,177,4}
71	{105,42,13,151,4}	{3,5,21,2,41}	{11,8,1,20,1}	{3,5,21,2,30}	{151,62,19,221,8}
72	{54,18,16,73,1}	{3,6,9,2,89}	{25,14,5,44,1}	{3,6,9,2,64}	{77,26,23,105,2}
73	{78,26,16,109,5}	{3,6,13,2,25}	{7,4,1,12,1}	{3,6,13,2,18}	{109,38,23,157,10}
74	{49,14,19,64,1}	{3,7,7,2,83}	{24,11,6,41,1}	{3,7,7,2,59}	{69,20,27,91,2}
75	{72,18,22,97,7}	{3,8,9,2,17}	{5,2,1,8,1}	{3,8,9,2,12}	{97,26,31,137,14}
76	{56,42,26,99,1}	{4,4,7,2,125}	{24,29,9,62,1}	{4,4,7,2,101}	{45,34,21,80,1}
77	{64,48,26,115,3}	{4,4,8,2,47}	{9,11,3,23,1}	{4,4,8,2,38}	{17,13,7,31,1}
78	{88,66,26,163,9}	{4,4,11,2,21}	{4,5,1,10,1}	{4,4,11,2,17}	{23,18,7,44,3}
79	{50,30,34,83,3}	{4,5,5,2,39}	{8,7,4,19,1}	{4,5,5,2,31}	{13,8,9,22,1}
80	{120,16,86,137,1}	{3,15,4,2,223}	{70,13,28,111,1}	{3,15,4,2,153}	{82,11,59,94,1}
81	{216,16,158,245,13}	{3,27,4,2,31}	{10,1,4,15,1}	{3,27,4,2,21}	{142,11,107,166,13}
82	{50,10,28,61,1}	{3,10,5,2,89}	{27,8,9,44,1}	{3,10,5,2,62}	{69,14,39,85,2}
83	{55,10,31,67,2}	{3,11,5,2,49}	{15,4,5,24,1}	{3,11,5,2,34}	{75,14,43,93,4}
84	{65,10,37,79,4}	{3,13,5,2,29}	{9,2,3,14,1}	{3,13,5,2,20}	{87,14,51,109,8}
85	{85,10,49,103,8}	{3,17,5,2,19}	{6,1,2,9,1}	{3,17,5,2,13}	{111,14,67,141,16}
86	{84,14,34,109,11}	{3,12,7,2,13}	{4,1,1,6,1}	{3,12,7,2,9}	{109,20,47,151,22}
87	{80,30,58,131,21}	{4,8,5,2,9}	{2,1,1,4,1}	{4,8,5,2,7}	{19,8,15,34,7}
88	{72,96,22,169,1}	{5,3,12,2,191}	{26,61,8,95,1}	{5,3,12,2,165}	{62,83,19,146,1}
89	{39,52,11,92,1}	{5,3,13,2,103}	{14,33,4,51,1}	{5,3,13,2,89}	{67,90,19,159,2}

90	{45,60,11,107,2}	{5,3,15,2,59}	{8,19,2,29,1}	{5,3,15,2,51}	{77,104,19,185,4}
91	{57,76,11,137,4}	{5,3,19,2,37}	{5,12,1,18,1}	{5,3,19,2,32}	{97,132,19,237,8}
92	{42,70,26,113,1}	{6,3,7,2,139}	{16,43,10,69,1}	{6,3,7,2,123}	{37,62,23,100,1}
93	{54,90,26,149,5}	{6,3,9,2,35}	{4,11,2,17,1}	{6,3,9,2,31}	{47,80,23,132,5}
94	{14,14,8,31,3}	{5,4,7,2,13}	{2,3,1,6,1}	{5,4,7,2,11}	{15,16,9,35,4}
95	{18,78,58,97,1}	{14,3,3,2,155}	{8,43,26,77,1}	{14,3,3,2,147}	{17,74,55,92,1}
96	{9,42,31,52,1}	{15,3,3,2,83}	{4,23,14,41,1}	{15,3,3,2,79}	{17,80,59,99,2}
97	{9,48,35,59,2}	{17,3,3,2,47}	{2,13,8,23,1}	{17,3,3,2,45}	{17,92,67,113,4}
98	{9,60,43,73,4}	{21,3,3,2,29}	{1,8,5,14,1}	{21,3,3,2,28}	{17,116,83,141,8}
99	{24,64,38,89,1}	{9,3,4,2,127}	{10,37,16,63,1}	{9,3,4,2,117}	{22,59,35,82,1}
100	{24,80,46,109,5}	{11,3,4,2,31}	{2,9,4,15,1}	{11,3,4,2,29}	{22,75,43,102,5}
101	{36,84,34,127,7}	{8,3,6,2,23}	{2,7,2,11,1}	{8,3,6,2,21}	{32,77,31,116,7}
102	{21,56,19,85,8}	{9,3,7,2,13}	{1,4,1,6,1}	{9,3,7,2,12}	{37,104,35,157,16}
103	{32,40,38,77,5}	{6,4,4,2,23}	{3,5,3,11,1}	{6,4,4,2,20}	{27,35,33,67,5}
104	{20,30,22,59,9}	{7,4,5,2,9}	{1,2,1,4,1}	{7,4,5,2,8}	{11,18,13,35,6}
105	{48,18,58,67,1}	{4,8,3,2,125}	{28,13,21,62,1}	{4,8,3,2,97}	{37,14,45,52,1}
106	{84,18,106,115,13}	{4,14,3,2,17}	{4,1,3,8,1}	{4,14,3,2,13}	{61,14,81,88,13}
107	{9,6,13,16,1}	{5,6,3,2,29}	{5,4,5,14,1}	{5,6,3,2,24}	{29,20,43,53,4}
108	{21,12,31,37,4}	{5,7,3,2,17}	{3,2,3,8,1}	{5,7,3,2,14}	{33,20,51,61,8}
109	{27,12,41,47,8}	{5,9,3,2,11}	{2,1,2,5,1}	{5,9,3,2,9}	{41,20,67,77,16}
110	{48,40,62,113,25}	{6,6,4,2,7}	{1,1,1,3,1}	{6,6,4,2,6}	{37,35,53,97,25}
111	{12,18,22,31,1}	{7,4,3,2,53}	{6,11,9,26,1}	{7,4,3,2,47}	{21,32,39,55,2}

Таблица: Here $\{k_i\}$ are weights arising simultaneously in loop and chain types, with the corresponding exponents $\{A_i\}^C$ and $\{A_i\}^L$. The $\{k_i(2)\}$ and $\{A_i\}^{C(mir)}$ are the weights and the exponents of the Mirror CY for the chain manifold $\{A_i\}^C$. The sets $\{k_i(1)\}$ and $\{A_i\}^{L(mir)}$ are the same for the loop manifold $\{A_i\}^L$. The notation $\{A_i\}^C\&\{A_i\}^{C(mir)}$ means that two sets coincide.