Thermal Large N Holography

Bulk tidal excitation from thermal mixing in the boundary

Bo Sundborg, Stockholm University, Work in progress with Julius Engelsöy

Quarks 2020 "Integrability, Holography, Higher-Spin Gravity and Strings" A.D. Sakharov's centennial 2021

Gravity, especially quantum gravity,
is entangled with thermodynamics.

Thermal large N holography and tidal excitation

- Is exotic, very stringy or higher spin symmetric gravity still recognisable as gravity? Are there universal features?

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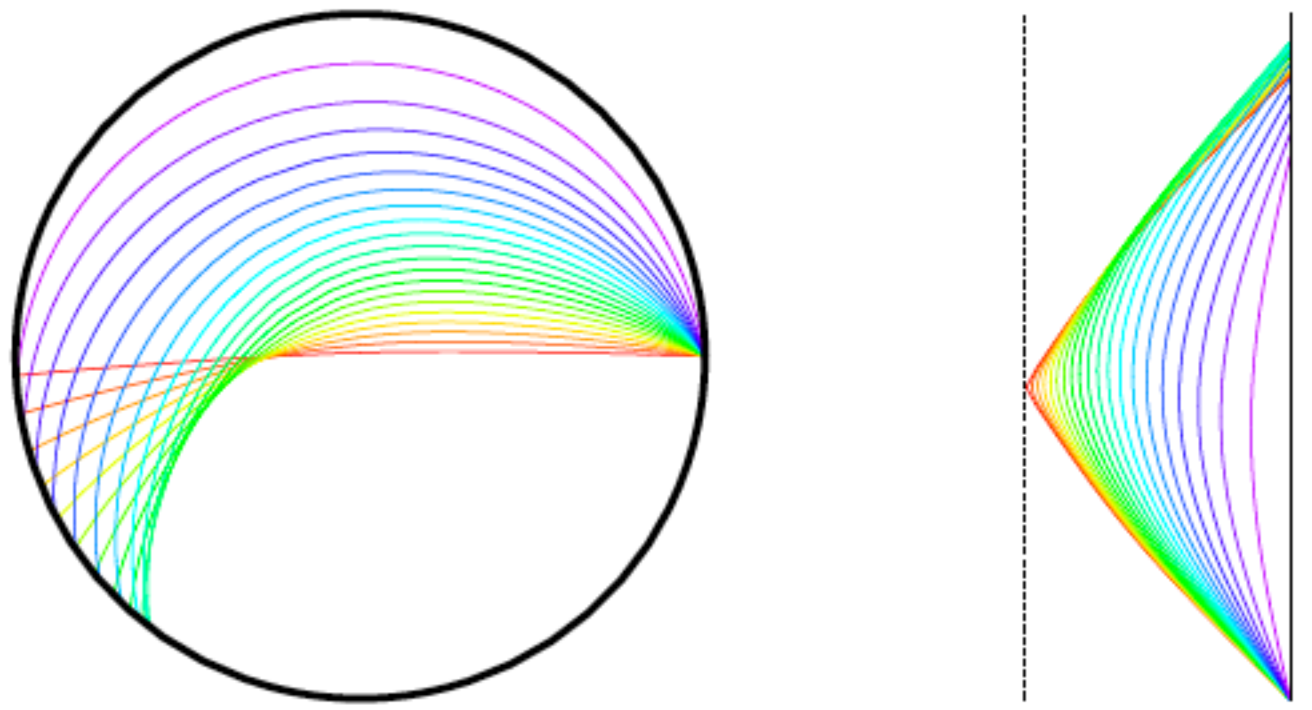
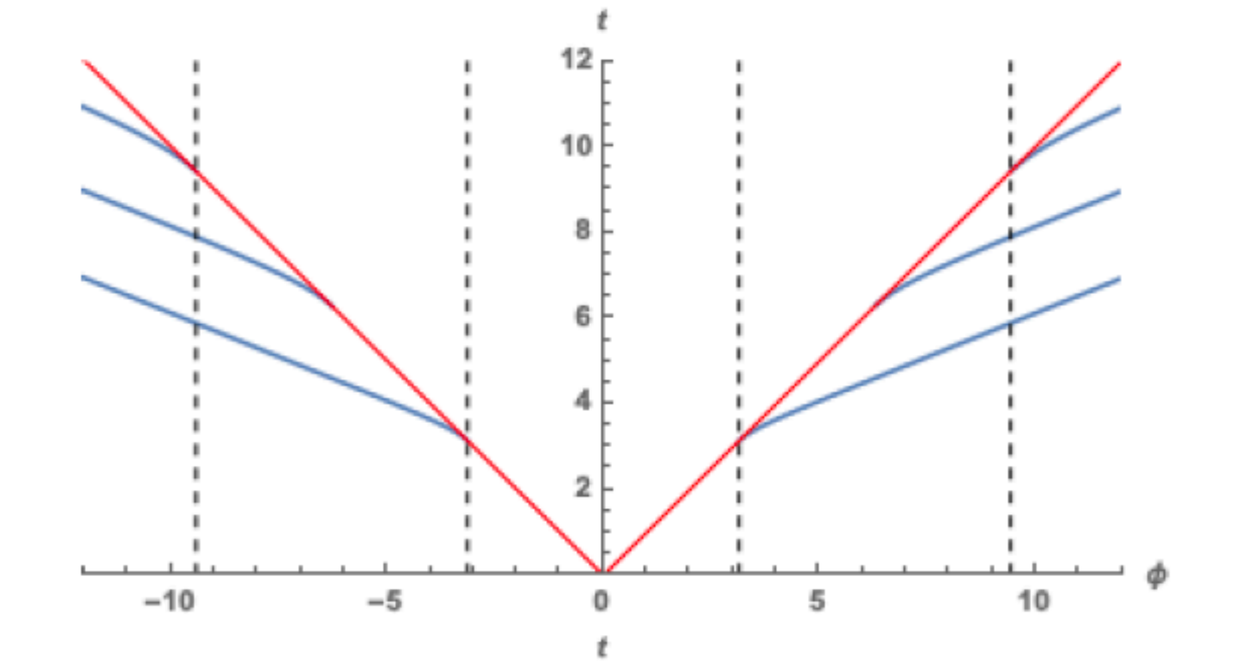


Fig. 4: Null geodesics in star with $\rho_0 = 10$ in AdS, projected onto a constant t slice and the $t - r$ plane, for varying angular momentum to energy ratio ($E = 10$ and $J = 0, 1, \ldots, 10$). On the

Hubeny, Liu and Rangamani: hep-th/0610041
“Bulk-cone singularities & signatures of horizon formation in AdS/CFT”

- Are there traces of bulk-cone singularities in exotic gravity?
- What breaks AdS periodicity?
- What is the boundary signature of strong tidal effects in the bulk?



Dodelson and Ooguri: 2010.09734
“Singularities of thermal correlators at strong coupling”

Related recent developments

- Dual of free super-Yang-Mills. Gaberdiel and Gopakumar 2104.08263, 2105.10496

- Summing over Geometries in String Theory Eberhardt 2102.12355

- The Harder They Fall, the Bigger They Become: Tidal Trapping of Strings by Microstate Geometries Martinec and Warner 2009.07847
- Singularities of thermal correlators at strong coupling Dodelson and Oogurii 2010.09734

- Proper time to the black hole singularity from thermal one-point functions Grinberg and Maldacena 2011.01004
- Correlation functions in finite temperature CFT and black hole singularities Rodriguez-Gomez and Russo 2102.11891

- Thermalization in Large-N CFTs Karlsson, Parnachev and Tadić 2102.04953

Recent interest in thermal CFTs

- There are limits to the convergence of the OPE due to topology.
- There are new simple but non-trivial observables:
  - One-point functions are non-zero.
  - Different operators mix.

Bulk physics from boundary physics

- Witten 1998: Hawking-Page black hole thermodynamics transition in asymptotic AdS → deconfinement transition in boundary large N gauge theory.

- 1999: Boundary ”free deconfinement” → to black hole thermodynamic phase transition in Higher Spin symmetric tensionless string theory?

- Boundary correlators probe the bulk to reveal ”evanescent modes” – whispering gallery modes – confined to the region inside the ”photon sphere” around black holes. Amado, Sundborg, Thorlacius, Wintergerst:1612.03009, 1712.06963

- Now: Tidal effects probed in the boundary. Engelsöy, Sundborg in progress

Bulk tidal excitation from thermal mixing in the boundary

Boundary correlators

Encoding a bulk thermal equilibrium/black hole?

- The typical thermal equilibrium geometry in asymptotically AdS spaces is believed to be a black hole or something similar.
- The boundary to global AdS_{d+1} is $S^{d-1} \times R^1$.
- The boundary field theory language for thermal equilibrium is a compact imaginary time circle. We get Euclidean $S^{d-1} \times S^1$.

$$Z[\beta] \approx \int_{\varphi \text{ on } S^{d-1} \times S^1} \mathscr{D}\varphi \mathscr{D}\varphi^\dagger e^{-S[\varphi, \varphi^\dagger; \beta]}$$

Singlet models

Bulk gravity as far from supergravity as possible

$$S = \int_{S^{d-1}\times S^1} \mathrm{d}^d x \sqrt{g}\, g^{\mu\nu}\partial_\mu \varphi^\dagger \partial_\nu \varphi$$

$$O = \varphi^\dagger \varphi = J^0 \qquad T_{\mu\nu} = J^2_{\mu\nu}$$

$$J^s_{\mu_1\ldots\mu_s} = \sum_{k=0}^{s} a_{sk}\, \partial_{\mu_1}\ldots\partial_{\mu_k}\varphi^\dagger \partial_{\mu_{k+1}}\ldots\partial_{\mu_s}\varphi \qquad \partial^\mu J^s_{\mu\mu_1\ldots\mu_s} = 0$$

- Free, integrable boundary theories are as far from strongly coupled gauge theory as possible.
- Large N counting and spectrum is ensured by the Gauss law - singlet - constraint, the only remnant of gauge symmetry.
- Conjuctured duals include
  - higher spin theory
  - tensionless string theory

Singlet models

Representing the Gauss law constraint

- Free, integrable boundary theories are as far from strongly coupled gauge theory as possible.

- Large N counting and spectrum is ensured by Gauss law - singlet - constraint, the only remnant of gauge symmetry, which is encoded in the path integral as topologically non-trivial A_0.

- Conjectured duals include
  - higher spin theory, and
  - tensionless string theory.

$$Z[\beta] = \int_{\varphi \text{ on } S^{d-1}\times S^1} \mathscr{D}A_\mu \mathscr{D}\varphi \mathscr{D}\varphi^\dagger e^{-S\left[A_\mu, \varphi, \varphi^\dagger; \beta\right]}$$

$$S = \int_{S^{d-1}\times S^1} \mathrm{d}^d x \sqrt{g} \left(g^{\mu\nu} (D_\mu \varphi)^\dagger D_\nu \varphi - \frac{(d-2)^2}{4R^2} \varphi^\dagger \varphi \right)$$

Thermal equilibrium

Large N saddle point

$$Z[\beta] = \int \mathcal{D}A_\mu \mathcal{D}\varphi \mathcal{D}\varphi^\dagger e^{-S\left[A_\mu, \varphi, \varphi^\dagger; \beta\right]}$$

- The A_0 integral can be written in terms of an integral over its N eigenvalues λ_i.

$$Z[\beta] = \frac{1}{N!}\int \left(\prod_i d\lambda_i\right) \exp\left[\sum_{i\neq j} \ln\left|\sin\left(\frac{\lambda_i - \lambda_j}{2}\right)\right| - \sum_i S_{s,\beta}\left[\lambda_i\right]\right]$$

$$Z[\beta] = \frac{1}{N!}\int \left(\prod_i d\lambda_i\right) \exp\left\{-S_{\text{eff}}[\beta; \lambda_i]\right\}$$

- Finding the large N saddle point is the dynamical principle.
  - $\frac{1}{N}$ relates to Newton's constant.

$$0 = \frac{\delta S_{\text{eff}}[\beta; \lambda_j]}{\delta \lambda_i}$$

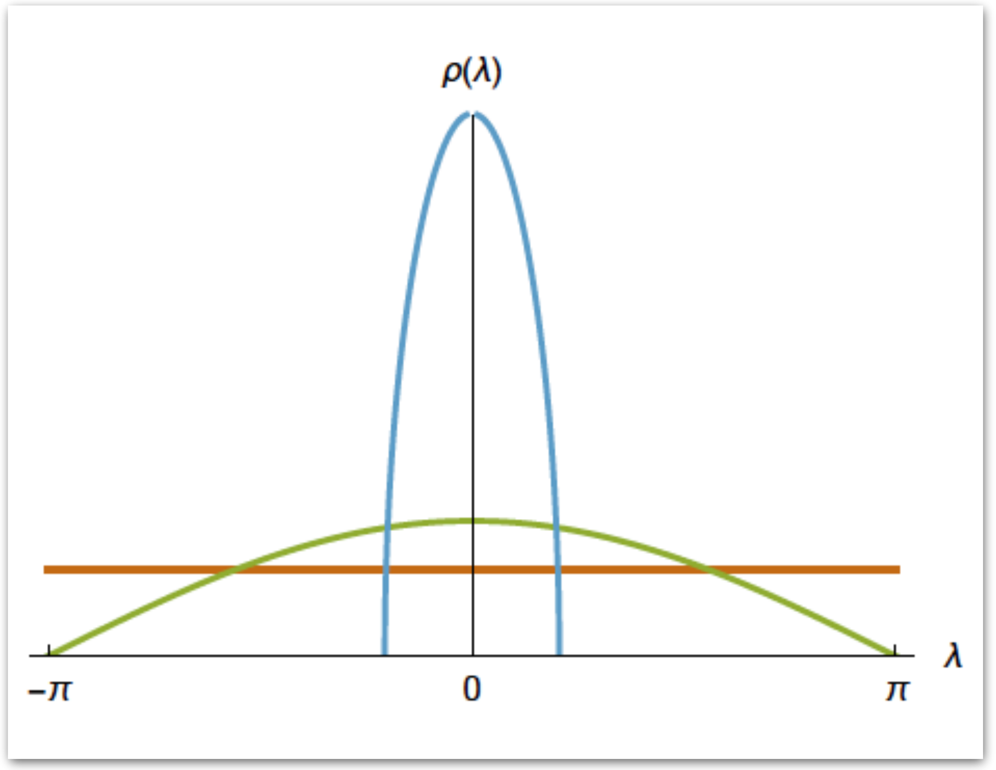
- Given the prominence of large N arguments in holography, we can expect to reproduce significant aspects of bulk gravity.

Saddle point densities

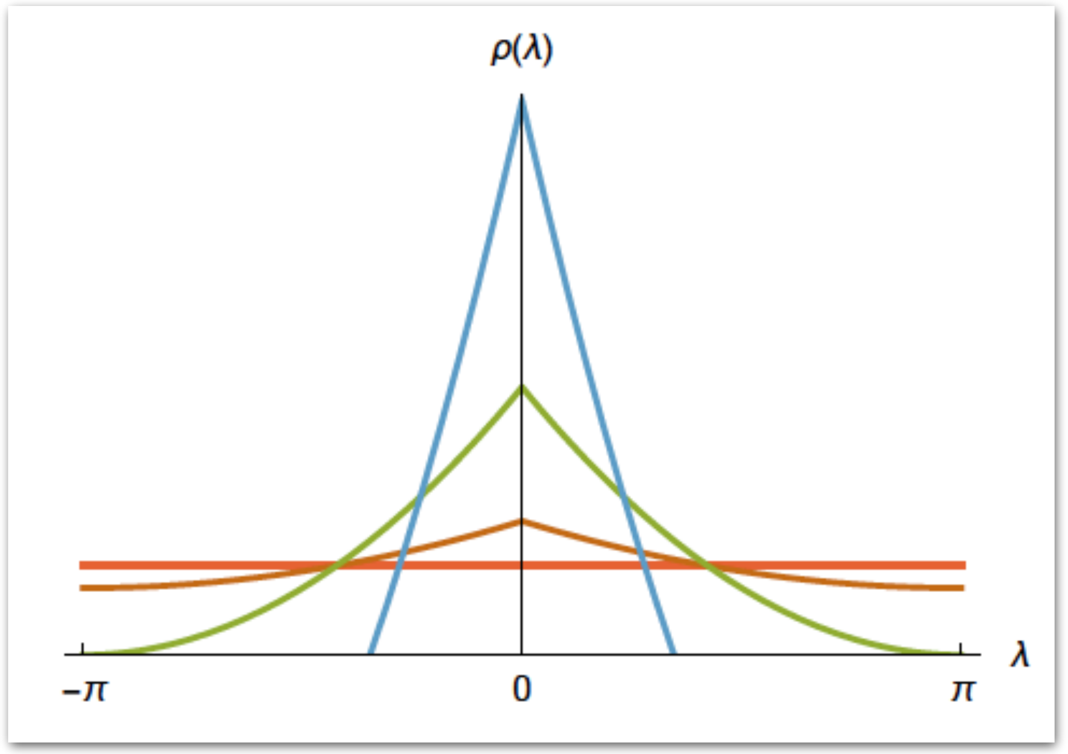
- The saddle point densities vary with temperature.
- Qualitative changes in the equilibrium density leads to phase transitions.
- The extreme high temperature limit typically simplifies the saddle point eigenvalue density to a delta function.

$$0 = \frac{\delta S_{\text{eff}}[\beta; \lambda_j]}{\delta \lambda_i}$$

Matrix model



Vector model



High T Critical T Low T Very low T

Correlators

- Correlators probe thermal equilibrium physics.
- Phase transitions affect correlators.
- The scale dependence of correlators probe the radial direction in the bulk.

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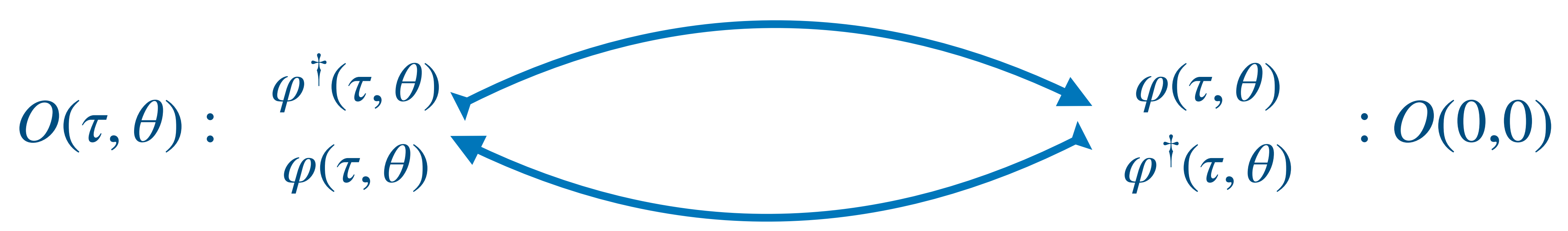
- The scale dependence of correlators probe the radial direction in the bulk.
- The momentum space two-point functions of singlet scalars can be obtained through constituents: fundamental and anti-fundamentals.
- To construct the propagator of the composite singlet scalar $O = \varphi^\dagger \varphi$, we sew two position space propagators together.

$$\Pi_{ab}(n,l) = \sum_{i=1}^{N} \frac{\Psi_i^a (\Psi_i^b)^*}{(\omega_n + \frac{\lambda_i}{\beta})^2 + E_l^2}$$

$$\Pi_{ab}(\tau,\theta) = \sum_{i=1}^{N} \Psi_i^a (\Psi_i^b)^* e^{-i\tau\frac{\lambda_i}{\beta}} \left(\sum_{n=-\infty}^{\infty} \frac{e^{-in\lambda_i}}{(\cosh(\tau+\beta n) - \cos\theta)^\sigma} \right)$$



$$\langle O(\tau,\theta) O(0,0) \rangle = \Pi_{ab}(\tau,\theta)(\Pi_{ba}(\tau,\theta))^*$$

$$\langle O(\tau,\theta) O(0,0) \rangle = \sum_{i=1}^{N} \sum_{n,m=-\infty}^{\infty} \frac{e^{-i(n-m)\lambda_i}}{(\cosh(\tau+\beta n) - \cos\theta)(\cosh(\tau+\beta m) - \cos\theta)}$$

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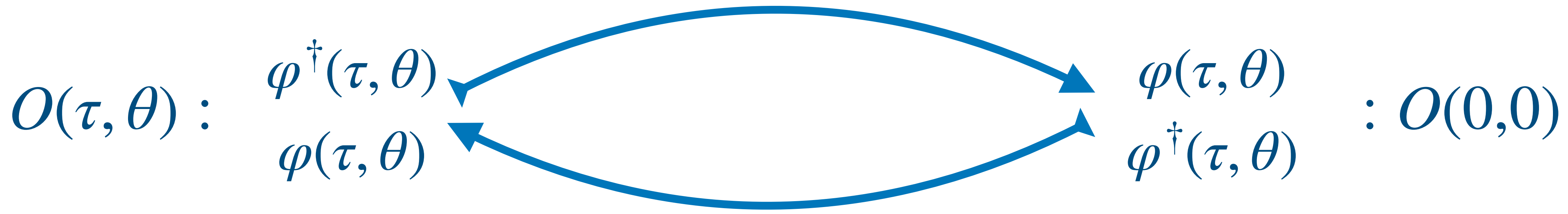
- To construct the propagator of the composite singlet scalar $O = \varphi^\dagger \varphi$, we sew two position space propagators together.
- The scale is set by β.



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The energy density

- The energy density is dual to the bulk graviton's time-time component.

- Euclidean time is compact: this graviton component measures fluctuations in this radius.

- It is analogous to a ”radion” scalar perturbing the circumference of a spatial dimension. It perturbs the compact imaginary time circumference related to temperature.

    - A “temporon”?

    - I will just call the operator T.

$$T_{\mu\nu} = \partial_{\{\mu}\varphi^\dagger\partial_{\nu\}}\varphi - g_{\mu\nu}\partial_\alpha\varphi^\dagger\partial^\alpha\varphi + 2\xi(G_{\mu\nu} + g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)|\varphi|^2$$

$$T_{00} \simeq \partial_0\varphi^\dagger\partial_0\varphi - \frac{1}{3}\left((\partial_0\partial_0\varphi^\dagger)\varphi + \varphi^\dagger(\partial_0\partial_0\varphi) + \sum_{i=1}^{3}\partial_i\varphi^\dagger\partial_i\varphi\right)$$

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More two-point functions

- The two-point functions of the singlet scalars are given by a sum over eigenvalue correlators.
- In the extreme high temperature limit, the eigenvalue distribution localises to $\lambda = 0$. The characteristic scale is $\beta \to 0$, so we focus on short distances.
- The correlator simplifies considerably.

$$\langle O(\tau,\theta)O(0,0)\rangle = \sum_{i=1}^{N} \sum_{n,m=-\infty}^{\infty} \frac{e^{-i(n-m)\lambda_i}}{(\cosh(\tau+\beta n)-\cos\theta)(\cosh(\tau+\beta m)-\cos\theta)}$$

$$\langle O(\tau,x)O(0,0)\rangle \simeq \frac{1}{4}\left(\sum_{m=-\infty}^{\infty} \frac{1}{(\tau+\beta m)^2+x^2}\right)^2 = \frac{\pi^2}{2x^2\beta^2}\frac{\sinh^2\frac{2\pi x}{\beta}}{\left(\cos\frac{2\pi\tau}{\beta}-\cosh\frac{2\pi x}{\beta}\right)^2} \equiv G_{OO}(\tau,x)$$

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- In this limit the higher dimension operators can be obtained from point-splitting constituents.

$$\langle T(\tau,x)O(0,0)\rangle \equiv \lim_{u\to v} D^T_{(u,v)}\langle O(\tau,x)O(0,0)\rangle \equiv G_{TO}(\tau,x)$$

- In practice we apply special differential operators to the basic singlet scalar-scalar correlator.

$$\langle T(\tau,x)T(0,0)\rangle \equiv \lim_{u\to v} D^T_{(u,v)}\langle O(\tau,x)T(0,0)\rangle \equiv G_{TT}(\tau,x)$$

Static correlators

- The full correlator contains all leading information about thermal equilibrium quantum gravity at $G_N \to 0$. Simplify even more:
    - Static thermal correlators represent how static perturbations respond to the presence of black holes/thermal equilibrium.
- We consider scalar perturbations $O_{\omega=0}(x)$ and temperature perturbations $T_{\omega=0}(x)$.

$$\left\langle O_0(x) O_0(0) \right\rangle \sim \int_0^\beta \mathrm{d}\tau \, \langle O(\tau, x) O(0,0) \rangle$$

$$= \frac{8\pi^2 \coth \frac{2\pi x}{\beta}}{\beta x^2}$$

$$\left\langle T_0(x) O_0(0) \right\rangle \sim \int_0^\beta \mathrm{d}\tau \, \langle T(\tau, x) O(0,0) \rangle$$

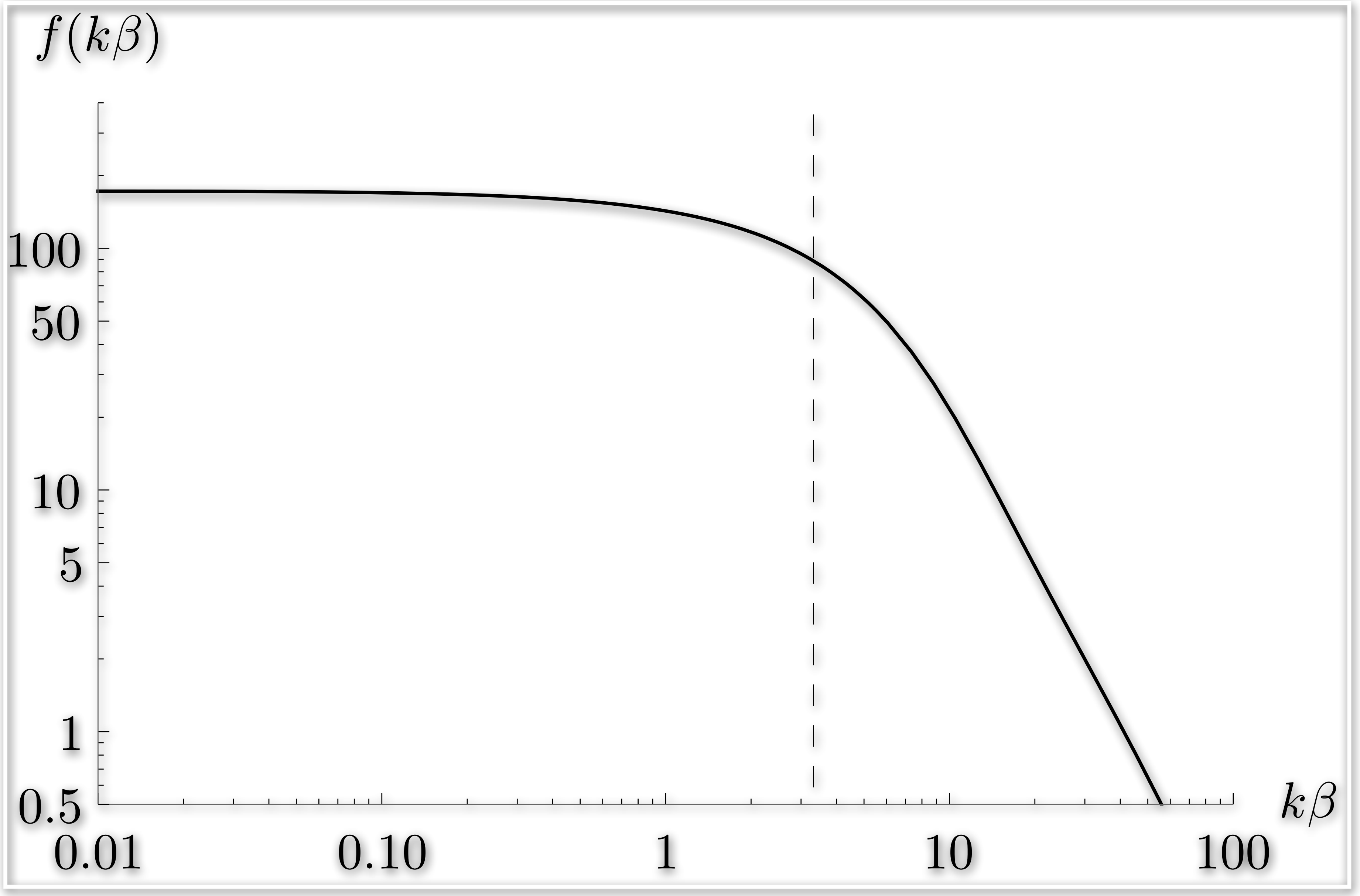
$$= -\frac{\pi^2}{6x^4 \beta^3 \sinh^2 \frac{2\pi x}{\beta}} \left[\beta \left(8\pi x + \beta \frac{\cosh \frac{6\pi x}{\beta}}{\sinh \frac{2\pi x}{\beta}} \right) - (32\pi^2 x^2 + \beta^2) \coth \frac{2\pi x}{\beta} \right]$$

The mixed correlator

- Scalar and temperature perturbations mix.
- The characteristic scale is $x \sim \beta$.
- Fourier transform to k space.
- $G_{TO}(\omega = 0, k) = -\dfrac{1}{\beta^2} f(k\beta)$
  - The mixed correlator is constant for $k\beta \ll 1$.
  - It falls as an inverse power law for $k\beta \gg 1$.

$$\langle T_0(x) O_0(0) \rangle \sim \int_0^\beta \mathrm{d}\tau \, \langle T(\tau, x) O(0,0) \rangle$$

$$= -\frac{\pi^2}{6x^4\beta^3 \sinh^2 \frac{2\pi x}{\beta}} \left[\beta \left(8\pi x + \beta \frac{\cosh \frac{6\pi x}{\beta}}{\sinh \frac{2\pi x}{\beta}} \right) - (32\pi^2 x^2 + \beta^2) \coth \frac{2\pi x}{\beta} \right]$$



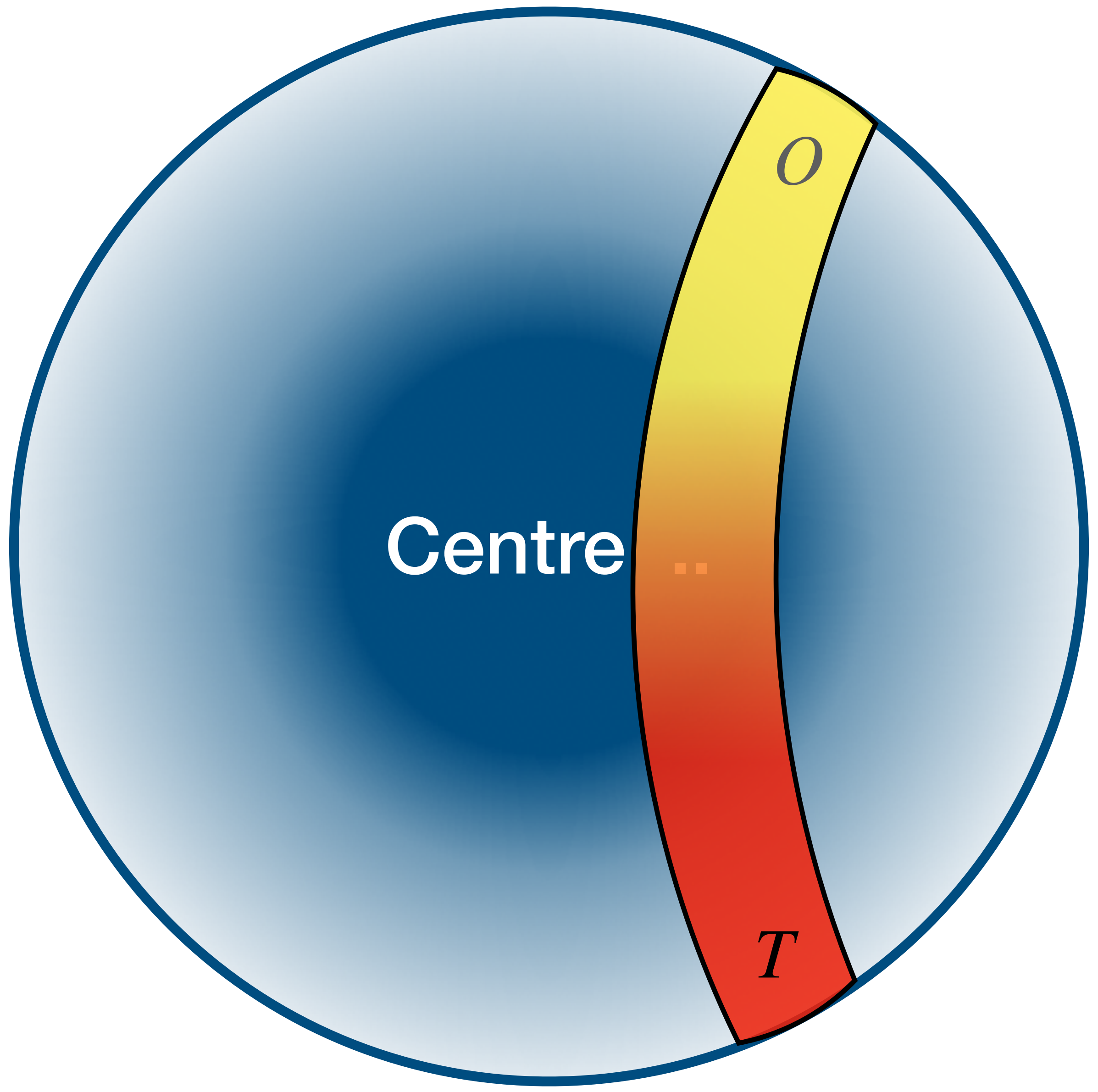
Tidal excitation

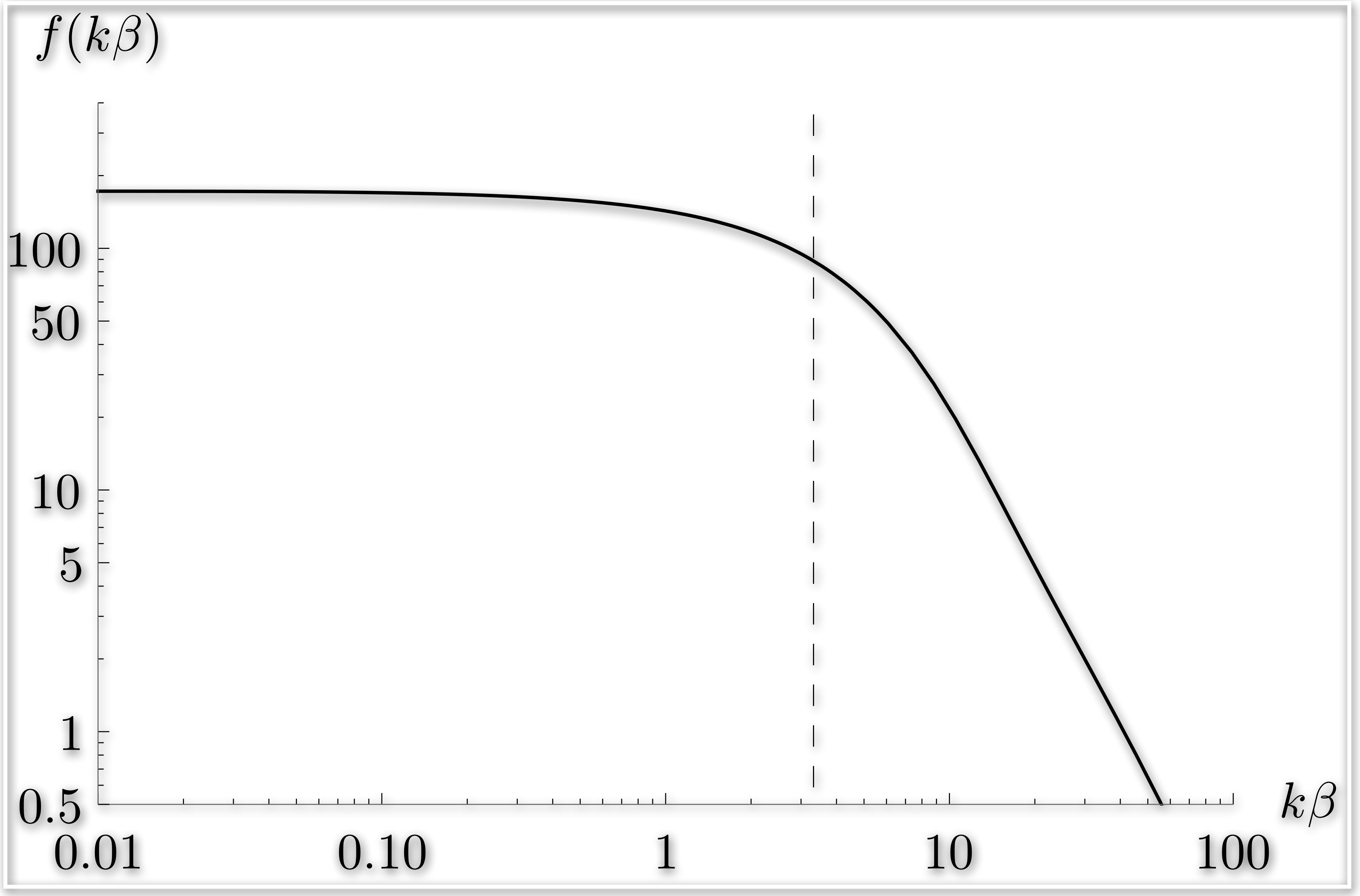
- Mixing of otherwise independent fields is often induced by non-trivial backgrounds. Here, a compact object seems to induce conversion between scalars and gravitons.

- In the bulk, a constant mixed correlator at $k\beta \ll 1$ means constant mixing of temperature and scalar fluctuations in the deep interior.

- The mixing grows as β^{-2} with temperature.

- For large $k\beta \gg 1$, the fall-off is the expected behaviour in thermal AdS. Only at large boundary distances can we detect the presence of a black hole.





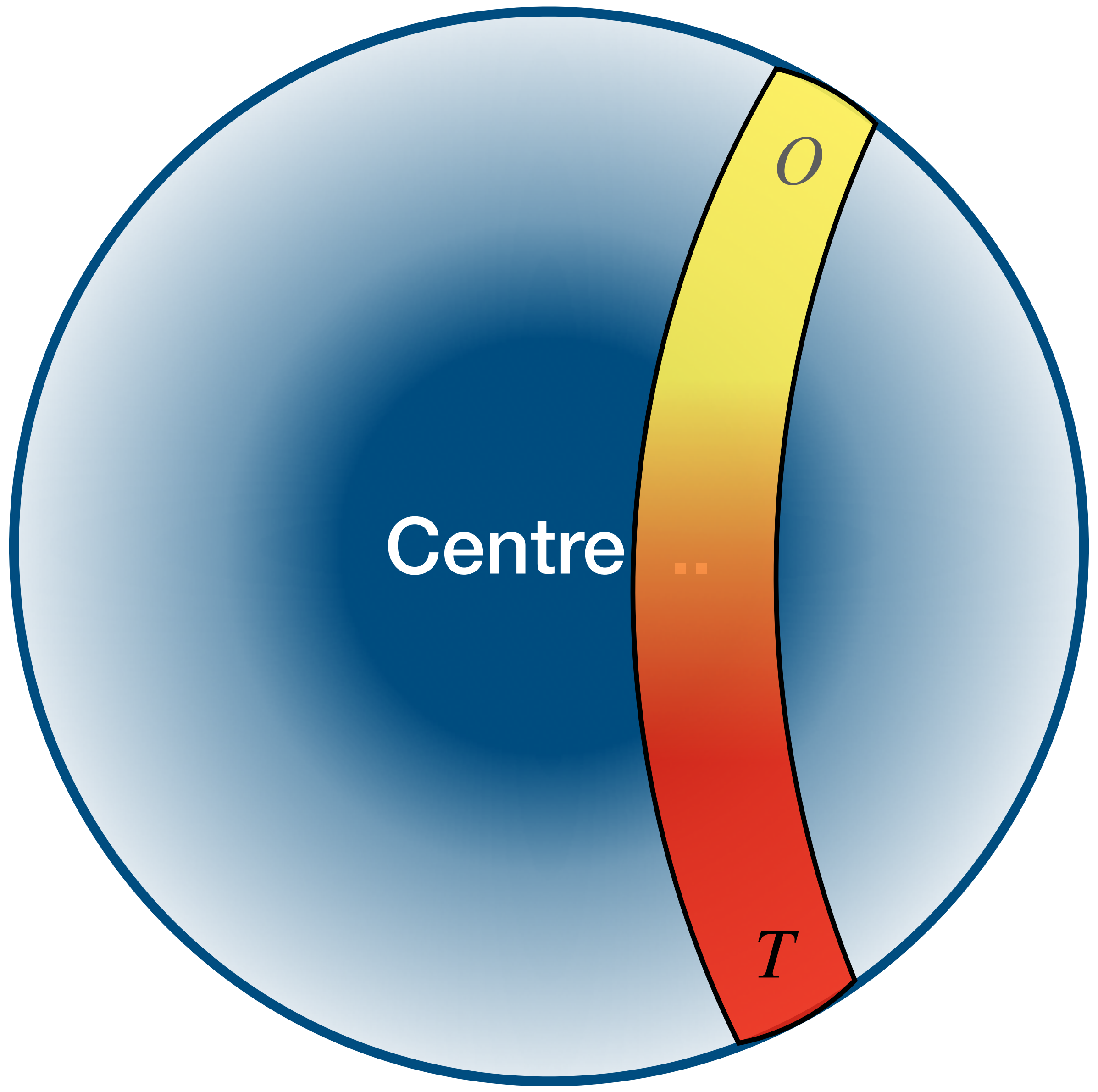
Tidal excitation

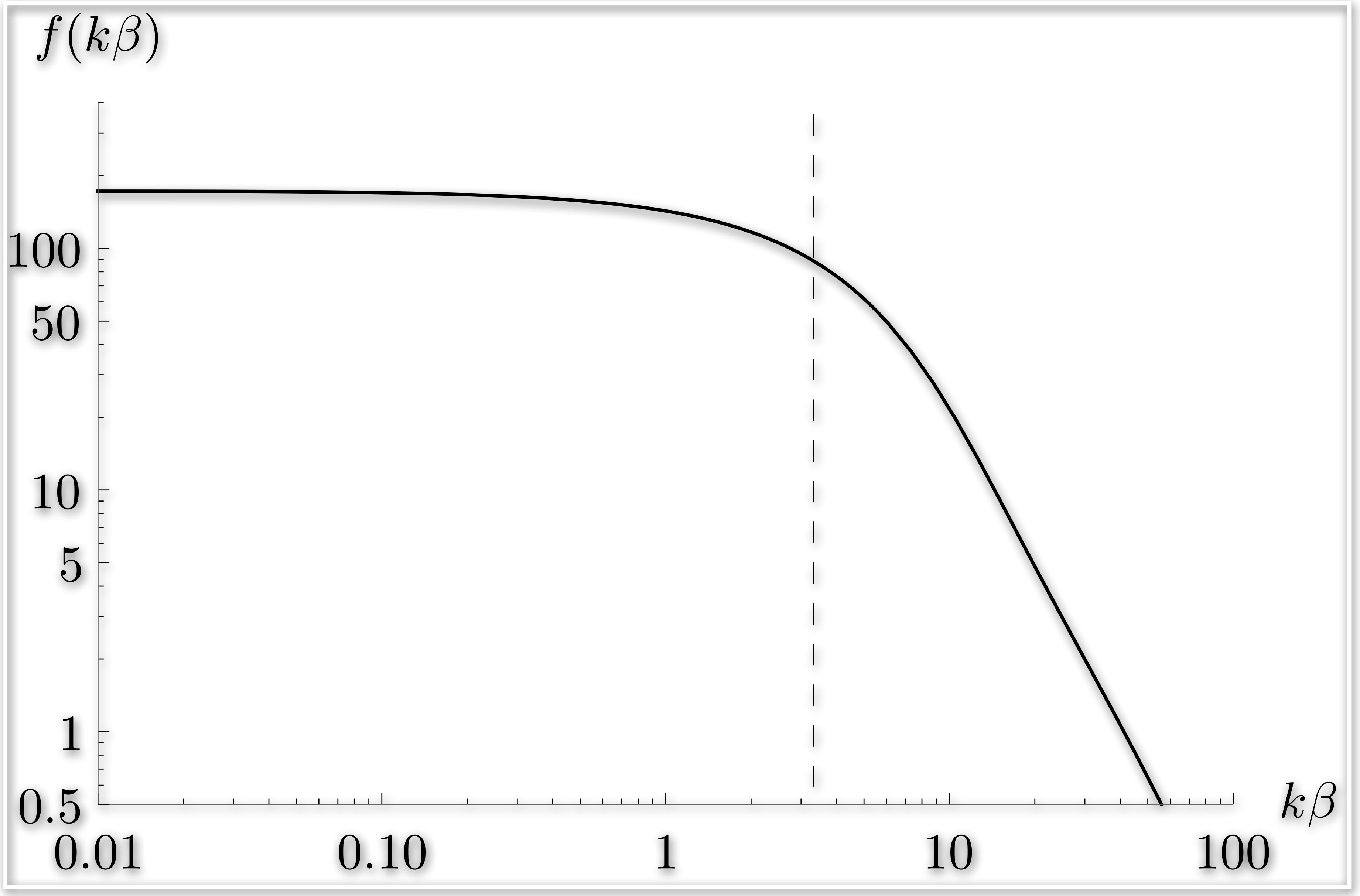
- Mixing of otherwise independent fields is often induced by non-trivial backgrounds. Here, a compact object seems to induce conversion between scalars and gravitons.

- Technically, the mixing arises because the same constituent fields build up both

  $O = \varphi^\dagger \varphi$ and $T = \lim_{u \to v} D^T_{(u,v)} \{\varphi^\dagger \varphi\}$

- This is leads to relations between large N theory operators, allowing for mixing.





Conclusions and Outlook

- ✓ Mixing of related large N operators is induced by a thermal equilibrium background.
- ✓ For extremely high temperature the static effect is large, especially for low boundary momentum/deep bulk interior.
- ✓ Correlators are consistent with a compact bulk object emulating a black hole, even in toy models of higher spin gravity or tensionless string theory.
- ✓ Strong gravity in the deep bulk appears to be vital for mixing.
- ✓ Mixing discloses relationships between different bulk fields, as expected in string theory, and also in higher spin theory.

- Exact correlator expressions are under way for all temperatures and separations on the sphere.
- If these expressions can be processed, the will yield full disclosure of the scattering by quantum black hole like objects.
- Tidal excitation of incident composite fields is likely to be substantial only in the presence of strong gravity.
- Then, mixing is an ideal probe of regions inside the photon sphere, around the horizon, and perhaps close to the resolved singularity of quantum black holes.