The string dual of free N=4 SYM

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AdS/CFT correspondence

The relation between the parameters of string theory on AdS and the dual CFT is

$$\left(\frac{R}{l_{\rm Pl}}\right)^4 = N \qquad g_{\rm string} = g_{\rm YM}^2 \qquad \left(\frac{R}{l_{\rm s}}\right)^4 = g_{\rm YM}^2 N = \lambda$$

AdS radius in Planck units

AdS radius in string units

't Hooft parameter

Tensionless limit

In particular, weakly coupled gauge theory corresponds to the tensionless regime of string theory

$$\left(\frac{R}{l_{\mathrm{Pl}}}\right)^4 = N \qquad g_{\mathrm{string}} = g_{\mathrm{YM}}^2 \qquad \left(\frac{R}{l_{\mathrm{s}}}\right)^4 = g_{\mathrm{YM}}^2 N = \lambda$$

large

small

$l_{\mathrm{s}} \longrightarrow \infty$ `tensionless strings'

[Sundborg '01] [Witten '01]
[Sezgin,Sundell '01]

Tensionless limit

This is the regime where AdS/CFT becomes perturbative:

tensionless strings on AdS	⟷	weakly coupled/free SYM theory

- very stringy (far from sugra)
- higher spin symmetry
- maximally symmetric phase of string theory

Tensionless limit

This is the regime where AdS/CFT becomes perturbative:

tensionless strings on AdS $\longleftrightarrow$ weakly coupled/free SYM theory

- very stringy (far from sugra)
- higher spin symmetry
- maximally symmetric phase of string theory

Could it have a free worldsheet description?

AdS3 review

For example, in the 3d case, the AdS/CFT duality relates string theory on

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$$

to a CFT that is on the same moduli space of CFTs as the symmetric orbifold theory

$$\mathrm{Sym}_N(\mathbb{T}^4) \equiv (\mathbb{T}^4)^N / S_N$$

[Maldacena '97], …. see e.g. [David et.al. '02]

AdS3 review

The analogue of free SYM is the symmetric orbifold theory itself. It has a tensionless (k=1) string dual with $\mathrm{AdS}_3 \times \mathrm{S}^3$ worldsheet theory described by

$$\boxed{\begin{array}{c}\text{4 symplectic bosons \& 4 free fermions} \\ \updownarrow \\ \text{free field realisation of } \mathfrak{psu}(1,1|2)_1\end{array}}$$

hybrid formalism of [Berkovits, Vafa, Witten '99]

Physical degrees of freedom come from spectrally flowed representations: matches precisely with single particle spectrum of dual symmetric orbifold.

[Eberhardt, MRG, Gopakumar '18]

AdS5 proposal

Similarly, free N=4 SYM in 4d should be dual to tensionless strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$: we **propose** `twistorial' worldsheet description via

$$\boxed{\begin{array}{c}\text{8 symplectic bosons \& 8 free fermions} \\ \updownarrow \\ \text{free field realisation of } \mathfrak{psu}(2,2|4)_1\end{array}}$$

similar to twistor string of [Berkovits '04]

Key ingredient: spectrally flowed representations.

Natural quantisation leads to a `reduced model' whose spectrum matches exactly that of free N=4 SYM.

[MRG, Gopakumar '21]

Key ingredients

More concretely, this worldsheet theory consists of what can be interpreted as components of ambitwistor fields

see also [Berkovits, '04]

$$\begin{aligned} Y_I &= (\mu^\dagger_\alpha, \lambda^\dagger_{\dot\alpha}, \psi^\dagger_a) \qquad && \alpha, \dot\alpha \in \{1,2\} \\ Z^I &= (\lambda^\alpha, \mu^{\dot\alpha}, \psi^a) && a \in \{1,2,3,4\} \end{aligned}$$

with defining relations

$$[\lambda^\alpha_r, (\mu^\dagger_\beta)_s] = \delta^\alpha_\beta\, \delta_{r,-s} \ , \qquad [\mu^{\dot\alpha}_r, (\lambda^\dagger_{\dot\beta})_s] = \delta^{\dot\alpha}_{\dot\beta}\, \delta_{r,-s} \ ,$$
$$\{\psi^a_r, (\psi^\dagger_b)_s\} = \delta^a_b\, \delta_{r,-s} \ .$$

There is an overall u(1) field that needs to be gauged

$$\mathcal{C} = \frac{1}{2} Y_I Z^I = \frac{1}{2}\left(\mu^\dagger_\gamma\, \lambda^\gamma + \lambda^\dagger_{\dot\gamma}\, \mu^{\dot\gamma} + \psi^\dagger_c \psi^c\right) \ .$$

Key ingredients

Postulate: in the w-spectrally flowed sector, physical state conditions remove all but the `wedge modes'

$$\boxed{\mu^{\dot\alpha}_r\ , \ (\mu^\dagger_\alpha)_r\ , \ (\psi^\dagger_{1,2})_r\ , \ \psi^{3,4}_r\ , \quad \left(-\tfrac{w-1}{2} \le r \le \tfrac{w-1}{2}\right)}$$

These modes act non-trivially on spectrally flowed vacuum $|0\rangle_w$.

On the resulting (wedge) Fock space, we then need to impose the residual gauge conditions

$$\mathcal{C}_n\phi = 0 \ \ (n \geq 0) \qquad (L_0 + pw)\phi = 0 \ \ (p \in \mathbb{Z}) \ .$$

[MRG, Gopakumar '21]

Key ingredients

Resulting spectrum reproduces exactly that of free N=4 SYM in 4d in planar limit.

More specifically, wedge modes can be thought of as momentum modes of w position space generators

$$\hat{Z}^I{}_j = \frac{1}{\sqrt{w}} \sum_{r=-(w-1)/2}^{(w-1)/2} Z^I_r \, e^{-2\pi i \frac{rj}{w}} \qquad (j = 1, \ldots, w) \ ,$$

and similarly for $(\hat{Y}_I)_j$. These position modes then satisfy

$$[\hat{Z}^I_{j_1}, (\hat{Y}^\dagger_J)_{j_2}]_\pm = \delta^I_J \, \delta_{j_1,j_2} \ .$$

[MRG, Gopakumar '21]

Key ingredients

The residual gauge conditions imply

$$\mathcal{C}_n \phi = 0 \ \ (n \geq 0) \qquad\qquad (L_0 + pw)\phi = 0 \ \ (p \in \mathbb{Z}) \ .$$

at each site j: $\hat{\mathcal{C}}_j = 0$

singleton rep

cyclic invariance

Get w-fold tensor product of singleton rep of $\mathfrak{psu}(2,2|4)$, subject to cyclicity condition: spectrum of free N=4 SYM.

[MRG, Gopakumar '21]

Key ingredients

Get w-fold tensor product of singleton rep of $\mathfrak{psu}(2,2|4)$, subject to cyclicity condition: spectrum of free N=4 SYM.

w-spectrally flowed sector $\longleftrightarrow$ $\mathrm{Tr}\Big(\underbrace{S_1 \cdots S_w}_{w \text{ letters}}\Big)$

$$S_i = \{\partial^s \phi^i, \partial^s \Psi_{\alpha a}, \partial^s \mathcal{F}_{\alpha\beta}, \partial^s \mathcal{F}_{\dot\alpha\dot\beta}\}$$

String bit picture!

$$\hat{Y} = (\hat{\mu}^\dagger_\alpha, \hat{\lambda}^\dagger_{\dot\alpha}, \hat{\psi}^\dagger_a)\ , \quad \hat{Z} = (\hat{\lambda}^\alpha, \hat{\mu}^{\dot\alpha}, \hat{\psi}^a)$$

twistor-valued string bits

[MRG, Gopakumar '21]

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2. Review of AdS3
3. Generalisation to AdS5
4. Conclusions and Outlook

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Hybrid formalism

[Berkovits, Vafa, Witten '99]

AdS3 theory at k=1 best described in hybrid formalism: for pure NS-NS flux, hybrid string consists of WZW model based on

$$\mathfrak{psu}(1,1|2)_k$$

together with the (topologically twisted) sigma model for T4. For generic k, this description agrees with the NS-R description a la Maldacena-Ooguri.

[Troost '11], [MRG, Gerigk '11]
[Gerigk '12]

Free field realisation

The level k=1 theory has a free field realisation

$$\mathfrak{u}(1,1|2)_1 \;\cong\; \begin{cases} 4 \ \textbf{symplectic bosons} \ \xi^{\pm}\,,\ \eta^{\pm} \\ 4 \ \textbf{real fermions} \ \psi^{\pm}\,,\ \chi^{\pm} \end{cases}$$

with

$$\{\psi^{\alpha}_r, \chi^{\beta}_s\} = \epsilon^{\alpha\beta}\,\delta_{r,-s}\ , \qquad [\xi^{\alpha}_r, \eta^{\beta}_s] = \epsilon^{\alpha\beta}\,\delta_{r,-s}\ .$$

Generators of $\mathfrak{u}(1,1|2)_1$ are bilinears in these free fields.

In order to reduce this to $\mathfrak{psu}(1,1|2)_1$ one has to gauge by the `diagonal' u(1) field

$$Z = \tfrac{1}{2}\big(\eta^-\xi^+ - \eta^+\xi^- + \chi^-\psi^+ - \chi^+\psi^-\big)\ .$$

Free field realisation

The only highest weight representations are:

- NS sector: all fields half-integer moded
- R sector: all fields integer moded

Here positive modes annihilate ground state.

In R sector, ground states form representation of zero modes: `singleton' representation of $\mathfrak{psu}(1,1|2)$.

Spectral flow

The full worldsheet spectrum consists of this R-sector representation, together with its spectrally flowed images. Here spectral flow comes from

[Henningson et.al. '91]
[Maldacena, Ooguri '00]

$$\tilde{\xi}^{\pm}_r = \xi^{\pm}_{r\pm\frac{w}{2}} \ , \qquad \tilde{\eta}^{\pm}_r = \eta^{\pm}_{r\pm\frac{w}{2}} \ , \qquad \tilde{\psi}^{\pm}_r = \psi^{\pm}_{r\mp\frac{w}{2}} \ , \qquad \tilde{\chi}^{\pm}_r = \chi^{\pm}_{r\mp\frac{w}{2}} \ ,$$

consider above R sector rep, for tilde modes $\longrightarrow$ interpret in terms of `untilde' modes

For w>1: not highest weight representation any longer.

[Spectral flow of NS-sector: R-sector.]

Physical spectrum

Since $\mathfrak{psu}(1,1|2)_1$ has many (!) null-vectors, it has effectively only 2 bosonic + fermionic oscillator degrees of freedom.

Thus after imposing the physical state conditions, only the degrees of freedom of $\mathbb{T}^4$ survive, and we get exactly the (single-particle) spectrum of

$$\mathrm{Sym}_N(\mathbb{T}^4)$$

in the large N limit, where w-cycle twisted sector comes from w spectrally flowed sector.

[Eberhardt, MRG, Gopakumar '18]

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Ansatz for worldsheet

Given the structure of the free field realisation for the case of $\mathrm{AdS}_3 \times \mathrm{S}^3$, we have proposed that the dual to free N=4 SYM in 4d should be described by a worldsheet theory consisting of

[MRG, Gopakumar '21]

8 symplectic bosons
8 real fermions

They generate $\mathfrak{u}(2,2|4)_1$. After removing again an overall u(1), we get $\mathfrak{psu}(2,2|4)$: guarantees that dual spacetime theory has the correct symmetry.

Ansatz for worldsheet

More concretely, the worldsheet theory consists of what can be interpreted as components of ambitwistor fields

see also [Berkovits, '04]

$$
\begin{aligned}
Y_I &= (\mu^\dagger_\alpha, \lambda^\dagger_{\dot\alpha}, \psi^\dagger_a) \qquad && \alpha, \dot\alpha \in \{1,2\} \\
Z^I &= (\lambda^\alpha, \mu^{\dot\alpha}, \psi^a) \qquad && a \in \{1,2,3,4\}
\end{aligned}
$$

with defining relations

$$
[\lambda^\alpha_r, (\mu^\dagger_\beta)_s] = \delta^\alpha_\beta\, \delta_{r,-s} \ , \qquad [\mu^{\dot\alpha}_r, (\lambda^\dagger_{\dot\beta})_s] = \delta^{\dot\alpha}_{\dot\beta}\, \delta_{r,-s} \ ,
$$

$$
\{\psi^a_r, (\psi^\dagger_b)_s\} = \delta^a_b\, \delta_{r,-s} \ .
$$

Free fields on worldsheet

The bilinears

$$\mathcal{J}^I_J =: Y_J Z^I : \qquad \begin{array}{rcl} Y_I & = & (\mu^\dagger_\alpha, \lambda^\dagger_{\dot\alpha}, \psi^\dagger_a) \\ Z^I & = & (\lambda^\alpha, \mu^{\dot\alpha}, \psi^a) \end{array}$$

generate $\mathfrak{u}(2,2|4)_1$, and in order to obtain $\mathfrak{psu}(2,2|4)_1$ we need to gauge by the overall u(1) field

$$\mathcal{C} = \frac{1}{2} Y_I Z^I = \frac{1}{2} \left(\mu^\dagger_\gamma \lambda^\gamma + \lambda^\dagger_{\dot\gamma} \mu^{\dot\gamma} + \psi^\dagger_c \psi^c \right) \, .$$

[MRG, Gopakumar '21]

This is the current algebra version of oscillator construction of $\mathfrak{psu}(2,2|4)$ which enters into spin chain discussion.

see e.g. [Beisert thesis]

Spectral flow

As in the case for AdS_3, all non-trivial aspects come from spectral flow where now

$$
\begin{aligned}
(\tilde{\lambda}^{\alpha})_r &= (\lambda^{\alpha})_{r-w/2} \ , & (\tilde{\lambda}^{\dagger}_{\dot{\alpha}})_r &= (\lambda^{\dagger}_{\dot{\alpha}})_{r-w/2} \ , & \\
(\tilde{\mu}^{\dot{\alpha}})_r &= (\mu^{\dot{\alpha}})_{r+w/2} \ , & (\tilde{\mu}^{\dagger}_{\alpha})_r &= (\mu^{\dagger}_{\alpha})_{r+w/2} \ , & \\
(\tilde{\psi}^{a}_r) &= \psi^{a}_{r-w/2} \ , & (\tilde{\psi}^{\dagger}_{a})_r &= (\psi^{\dagger}_{a})_{r+w/2} & (a = 1,2) \ , \\
(\tilde{\psi}^{b}_r) &= \psi^{b}_{r+w/2} \ , & (\tilde{\psi}^{\dagger}_{b})_r &= (\psi^{\dagger}_{b})_{r-w/2} & (b = 3,4) \ .
\end{aligned}
$$

Starting from the usual NS-sector representation, the `untilde' modes act as

$$
\begin{aligned}
\mu^{\dot{\alpha}}_r \, |0\rangle_w = (\mu^{\dagger}_{\alpha})_r \, |0\rangle_w = (\psi^{\dagger}_{1,2})_r \, |0\rangle_w = \ \psi^{3,4}_r \, |0\rangle_w = 0 \ , \qquad & (r \geq \tfrac{w+1}{2}) \\
\lambda^{\alpha}_r \, |0\rangle_w = (\lambda^{\dagger}_{\dot{\alpha}})_r \, |0\rangle_w = (\psi^{1,2})_r \, |0\rangle_w = (\psi^{\dagger}_{3,4})_r \, |0\rangle_w = 0 \ , \qquad & (r \geq -\tfrac{w-1}{2})
\end{aligned}
$$

Wedge modes

Since

$$\mu_r^{\dot\alpha}\,|0\rangle_w = (\mu_\alpha^\dagger)_r\,|0\rangle_w = (\psi_{1,2}^\dagger)_r\,|0\rangle_w = \ \psi_r^{3,4}\,|0\rangle_w = 0\ , \qquad (r \geq \tfrac{w+1}{2})$$
$$\lambda_r^{\alpha}\,|0\rangle_w = (\lambda_{\dot\alpha}^\dagger)_r\,|0\rangle_w = (\psi^{1,2})_r\,|0\rangle_w = (\psi_{3,4}^\dagger)_r\,|0\rangle_w = 0\ , \qquad (r \geq -\tfrac{w-1}{2})$$

the non-zero modes acting on $|0\rangle_w$ are the wedge modes

$$\mu_r^{\dot\alpha}\ ,\ (\mu_\alpha^\dagger)_r\ ,\ (\psi_{1,2}^\dagger)_r\ ,\ \psi_r^{3,4}\ ,\quad (-\tfrac{w-1}{2} \leq r \leq \tfrac{w-1}{2})$$

as well as the `out-of-the-wedge' modes

$$Z_r^I \text{ and } (Y_J)_r \qquad \text{with } r \leq -\tfrac{w+1}{2}$$

Wedge modes

the non-zero modes acting on $|0\rangle_w$ are the wedge modes

$$\boxed{\mu^{\dot\alpha}_r \ , \ (\mu^\dagger_\alpha)_r \ , \ (\psi^\dagger_{1,2})_r \ , \ \psi^{3,4}_r \ , \qquad (-\tfrac{w-1}{2} \leq r \leq \tfrac{w-1}{2})}$$

as well as the `out-of-the-wedge' modes

$$Z^I_r \text{ and } (Y_J)_r \qquad \text{with } r \leq -\tfrac{w+1}{2}$$

<u>Postulate</u>: physical state conditions (N=4 critical string) remove all out-of-the-wedge modes. [MRG, Gopakumar '21]

Retain only generalised zero modes = wedge modes.

cf. [Dolan, Goddard '07], [Nair '08]

Wedge modes

On the resulting (wedge) Fock space, we finally need to impose the residual gauge conditions [MRG, Gopakumar '21]

$$\mathcal{C}_n \phi = 0 \;\; (n \geq 0) \qquad (L_0 + pw)\phi = 0 \;\; (p \in \mathbb{Z}) \; .$$

similar to Virasoro condition
in light-cone gauge

$$L_0 \sim -2\, p^- p^+$$

with

$$2\, p^- = \sum_{n=-\infty}^{\infty} N_n \sqrt{\mu^2 + \frac{n^2}{(\alpha' p^+)^2}} \sim N_{\text{tot}} \quad \text{and} \quad p^+ \cong w$$

[Berenstein, Maldacena, Nastase '02]

Spacetime spectrum

[MRG, Gopakumar ’21]

As explained before, this **reproduces then exactly the single-trace spectrum of free SYM in 4d**, where now the spectral flow parameter w is to be identified with

$$w \text{ units of spec. flow} \longleftrightarrow \mathrm{Tr}\big(\underbrace{S_1 \cdots S_w}_{\text{w fundamental SYM fields}}\big)$$

For small levels we have also confirmed this by explicit calculations.

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Conclusions and Outlook

The free field realisation of the $\mathrm{AdS}_3 \times \mathrm{S}^3$ worldsheet theory dual to the symmetric orbifold suggests a natural generalisation to $\mathrm{AdS}_5 \times \mathrm{S}^5$.

With some assumptions about the structure of the physical state conditions, we have managed to reproduce the exact single-trace spectrum of free SYM in 4d from our worldsheet model.

This **opens the door for a proof of the AdS/CFT correspondence** for this most relevant case.

Future directions

- Understand physical state condition from first principles.
- Study structure of correlation functions for AdS_5 .
- Explore novel perspective on N=4 spectrum.
  cf [Basso, Komatsu, Vieira '15]
- Analyse perturbation away from free case.
- ….

Thank you!

Explicit states

[MRG, Gopakumar '21]

From this worldsheet perspective, the physical states all seem to be generated by DDF-like operators

$$S_m^{\mathbf{a}} \equiv ({S_I}^J)_m = \sum_{r=m-\frac{w-1}{2}}^{\frac{w-1}{2}} (Y_I)_r \, (Z^J)_{m-r}$$

interesting algebraic structure similar to Yangian

In particular, zero modes generate $\mathfrak{u}(2,2|4)$: physical states fall into representations of $\mathfrak{psu}(2,2|4)$.

Acting on ground state $|0\rangle_w$ generate full BPS multiplet

$$L_0 = 0 : \qquad \big(\underbrace{0,0}_{\mathfrak{su}(2)\oplus\mathfrak{su}(2)} ; \underbrace{[0,w,0]}_{\mathfrak{su}(4)} \big)_w$$

$\mathcal{D}_0$ eigenvalue

Explicit states

[MRG, Gopakumar '21]

- w=0: only the vacuum state survives — $\mathbf{1}$ in SYM
- w=1: wedge modes = zero modes: BPS singleton representation — absent in su(N).
- w=2: $L_0 = 0 :$ BPS rep. $\big(0,0;[0,2,0]\big)_2$

  $L_0 = -2 :$ Konishi multiplet $\big(0,0;[0,0,0]\big)_2$

  generated from hwv $|\mathrm{K}\rangle = (\psi_1^\dagger)_{\frac{1}{2}}(\psi_2^\dagger)_{\frac{1}{2}}\psi^3_{\frac{1}{2}}\psi^4_{\frac{1}{2}}|0\rangle_2 \sim S_1^{\mathbf{a}}S_1^{\mathbf{b}}|0\rangle_2$

  $L_0 = -2p :$ hs multiplet $\big(p-1,p-1;[0,0,0]\big)_{2p}$

  generated from hwv $\prod_{i=1}^{2p-2}(\mu^\dagger_{\alpha_i})_{\frac{1}{2}}\,\mu^{\dot\alpha_i}_{\frac{1}{2}}\,|\mathrm{K}\rangle = \prod_{i=1}^{2p-2} S_1^{\alpha_i\dot\alpha_i}\,|\mathrm{K}\rangle$

Explicit states

[MRG, Gopakumar '21]

- w=3: structure is quite complicated…

  but we have enumerated the low-lying states and compared to the N=4 SYM spectrum (for $\mathcal{D}_0 \leq 4$)

Δ	$(j,\bar{j})$	$SU(4)$	$\mathcal{O}$
2	(0,0)	$[0,0,0]+[0,2,0]=\mathbf{1}+\mathbf{20}$	$\mathrm{Tr}\,\phi^{((i_1}\phi^{i_2))}$
3	(0,0)	$[0,1,0]+[0,3,0]=\mathbf{6}+\mathbf{50}$	$\mathrm{Tr}\,\phi^{((i_1}\phi^{i_2}\phi^{i_3))}$
	(0,0)	$[0,0,2]+[0,0,2]=\mathbf{10}_s+\mathbf{10}_c$	$\mathrm{Tr}\,\phi^{[i_1}\phi^{i_2}\phi^{i_3]}$
	(0,0)	$[2,0,0]+[0,0,2]=\mathbf{10}_s+\mathbf{10}_c$	$\mathrm{Tr}\,\lambda_\alpha^{(A}\lambda^{B)\alpha}+\mathrm{h.c.}$
	(1,0)	$[0,1,0]=\mathbf{6}$	$\mathrm{Tr}\,F_{\alpha\beta}\,\phi^i$
	(1,0)	$[0,1,0]=\mathbf{6}$	$\mathrm{Tr}\,\lambda_{(\alpha}^{[A}\lambda_{\beta)}^{B]}$
	$(\frac{1}{2},\frac{1}{2})^*$	$[1,0,1]=\mathbf{15}$	$\mathrm{Tr}\,\phi^{[i_1}\partial_\mu\phi^{i_2]}$
	$(\frac{1}{2},\frac{1}{2})^*$	$[0,0,0]+[1,0,1]=\mathbf{1}+\mathbf{15}$	$\mathrm{Tr}\,\lambda_\alpha^A\bar{\lambda}_{\dot{\beta}B}$
4	(0,0)	$[0,0,0]+[0,2,0]+[0,4,0]=\mathbf{1}+\mathbf{20}+\mathbf{105}$	$\mathrm{Tr}\,\phi^{((i_1}\phi^{i_2}\phi^{i_3}\phi^{i_4))}$
	(0,0)	$[0,0,0]+[0,2,0]+[2,0,2]=\mathbf{1}+\mathbf{20}+\mathbf{84}$	$\mathrm{Tr}\,\phi^{[i_1}\phi^{((i_2]}\phi^{[i_3))}\phi^{i_4]}$
	(0,0)	$[1,0,1]+[0,1,2]+[2,1,0]=\mathbf{15}+\mathbf{45}_s+\mathbf{45}_c$	$\mathrm{Tr}\,\phi^{[i_1}\phi^{i_2}\phi^{((i_3]}\phi^{i_4))}$
	(0,0)	$2([000]+[1,0,1]+[0,2,0])=2(\mathbf{1}+\mathbf{15}+\mathbf{20})$	$\mathrm{Tr}\,\lambda_\alpha^{[A}\lambda^{B]\alpha}\phi^i+\mathrm{h.c.}$
	(0,0)	$[1,0,1]+[0,1,2]+[2,1,0]=2\cdot\mathbf{15}+\mathbf{45}_s+\mathbf{45}_c$	$\mathrm{Tr}\,\lambda_\alpha^{(A}\lambda^{B)\alpha}\phi^i+\mathrm{h.c.}$
	(0,0)	$2[0,0,0]=2\cdot\mathbf{1}$	$\mathrm{Tr}\,F^2,\,\mathrm{Tr}\,F\widetilde{F}$
	(1,0)	$[000]+[1,0,1]+[0,2,0]=\mathbf{1}+\mathbf{15}+\mathbf{20}$	$\mathrm{Tr}\,\lambda_{(\alpha}^{[A}\lambda_{\beta)}^{B]}\phi^i$
	(1,0)	$[1,0,1]+[2,1,0]=\mathbf{15}+\mathbf{45}_s$	$\mathrm{Tr}\,\lambda_{(\alpha}^{(A}\lambda_{\beta)}^{B)}\phi^i$
	(1,0)	$[0,0,0]+[1,0,1]+[0,2,0]=\mathbf{1}+\mathbf{15}+\mathbf{20}$	$\mathrm{Tr}\,F_{\alpha\beta}\,\phi^{i_1}\phi^{i_2}$
	$(\frac{1}{2},\frac{1}{2})$	$2[1,1,1]+2[0,1,0]=2\cdot\mathbf{6}+2\cdot\mathbf{64}$	$\mathrm{Tr}\,\partial_\mu\phi^{((i_1}\phi^{[i_2))}\phi^{i_3]}$
	$(\frac{1}{2},\frac{1}{2})$	$4[010]+2[0,0,2]+2[2,0,0]+2[1,1,1]$ $=4\cdot\mathbf{6}+2\cdot\mathbf{10}_s+2\cdot\mathbf{10}_c+2\cdot\mathbf{64}$	$\mathrm{Tr}\,\lambda_\alpha^A\bar{\lambda}_{\dot{\beta}B}\phi^i,\,\mathrm{Tr}\,\lambda_\alpha^A\phi^i\bar{\lambda}_{\dot{\beta}B}$
	$(\frac{3}{2},\frac{1}{2})^*$	$[2,0,0]=\mathbf{10}_s$	$\mathrm{Tr}\,\lambda_{(\alpha}^{(A}\partial_{\dot{\alpha}\beta}\lambda_{\gamma)}^{B)}$
	$(\frac{3}{2},\frac{1}{2})^*$	$[0,1,0]=\mathbf{6}$	$\mathrm{Tr}\,\partial_{\dot{\alpha}(\gamma}F_{\alpha\beta)}\phi^i$
	(2,0)	$[0,0,0]=\mathbf{1}$	$\mathrm{Tr}\,F_{(\alpha\beta}F_{\gamma\delta)}$
	$(1,1)^*$	$[0,0,0]=\mathbf{1}$	$\mathrm{Tr}\,F_{\alpha\beta}F_{\dot{\alpha}\dot{\beta}}$
	$(1,1)^*$	$[0,0,0]+[0,2,0]=\mathbf{1}+\mathbf{20}$	$\mathrm{Tr}\,\partial_{(\mu}\phi^{((i_1}\partial_{\nu)}\phi^{i_2))}$
	$(1,1)^*$	$[0,0,0]+[1,0,1]=\mathbf{1}+\mathbf{15}$	$\mathrm{Tr}\,\lambda_{(\alpha}^A\partial_{\beta)(\dot{\beta}}\bar{\lambda}_{\dot{\alpha})B}$

Table 3: N=4 SYM at $\lambda=0$. Brackets denote antisymmetrization. Parentheses denote complete symmetrization when traces cannot appear. Double parentheses denote complete symmetrization not excluding traces.

and answer reproduces intricate spectrum of BPS & non-BPS $\mathfrak{psu}(2,2|4)$ multiplets.

from [Bianchi, Morales, Samtleben, '03]

Physical states for AdS3

As a consistency check we have also imposed this wedge construction in the case of AdS_3.

reproduces a subset of `compactification independent' states, e.g. the BPS state is the `upper' BPS state in the (w-1)-cycle twisted sector

$$\bar{\psi}^{1}_{-\frac{1}{2}}\bar{\psi}^{2}_{-\frac{1}{2}}|\mathrm{BPS}_{\mathrm{lower}}\rangle^{(w-1)}$$ with $$h = j = \frac{w}{2}$$

$h = j = \frac{w-2}{2}$

zero'th cohomology of 4d manifold

[MRG, Gopakumar '21]

see e.g. [David et.al. '02]