On massive spin 2 in the Fradkin-Vasiliev formalism

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Outlook

1. FV-formalism: massless versus massive
2. Massive spin 2 kinematics
3. Self-interaction
4. Gravitational interaction

FV-formalism: massless case

- Main objects are one-form (gauge) fields Φ and their gauge invariant two-forms (curvatures) $\mathcal{R}$. Free Lagrangian:

$$\mathcal{L}_0 \sim \sum \mathcal{R}\mathcal{R}$$

- The most general quadratic deformations for curvatures $\hat{\mathcal{R}} = \mathcal{R} + \Delta\mathcal{R}$:

$$\Delta\mathcal{R} \sim \sum \Phi\Phi \quad \Rightarrow \quad \delta\Phi \sim \Phi\xi$$

  Consistency conditions:

$$\delta\hat{\mathcal{R}} \sim \mathcal{R}\xi$$

- Interacting Lagrangian:

$$\mathcal{L} \sim \sum [\hat{\mathcal{R}}\hat{\mathcal{R}} + \mathcal{R}\mathcal{R}\Phi]$$

FV-formalism: massive case

- Besides the one-form fields Φ and their two-forms $\mathcal{R}$ we have zero-form (Stueckelberg) fields W and their gauge invariant one-forms $\mathcal{C}$. Free Lagrangian:

$$\mathcal{L}_0 \sim \sum [\mathcal{R}\mathcal{R} \oplus \mathcal{R}\mathcal{C} \oplus \mathcal{C}\mathcal{C}]$$

- Deformations:

$$\begin{aligned} \Delta\mathcal{R} \sim \Phi\Phi + \Phi W + WW \qquad &\Rightarrow \qquad \delta\Phi \sim \Phi\xi + W\xi \\ \Delta\mathcal{C} \sim \Phi W + WW \qquad &\Rightarrow \qquad \delta W \sim W\xi \end{aligned}$$

- There exists a lot of possible field redefinitions:

$$\Phi \Rightarrow \Phi + \Phi W + WW, \qquad W \Rightarrow W + WW$$

which can drastically change the representation of the interacting Lagrangian.

General statements

- General analysis of the metric-like gauge invariant approach to massive fields interactions (Boulanger et al 2018) provided two general statements:
  - There are always exist enough redefinitions of fields and gauge parameters to transform the vertex into completely abelian form:

$$\mathcal{L} \sim \sum \mathcal{R}\Phi\Phi \quad \Rightarrow \quad \sum \mathcal{R}\mathcal{R}\Phi$$

  - Using further (having even more derivatives) on can bring the vertex to the trivially gauge invariant form

$$\mathcal{L} \sim \sum \mathcal{R}\mathcal{R}\Phi \quad \Rightarrow \quad \sum \mathcal{R}\mathcal{R}\mathcal{R}$$

- Let us stress that all these redefinitions contain Stueckelberg fields and so do not change the results in the unitary gauge $W = 0$
- It was important that all fields are massive so that each gauge field Φ has its own Stueckelberg one W

Fields and gauge transformations

- In the frame-like gauge invariant formalism we use one-forms f^a, $\Omega^{[ab]}$ and Stueckelberg zero-forms π^a, $B^{[ab]}$
- There exists a number of possible field redefinitions already at quadratic level. We use them to separate the roles of the fields (gauge $\Leftrightarrow$ Stueckelberg)
- Gauge transformations:

$$
\begin{aligned}
\delta\Omega^{ab} &= D\eta^{ab} + \kappa e^{[a}\xi^{b]} \\
\delta f^a &= D\xi^a + \eta^{ab} e_b \\
\delta B^{ab} &= m\eta^{ab}, \qquad \delta\pi^a = m\xi^a
\end{aligned}
$$

Here e^a is a background frame and D is a Lorentz covariant derivative, $\kappa \sim \Lambda$

Gauge invariant curvatures and Lagrangian

- Each field has its own gauge invariant curvature

$$\begin{aligned} \mathcal{R}^{ab} &= D\Omega^{ab} + \kappa e^{[a} f^{b]} \\ \mathcal{T}^a &= Df^a + \Omega^{ab} e_b \\ \mathcal{B}^{ab} &= DB^{ab} - m\Omega^{ab} + \kappa e^{[a} \pi^{b]} \\ \Pi^a &= D\pi^a - mf^a + B^{ab} e_b \end{aligned}$$

- They satisfy the corresponding differential identities and as a result the form of the free Lagrangian is not unique
- Smart choice:

$$\mathcal{L}_0 = a_0 \hat{E}_{abcd} \mathcal{R}^{ab} \mathcal{R}^{cd} + \frac{1}{2} \hat{E}_{ab} \Pi^a \Pi^b$$

where $\hat{E}_{ab} \sim \hat{e}_a \hat{e}_b$ and so on

General statements

- We construct the most general quadratic deformations for all four curvatures and find the general solution of the consistency condition
- It appears that the number of free parameters in this solution is equal to the number of possible field redefinitions and it is indeed possible to transform the whole vertex into the pure abelian form

$$\mathcal{L}_1 \sim \sum \mathcal{R}\mathcal{R}\Phi$$

- We also consider the most general abelian vertices and show that all the gauge invariant ones are on-shell equivalent to some trivially gauge invariant vertex

Trivially gauge invariant form

- There exist four such vertices that do not contain terms with more than four derivatives and do not vanish on-shell:

$$\begin{aligned} \mathcal{L}_1 &= h_1 \hat{E}_{abcd} \mathcal{R}^{ab} \mathcal{B}^{ce} \mathcal{B}^{de} \\ &+ \hat{E}_{abc} [h_2 \mathcal{B}^{ad} \mathcal{B}^{bd} \Pi^c + h_3 \mathcal{B}^{ab} \mathcal{B}^{cd} \Pi^d + h_4 \Pi^a \Pi^b \Pi^c] \end{aligned}$$

- In the unitary gauge $\mathcal{B}^{ab} \to m\Omega^{ab}$, $\Pi^a \to mf^2$ this gives

$$\begin{aligned} \mathcal{L}_1 &\sim m^2 h_1 \hat{E}_{abcd} \mathcal{R}^{ab} \Omega^{ce} \Omega^{de} \\ &+ m^3 \hat{E}_{abc} [h_2 \Omega^{ad} \Omega^{bd} f^c + h_3 \Omega^{ab} \Omega^{cd} f^d + h_4 f^a f^b f^c] \end{aligned}$$

- Vertex with four derivatives exists only in $d > 4$
- For the appropriate values of $h_{2,3,4}$ this reproduces the minimal interaction with no more than two derivatives

Non-abelian form

- We return back to the non-abelian formulation such that the deformations for the gauge fields have the same form as in the massless gravity:

$$
\begin{aligned}
\Delta \mathcal{R}^{ab} &= 2b_0 \Omega^{c[a}\Omega^{b]c} + 2b_0 \kappa f^{[a} f^{b]} \\
\Delta \mathcal{T}^{a} &= 2b_0 \Omega^{ab} f^{b} \\
\Delta \mathcal{B}^{ab} &= b_0 B^{c[a}\Omega^{b]c} + \kappa f^{[a}\pi^{b]} \\
\Delta \Pi^{a} &= b_0 \Omega^{ab}\pi^{b} + b_0 B^{ab} f^{b}
\end{aligned}
$$

- We still need some abelian terms to achieve gauge invariance:

$$
\begin{aligned}
\mathcal{L} &= a_0 \hat{E}_{abcd}\hat{\mathcal{R}}^{ab}\hat{\mathcal{R}}^{cd} + d_1 \hat{E}_{abcde}\mathcal{R}^{ab}\mathcal{R}^{cd} f^{e} \\
&\quad + \frac{1}{2}\hat{E}_{ab}\hat{\Pi}^{a}\hat{\Pi}^{b} + b_0 \hat{E}_{abc}\Pi^{a}\Pi^{b} f^{c}
\end{aligned}
$$

- Note that the term with coefficient d_1 exists only in $d > 4$

General statements

- Now we also have massless graviton with one-forms h^a, ω^{ab} and two-forms R^{ab}, T^a
- Both for the massive spin 2 as well as massless one the most general consistent deformations can be transformed with the appropriate field redefinitions into the abelian form
- At the same time, there exist two abelian vertices which are not on-shell equivalent to any trivially gauge invariant ones:

$$\begin{aligned}\mathcal{L}_1 \quad \sim \quad & g_1 \hat{E}_{abc} \mathcal{B}^{ab} \Pi^d \omega^{cd} \\ & + g_2 \hat{E}_{abc} [\mathcal{B}^{ad} \mathcal{B}^{bd} h^c + \mathcal{B}^{ab} \mathcal{B}^{cd} h^d - M^2 \Pi^a \Pi^b h^c]\end{aligned}$$

Moreover, it is these abelian vertices that allows one to reproduce the minimal gravitational interaction with two derivatives

Non-abelian version

- Deformations for graviton

$$
\begin{aligned}
\Delta R^{ab} &= c_1 \Omega^{c[a} \Omega^{b]c} + c_1 \kappa f^{[a} f^{b]} \\
\Delta T^a &= c_1 \Omega^{ab} f^b
\end{aligned}
$$

- Deformations for massive spin 2

$$
\begin{aligned}
\Delta \mathcal{R}^{ab} &= b_1 \Omega^{c[a} \omega^{b]c} + b_1 \kappa f^{[a} h^{b]} \\
\Delta \mathcal{T}^a &= b_1 \omega^{ab} f^b + b_1 \Omega^{ab} h^b \\
\Delta \mathcal{B}^{ab} &= b_1 \omega^{c[a} B^{b]c} + b_1 \kappa h^{[a} \pi^{b]} \\
\Delta \Pi^a &= b_1 \omega^{ab} \pi^b - b_1 B^{ab} h_b
\end{aligned}
$$

- Lagrangian

$$
\begin{aligned}
\mathcal{L} &= a_0 \hat{E}_{abcd} [\hat{\mathcal{R}}^{ab} \hat{\mathcal{R}}^{cd} + \hat{R}^{ab} \hat{R}^{cd}] + g_1 \hat{E}_{abcde} \mathcal{R}^{ab} \mathcal{R}^{cd} h^e \\
&+ \frac{1}{2} \hat{E}_{ab} \hat{\Pi}^a \hat{\Pi}^b + \frac{b_1}{2} \hat{E}_{abc} \Pi^a \Pi^b h^c
\end{aligned}
$$