Interacting gauge fields from quantized spinning particles

Maxim Grigoriev

Lebedev Physical Institute
&
Institute for Theoretical and Mathematical Physics, Lomonosov MSU

Based on:

M.G., Adiel Meyer, Ivo Sachs   to appear

also on:

M.G. 2006; Bekaert, M.G. 2013

Bekaert, M.G., Skvortsov 2017; M.G., Skvortsov 2018

QUARKS ONLINE WORKSHOPS-2021
dedicated to A.D. Sakharov's centennial

Motivations

- String (field) theory: spectrum of fields − general background. Equations of motion ensure consistent propagation.
- Background-independence is difficult to achieve in SFT because quantization requires specific background geometry.
- Digression: strings $\to$ particles which are background indep.
  - CHS theories from quantized particles *Segal 2002*
  - AdS HS theories reconstructions from (super)particles, *Bekaert, M.G. 2013; Bekaert, M.G., Skvortsov 2017; M.G., Skvortsov 2018*
  - YM equations from a sector of $N = 2$ superparticle *Dai, Huang, Siegel 2008*
  - Gravity equations from $N = 4$ superparticle *Bonezzi, Meyer, Sachs 2018*
- SFT-like BV-BRST graded algebraic setup for gauge theories associated to superparticles.

First-quantized BRST approach

Representation space: $\boldsymbol{\mathcal{H}} = \mathcal{H} \otimes \mathcal{C}^{\infty}(X)$; $X-$ space-time, $\mathcal{H}$ is a $\mathbb{Z}$-graded linear space. Generic element $\Phi = e_A \phi^A(x)$. $\phi^A(x)$-components of the wave function (fields, antifields, ghosts,....).

BRST operator:

$$\Omega\Phi = \Omega^A_B(x, \frac{\partial}{\partial x})\phi^B(x) e_A \,, \qquad \Omega^2 = 0 \,, \quad \mathrm{gh}(\Omega) = 1$$

Physical fields: $\mathrm{gh}(\Phi_0) = 1$, gauge parameters $\mathrm{gh}(\Phi_{-1}) = 0$, ...
Equations of motion and gauge symmetries:

$$\Omega\Phi_0 = 0 \,, \qquad \delta\Phi_0 = \Omega\Phi_{-1} \,, \qquad \ldots$$

If there is an invariant inner product of ghost degree -3:

$$S = \frac{1}{2}\langle \Phi_0, \Omega\Phi_0 \rangle$$

gauge-invariant action. Used in SFT and HS.

Example: relativistic particle. Operator algebra:

$$[x^a, p_b] = i\delta^a_b\,, \qquad [c, b] = 1\,, \qquad \mathrm{gh}(c) = -\mathrm{gh}(b) = 1\,.$$

Generic $\Phi \in \boldsymbol{\mathcal{H}}$ is given by $\Phi = \phi(x) + c\phi^*(x)$. ϕ^* corresponds to BV antifield.

BRST operator $\Omega = c(p^2 + m^2)$. No gauge symmetries, EOM's:

$$(p^2 + m^2)\phi(x) = 0$$

Interactions:

$$\mu_2 : \boldsymbol{\mathcal{H}} \times \boldsymbol{\mathcal{H}} \to \boldsymbol{\mathcal{H}}\,, \quad \mu_3 : \boldsymbol{\mathcal{H}} \times \boldsymbol{\mathcal{H}} \times \boldsymbol{\mathcal{H}} \to \boldsymbol{\mathcal{H}}\,, \quad \ldots$$

$\mathrm{gh}(\mu_i) = 2 - i$. Consistency: $(\boldsymbol{\mathcal{H}}, \Omega, \mu_i, i > 1)$ is an L_∞-algebra. For instance, if $\mu_i = 0$ for $i > 2$ then

$$\Omega^2 = 0\,, \qquad \Omega\mu_2(\phi, \psi) = \mu_2(\Omega\phi, \psi) + (-1)^{|\phi|}\mu_2(\phi, \Omega\psi)$$

i.e. differential graded Lie algebra.

General case is better formulated in the BV language: promote each component field $\phi^A(x)$ to the BV field $\psi^A(x)$ with $\text{gh}(\psi^A) = 1 - \text{gh}(e_A)$. String field:

$$\Psi = e_A \psi^A(x)$$

BRST differential $s = \int s^A \frac{\partial}{\partial \psi^A}$

$$s\Psi = \Omega\Psi + \mu_2(\Psi, \Psi) + \mu_3(\Psi, \Psi, \Psi) + \dots$$

In other words Ω, μ_i are Taylor components of s. L_∞-relations are $s^2 = 0$. BV master action:

$$S_{BV} = \frac{1}{2}\langle\Psi, \Omega\Psi\rangle + \frac{1}{6}\langle\Psi, \mu_2(\Psi, \Psi)\rangle + \dots .$$

Algebaric structure of perturbative BV.

Zwiebach 1993; Alexandrov, Kontsevich, Schwartz, Zaboronsky, 1995

Background independence

Suppose that $\mu_i = 0$ for $i > 2$ and moreover there exists (e.g. thanks to operator-state correspondence) a particular solution $\Phi_{vac}, \mathrm{gh}(\Phi_{vac}) = 1$ such that:

$$\mu_2(\Phi_{vac}, \chi) = \Omega\chi \qquad \forall\chi \in \boldsymbol{\mathcal{H}}\,.$$

Field redefinition $\Phi \to \bar{\Phi} = \Phi - \Phi_{vac}$:

$$\frac{1}{2}\langle\Phi, \Omega\Phi\rangle + \frac{1}{6}\langle\Phi, \mu_2(\Phi, \Phi)\rangle \quad \Rightarrow \quad \frac{1}{6}\langle\bar{\Phi}, \mu_2(\bar{\Phi}, \bar{\Phi})\rangle$$

"puerly cubic SFT action" *Horowitz, Lykken, Rohm, Strominger*.

Equations of motion:

$$\bar{\Phi}^2 \equiv \frac{1}{2}\mu_2(\bar{\Phi}, \bar{\Phi}) = 0\,.$$

Nilpotency of BRST operator. Formal background independence. Unfortunately only formal in SFT.

Background fields from constrained systems

M.G. 2006

Given a quantum constrained system:

$F_i|\Phi\rangle = 0\,,$ (More generally: $F_a|\Phi\rangle = 0\,, \quad |\Phi\rangle \sim |\Phi\rangle + F_\alpha|\Xi\rangle$)

The consistency condition:

$$[F_i, F_j] = U_{ij}^k F_k$$

Natural equivalence transformations for the constraints:

$$\delta F_i = \lambda_i^j F_j + [\epsilon, F_i]$$

$F_i(x,p)$ can be seen as a generating function of background fields:

$$F_i = F_i(x) + F_i^a(x)p_a + F_i^{ab}(x)p_a p_b + \ldots$$

while the above consistency and the equivalence, as resp. equations of motion and gauge symmetries.

Background fields from BRST

Introducing BRST operator Ω understood as a generating function of background fields:

$$\Omega = c^i F_i + \frac{1}{2} c^i c^j U_{ij}^k F_k + \dots$$

the equations of motion and gauge symmetries are:

$$[\Omega, \Omega] = 0\,, \qquad \delta\Omega = [\Omega, \Xi]$$

If in addition there is an invariant inner product of ghost degree -3

$$S = \frac{1}{6}\langle \Omega, [\Omega, \Omega]\rangle = \frac{1}{3} Tr(\Omega^3)$$

and we are back to background-independent action. Is well defined for (super) particles but the underlying gauge theory is off-shell.

Background fields for a scalar

Phase space:

$$[x^a, p_b] = \delta^a_b\,, \qquad [c, b] = 1$$

First class constraint $F(x,p)$. Although the consistency condition is trivial the gauge symmetries are

$$\delta F = [F, \xi]_\star + \omega F\,, \qquad F = D(x) + A^a(x)p_a + g^{ab}(x)p_a p_b + \dots$$

gauge symmetries for background fields for a scalar *Segal 2002*. In the BRST terms: $\Omega = cF$, $\Xi = \widetilde{\xi} + cb\widetilde{\omega}$

$$\delta\Omega = [\Omega, \Xi]$$

Upon linearizing around $\Omega^{init} = cp^2 \equiv \eta^{ab}p_a p_b$ one gets linear gauge symmetries equivalent to those of Fradkin-Tseytlin fields. HS algebra arise as that of global reducibility parameters.

Low spin sector:

Consider the following degree on the operator algebra: *M.G. 2012*

$$\deg x^a = 0\,, \quad \deg p_a = 1\,, \quad \deg c = -1\,, \quad \deg b = 2\,,$$

deg $\leqslant 0$ subspace (filtration-0) is an associative subalgebra
deg $\leqslant 1$ (filtration-1) is a Lie subalgbera.
The background field theory associated to deg $\leqslant 1$ subspace is off-shell conformal gravity+spin-1+ spin-0. In particular,

$$\deg \Omega^{init} = \deg cp^2 = 1$$

Segal-Tseytlin CHS theory and AdS HS theories

In the spirit of Sakharov's induced gravity approach consider:

$$\int D\varphi \exp\left(\frac{i}{2\hbar}\langle\varphi, \Omega\varphi\rangle\right)$$

CHS action arises as a log-divergent term in the effective action

Alternatively, the off-shell background fields encoded in Ω can be identified with the boundary values of the on-shell Fronsdal fields in AdS_{d+1}.

Diff-invariant local theory is determined by its on-shell gauge transformation *Barnich, M.G. 2010;*. Background fields on the boundary are 1:1 with on-shell bulk fields and we know non-linear gauge transformations for the boundary values so that in some sense we can reconstruct the bulk *Vasiliev* HS theory *Bekaert, M.G. Skvortsov 2017*
and construct Type-B HS theory *M.G. Skvortsov, 2018*

Constraint reduction of operator algebras

On-shell fields without holography or inducing: *Dai, Huang, Siegel*
- relax master equation in a consistent way
- supersymmetry

Let $\mathcal{A}$ be an associative algebra and $\boldsymbol{\mathcal{H}}$ its module. Interested in a subspace $\boldsymbol{\mathcal{H}}_0 \subset \boldsymbol{\mathcal{H}}$. Consider a subalgbera:

$$\mathcal{A}'' = \{a \in \mathcal{A} : a\boldsymbol{\mathcal{H}}_0 \subset \boldsymbol{\mathcal{H}}_0\}$$

Consider the left-right ideal $\mathcal{I}$ of elements acting trivially on $\mathcal{H}_0$:

$$\mathcal{I} = \{a \in \mathcal{A}'' : a\mathcal{H}_0 = 0\}$$

Finally one gets the associative algebra:

$$\mathcal{A}' = \mathcal{A}''/\mathcal{I}$$

Standard way to define algebras of quantum observables of constrained systems, e.g. HS algebra of *Eastwood, Vasiliev, ...*

$N = 2$ - supersymmetric particle

Operator algebra $\mathcal{A}$:

$$[x^a, p_b] = i\delta^a_b\,, \qquad [c, b] = 1\,, \quad \mathrm{gh}(c) = -\mathrm{gh}(b) = 1$$

$$[\theta^a, \bar{\theta}_b] = \delta^a_b\,, \qquad [\gamma, \bar{\beta}] = 1\,, \qquad [\bar{\gamma}, \beta_j] = 1\,,$$

$$\mathrm{gh}(\gamma) = \mathrm{gh}(\bar{\gamma}) = -\mathrm{gh}(\beta) = -\mathrm{gh}(\bar{\beta}) = 1$$

Represented on wave functions of $x, \theta, c, \beta, \gamma$ as:

$$p_a = -i\frac{\partial}{\partial x^a}\,, \qquad \bar{\theta}_a = \frac{\partial}{\partial \theta^a}\,, \qquad b = \frac{\partial}{\partial c}, \qquad \bar{\gamma} = \frac{\partial}{\partial \beta}, \qquad \bar{\beta} = -\frac{\partial}{\partial \gamma}$$

BRST operator:

$$\Omega^{init}\phi = \left(c\Box - i\gamma\frac{\partial}{\partial x}\cdot\frac{\partial}{\partial \theta} - i\theta\cdot\frac{\partial}{\partial x}\frac{\partial}{\partial \beta} + \gamma\frac{\partial}{\partial \beta}\frac{\partial}{\partial c}\right)\phi\,.$$

Generic background metric is also allowed, giving background independence

Inner product:

$$\langle , \rangle = \int d^d x dc \langle , \rangle_0 \,,$$

where $\langle , \rangle_0$ is the Fock space inner product determined by: $\theta^\dagger = \bar{\theta}$, $\gamma^\dagger = \bar{\gamma}$, $\beta^\dagger = -\bar{\beta}$. The quadratic action can be written as

$$S[\phi] = \frac{1}{2}\langle \phi, \Omega^{init}\phi \rangle \,, \qquad \mathrm{gh}(\phi) = 1 \,.$$

Well-known to describe totally-antisymmetric gauge fields.

Henneaux, Teitelboim, Buchbinder, Pashnev, Bastianelli, Krykhtin ....

Ω^{init}-invariant spin-1 subspace $\boldsymbol{\mathcal{H}}_0 \subset \boldsymbol{\mathcal{H}}$:

$$(N-1)\phi = 0 \,, \quad N = \theta \cdot \frac{\partial}{\partial \theta} + \gamma \frac{\partial}{\partial \gamma} + \beta \frac{\partial}{\partial \beta}$$

defines a reduced algebra $\mathcal{A}'$ of operators on $\mathcal{H}_0$.

Gauge theory associated to $\mathcal{A}'_1$

The theory of background fields determined by

$$\Omega^2 = 0\,, \qquad \mathrm{gh}(\Omega) = 1\,, \quad \Omega \in \mathcal{A}'$$

contains on-shell spin-1 field along with off-shell metric, dilaton and HS modes. $N = 2$ version of the degree:

$$\deg p_a = 1\,, \quad \deg x^a = 0\,, \quad \deg c = -1\,, \quad \deg b = 2\,,$$
$$\deg \theta = \deg \bar{\theta} = \frac{1}{2} \quad \deg \gamma = \deg \bar{\gamma} = -\frac{1}{2}\,, \quad \deg \beta = \deg \bar{\beta} = \frac{3}{2}$$

Again $\mathcal{A}'_1$ ($\deg \leqslant 1$) is a Lie subalgabera, giving a consistent gauge theory with finite number of fields and without HS fields.

For simplicity fix a vacuum solution Ω^{init} correspondin to Minskowski background:

$$\Omega = \Omega^{init} + \Omega^{ext}, \qquad \Omega^{ext} \in \mathcal{A}'_0$$

Generic expression for $\Omega^{ext} \in \mathcal{A}'_0$ reads:

$$\Omega^{ext} = c(B^a(x)p_a + \mathbf{F}^{ab}(x)\theta_a\bar{\theta}_b + D(x) + \\ + E(x)(\beta\bar{\gamma} + \gamma\bar{\beta})) + \gamma(\mathbf{A}^a(x)\bar{\theta}_a) + \bar{\gamma}(\bar{\mathbf{A}}^a(x)\theta_a)\,.$$

Then $\Omega^2 = 0$ implies that all the fileds save for $\mathbf{A}$ and Φ defined through $\bar{\mathbf{A}} = \mathbf{A} - 2i\partial_a\Phi$ are auxiliary while:

$$\partial^a(\partial_a\mathbf{A}_b - \partial_b\mathbf{A}_a) = 2(\partial_a\mathbf{A}_b - \partial_b\mathbf{A}_a)\partial^a\Phi\,, \qquad \bar{\mathbf{A}}_\mathbf{a} = \mathbf{A}_\mathbf{a} - 2i\partial_a\Phi$$

Upon imposing real form via $\Omega^\dagger = \Omega$ this reduces to real Maxwell-dilaton system:

$$\partial_a F^{ab} = 2F^{ab}\partial_a\varphi$$

Note that φ is off-shell.

Non-abelian gauge theory

Take a universal enveloping $\mathcal{U}^{\mathfrak{g}}$ of a given Lie algebra $\mathfrak{g}$. Algebra $U^{\mathfrak{g}}$ is equipped with the standard filtration

$$U_0^{\mathfrak{g}} \subset U_1^{\mathfrak{g}} \subset U_2^{\mathfrak{g}} \subset \ldots$$

where $U_k^{\mathfrak{g}}$ contains elements of order at most k in the generators T_a (basis elements of $\mathfrak{g}$). Furthermore, just like for $\mathcal{A}$

$$U_m^{\mathfrak{g}} U_n^{\mathfrak{g}} \subset U_{m+n}^{\mathfrak{g}}\,, \qquad [U_m^{\mathfrak{g}}, U_n^{\mathfrak{g}}] \subset U_{m+n-1}^{\mathfrak{g}}\,. \tag{1}$$

In place of $\mathcal{A}$ take

$$\hat{\mathcal{A}} = \mathcal{A} \otimes \mathcal{U}^{\mathfrak{g}}$$

equipped with the total filtration. In particular, $\widehat{\mathcal{A}}_1'$ is again a Lie subalgebra, giving a consistent gauge theory determined by $\Omega^2 = 0$ for $\Omega \in \widehat{\mathcal{A}}_1'$.

Finally, take as $\Omega^{init} \otimes 1$ as a new Ω^{init} and $\Omega^{ext} \in \widehat{\mathcal{A}}'_0$. Together with extra reality condition $A_a^\dagger = A_a$ one finds that Ω^2 encodes YM-equations on A_a. Without extra reality cond. the exhaustive description is not known.

Off-shell gravity

Although we restricted the analysis to Ω^{init} of Minskowskian background a generic off-shell gravity is also admissible:

$$\Omega_g^{init} = c\mathcal{D} + \bar{\gamma}\theta^\mu \Pi_\mu + \gamma\bar{\theta}^\mu \Pi_\mu + \gamma\bar{\gamma} b$$

where $\Pi_\mu = p_\mu - i\omega_{\mu ab}\theta^a\bar{\theta}^b$ and

$$\mathcal{D} = \Pi^2 + R_{\mu\nu\lambda\sigma}\theta^\mu\bar{\theta}^\nu\theta^\lambda\bar{\theta}^\sigma$$

One finds that Ω_g^{init} is nilpotent, provided the connection is torsionless. This ensures the background independence.

Towards Lagrangians

Full Lagrangian requires a ghost degree -3 invariant inner product which is not known. Lagrangian for the linearized system: consider a map: *Dai, Huang, Siegel*

$$\mu : \mathcal{A}'_0 \to \boldsymbol{\mathcal{H}}_0 \qquad \mu(O) = O\beta \quad O \in \mathcal{A}'_0$$

and pull-back $\langle , \rangle$ from $\mathcal{H}_0$ to $\mathcal{A}'_0$:

$$\langle a, b \rangle_{\boldsymbol{\beta}} := \langle a\boldsymbol{\beta}, b\boldsymbol{\beta} \rangle \,.$$

This is Ω^{init}-invariant because $\Omega^{init}\beta = 0$ so that

$$S[\Omega^{ext}] = \frac{1}{2}\langle \Omega^{ext}, \Omega^{init}\Omega^{ext} \rangle_{\boldsymbol{\beta}}$$

is gauge invariant. Being suplemented with algebraic constraints encoded in $\Omega^2 = 0$ it is equiavlent to a complexified Maxwell action. Indeed, $\psi = \mu(\Omega^{ext}) = \Omega^{ext}\beta = A_b(x)\theta^b + D(x)c\beta$ and is a generic vector of $\boldsymbol{\mathcal{H}}_0$ so the action is just standard $\langle \psi, \Omega^{init}\psi \rangle$ action.

Conclusions

- Using graded algebras of quantum observables of constrained systems is fruitfull approach in String theory, HS theories, and usual low spin models

- Background-independent toy model of SFT with finite amount of fields

- Background independent algebraic structures such as operator algebra, its constraint reduction and filtration

Open problems

- Exhaustive desciption of the spectrum of fields, i.e. $[\Omega^{init}, \cdot]$ cohomology in $\mathcal{A}'_1$.

- Extension to $N = 4$ along the lines of *Bonezzi, Meyer, Sachs 2018*

- Relation to HS algebras associated to generic N susy conformal particles. *Bekaert, MG 2009*. In particular Type-C HS theory determined by $N = 2$.