Non-abelian fermionic T-duality

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Key features of T-duality

- Bosonic T-duality is a symmetry of string theory and supergravity.
- T-duality links together different string and supergravity theories ($IIA \leftrightarrow IIB$, heterotic $E_8 \times E_8 \leftrightarrow SO(32)$).
- It is a powerful solution generating technique.
- Buscher procedure: Method of obtaining T-dual background fields, starting from a solution with an isometry.

Common sector of $d = 10$ $\mathcal{N} = 2$ supergravities

$$S = \frac{1}{2\kappa^2} \int d^{10}x \, G^{1/2} e^{-2\phi} \left[R(G) + 4(\partial\phi)^2 - \frac{1}{2}\frac{1}{3!}(\partial B)^2 \right] \tag{1}$$

String sigma-model action

$$S = \frac{1}{4\pi\alpha'} \int d^2z \, [G_{\mu\nu}(X) + B_{\mu\nu}(X)] \, \partial X^\mu \bar{\partial} X^\nu + \frac{1}{4\pi} \int d^2z \, \phi \mathcal{R}^{(2)} \tag{2}$$

Buscher procedure

$$S = \int d^2z \, [G_{\mu\nu}(X) + B_{\mu\nu}(X)] \, \partial X^\mu \bar{\partial} X^\nu$$

$$X^1 \to X^1 + a \quad : \quad \text{shift isometry}$$

$$\partial_\alpha X^1 \to \partial_\alpha X^1 + A_\alpha \quad : \quad \text{worldsheet gauge field}$$

$$\delta \mathcal{L} = \tilde{X}^1 \left(\partial \bar{A} - \bar{\partial} A \right) \quad : \quad \text{pure gauge restriction}$$

Buscher rules:

$$\begin{gathered} G'_{11} = (G_{11})^{-1}, \quad G'_{1\mu} = (G_{11})^{-1} B_{1\mu}, \quad B'_{1\mu} = -(G_{11})^{-1} G_{1\mu}, \\ G'_{\mu\nu} = G_{\mu\nu} - (G_{11})^{-1}(G_{\mu 1} G_{1\nu} + B_{\mu 1} B_{1\nu}), \\ B'_{\mu\nu} = B_{\mu\nu} - (G_{11})^{-1}(G_{\mu 1} B_{1\nu} + B_{\mu 1} G_{1\nu}), \\ \phi' = \phi - \frac{1}{2} \log G_{11}. \end{gathered} \tag{3}$$

Generalization to superspace

Berkovits' superstring

$$S = \frac{1}{2\pi\alpha'}\int d^2z\left[L_{MN}\partial Z^M\bar{\partial}Z^N + P^{\alpha\hat{\beta}}d_\alpha\hat{d}_{\hat{\beta}} + E_M^\alpha d_\alpha\bar{\partial}Z^M + \ldots\right]$$

$Z^M = \left\{X^\mu, \theta^\alpha, \hat{\theta}^{\hat{\alpha}}\right\}$: $\mathbb{R}^{10|32}$ superspace coordinates

$\theta^1 \to \theta^1 + \epsilon^1$: supersymmetry generated by $\exp\bar{\epsilon}Q$

$\delta\mathcal{L} = \tilde{\theta}^1\left(\partial\bar{A} - \bar{\partial}A\right)$, etc. : fermionic Buscher procedure

RR bispinor:

$$P^{\alpha\hat{\beta}} \sim F^{\alpha\hat{\beta}} = F_\mu(\gamma^\mu)^{\alpha\hat{\beta}} + \frac{1}{3!}F_{\mu\nu\kappa}(\gamma^{\mu\nu\kappa})^{\alpha\hat{\beta}} + \frac{1}{2}\frac{1}{5!}F_{\mu\nu\kappa\lambda\rho}(\gamma^{\mu\nu\kappa\lambda\rho})^{\alpha\hat{\beta}}. \quad (4)$$

Summary of fermionic T-duality

- Buscher procedure generalised to a superspace setup.
  [Berkovits, Maldacena 2008] [Beisert, Ricci, Tseytlin, Wolf 2008]
- Metric $g_{\mu\nu}$ and the 2-form $b_{\mu\nu}$ are not affected.
- As compared to bosonic T-duality:
  - Transformation of the RR forms is totally different;
  - Opposite sign in the dilaton transformation.

Fermionic Buscher rules

$$F'^{\alpha\hat{\beta}} = C^{-1/2}\left(F^{\alpha\hat{\beta}} + 16i\, C^{-1} e^{-\phi} \epsilon^{\alpha}\hat{\epsilon}^{\hat{\beta}}\right), \quad \phi' = \phi + \frac{1}{2}\log C. \tag{5}$$

The auxiliary function C is constructed from the Killing spinor $(\epsilon^{\alpha}, \hat{\epsilon}^{\hat{\alpha}})$:

$$\partial_\mu C = i\left(\epsilon\gamma_\mu\epsilon - \hat{\epsilon}\gamma_\mu\hat{\epsilon}\right) \tag{6}$$

Applications of fermionic T-duality so far

- Self-duality under combined bosonic and fermionic T-dualities:
  - $AdS_5 \times \mathbb{S}^5$,
    [Berkovits, Maldacena 2008] [Beisert, Ricci, Tseytlin, Wolf 2008]
  - pp-waves (32, 24, 16 supersymmetries),
    [IB, Berman 2009] [IB, Ó Colgáin, Yavartanoo 2011]
  - $AdS_3 \times \mathbb{S}^3 \times \mathbb{T}^4$,
    [Ó Colgáin 2012]
  - $AdS_k \times \mathbb{S}^k \times \mathbb{T}^{10-2k}$, $AdS_k \times \mathbb{S}^k \times \mathbb{S}^k \times \mathbb{T}^{10-3k}$ $(k = 2, 3)$;
    [Abbott, Murugan, Penati, Pittelli, Sorokin, Sundin, Tarrant, Wolf, Wulff 2015]
- Solution generation in $d = 10$ supergravity;
- Supersymmetry preserving deformation of the supergravity solutions.
  [IB, Berman 2009]

Abelian constraint

Bosonic isometries can be abelian or non-abelian.

Fermionic isometries (supersymmetries) are intrinsically non-abelian

$$[\delta_{\epsilon,\hat{\epsilon}}, \delta_{\epsilon,\hat{\epsilon}}] = [\bar{\epsilon}Q, \bar{\epsilon}Q] + \left[\bar{\hat{\epsilon}}\hat{Q}, \bar{\hat{\epsilon}}\hat{Q}\right] = (\epsilon\gamma^\mu\epsilon + \hat{\epsilon}\gamma^\mu\hat{\epsilon})\, P_\mu \tag{7}$$

The supersymmetric Killing vector $\epsilon\gamma^\mu\epsilon + \hat{\epsilon}\gamma^\mu\hat{\epsilon}$ must vanish for abelian fermionic T-duality

$$\epsilon\gamma^\mu\epsilon + \hat{\epsilon}\gamma^\mu\hat{\epsilon} \overset{!}{=} 0 \qquad \text{abelian constraint.} \tag{8}$$

However, this condition cannot be satisfied by real valued Killing spinors.

The Abelian constraint requires complexification of the Killing spinors

$$\epsilon \to \epsilon + i\epsilon' \tag{9}$$

Non-abelian fermionic T-duality

T-dualize a subalgebra $(\epsilon, \hat{\epsilon}, \tilde{K})$:

- Non-abelian fermionic T-duality w.r.t. $(\epsilon, \hat{\epsilon})$;
- Bosonic T-duality w.r.t. $\tilde{K}$.

Can be worked out using the formulation of non-abelian duality for supergroups by [Borsato, Wulff 2018].

Our conjecture

For a generic background the first step above produces solutions of Double Field Theory according to:

$$
\begin{aligned}
\partial_\mu C &= i(\epsilon\gamma^\mu\epsilon - \hat{\epsilon}\gamma^\mu\hat{\epsilon}), \\
\tilde{\partial}^\mu C &= i(\epsilon\gamma^\mu\epsilon + \hat{\epsilon}\gamma^\mu\hat{\epsilon}),
\end{aligned}
\tag{10}
$$

using the same fermionic Buscher formulae.

DFT Summary

Double Field Theory: manifestly T-duality covariant form of supergravity

[Tseytlin 1990; Siegel 1993; Hohm, Hull, Zwiebach 2010]

Doubled coordinates: $X^M = (x^\mu, \tilde{x}_\mu)$

Dynamical variable: generalized metric $\mathcal{H}_{MN}$

$$\mathcal{H}_{MN} = \begin{bmatrix} G + BG^{-1}B & GB \\ BG & G^{-1} \end{bmatrix}$$

T-duality covariant: $\mathcal{H}' = \mathcal{O}\mathcal{H}\mathcal{O}^T$

Action ($d = \phi + \frac{1}{4}\log G_{\mu\nu}$):

$$S = \int dx\, d\tilde{x}\, e^{-2d} \left(\frac{1}{8}\mathcal{H}^{MN}\partial_M \mathcal{H}^{KL}\partial_N \mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{KL}\partial_L \mathcal{H}^{MN}\partial_N \mathcal{H}_{KM} - \right.$$
$$\left. -2\partial_M d \partial_N \mathcal{H}^{MN} + 4\mathcal{H}^{MN}\partial_M d \partial_N d \right).$$

Our suggestion

$$\begin{aligned}\partial_\mu C &= i(\epsilon\gamma^\mu\epsilon - \hat{\epsilon}\gamma^\mu\hat{\epsilon}),\\ \tilde{\partial}^\mu C &= i(\epsilon\gamma^\mu\epsilon + \hat{\epsilon}\gamma^\mu\hat{\epsilon}),\end{aligned} \tag{11}$$

Usual fermionic Buscher rules

$$F'^{\alpha\hat{\beta}} = C^{-1/2}\left(F^{\alpha\hat{\beta}} + 16i\, C^{-1}e^{-\phi}\epsilon^\alpha\hat{\epsilon}^{\hat{\beta}}\right), \quad \phi' = \phi + \frac{1}{2}\log C. \tag{12}$$

- DFT *solutions* with dependence on both x^μ and $\tilde{x}_\mu$;
- Non-geometric coordinate $\tilde{x}_\mu$ dependence may be removed by a bosonic T-duality in the x^μ direction;
- This corresponds to T-dualizing the supersymmetric Killing vector $\epsilon\gamma^\mu\epsilon + \hat{\epsilon}\gamma^\mu\hat{\epsilon}$;
- There are genuinely non-geometric solutions, where $C \sim x^\mu + \tilde{x}_\mu$;
- We still have to complexify the Killing spinors, or deal with timelike T-duality.