Boundary conditions, orderings and Fronsdal fields in Vasiliev's higher-spin gravity

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Motivations

- Higher-spin gravity = dynamics of an infinite multiplet of gauge fields of all spins, conjectured to be holographic dual to free CFT.

- A system of intermediate complexity between the full String Theory and SUGRA + higher-derivative corrections: very constrained, due to infinite-dimensional local symmetries, yet quite far from ordinary field theories, since

  - Order of derivatives in a vertex grows linearly with the spins involved → expected to contain *non-local interactions* (among physical spin-s fields)

  - All fields are on equal footing. Concepts of the standard riemannian geometry lose meaning as they are *not HS-invariant*. A gauge-invariant description of bulk dynamics requires developing and understanding proper HS invariants, leading to a stringy generalization of geometry.

Motivations

- Remarkable that one can still control many features of the theory, essentially due to the infinite-dimensional symmetry + compact form of the non-linear eqs.

- As non-locality is expected in some degree, certain issues (allowed field redefinitions, large vs small gauge transformations, ...) seem hard to tackle at least by standard means. The language of spin-*s* component fields is not the most appropriate one to address such questions [and the very concept only makes sense in weak field regime].

- There are indications (from study of cubic and quartic vertices, of singularities of bh-like solutions, ...) that much insight is to be gained by addressing currently open questions within the natural framework of the eqs., in terms of HS-covariant variables and corresponding HS-invariant observables.

(Vasiliev; Didenko, Gelfond, Korybut, Vasiliev; Sezgin, Sundell; C.I., Sundell; Boulanger, Sundell; Colombo, Sundell; Didenko, Skvortsov; Bonezzi, Boulanger, De Filippi, Sundell...)

4D bosonic Vasiliev's equations

- Vasiliev's eqs = generating system for nonlinear eqs involving gauge fields of all spins .

  Formulated in terms of *master-fields* on *correspondence space*, locally $\mathcal{X}$ x $\mathcal{Z}$ x $\mathcal{Y}$:

$$\widehat{U} = dx^\mu \widehat{U}_\mu(\widehat{Y},\widehat{Z}|x) \longrightarrow \text{gauge fields of all spins + auxiliary}$$

$$\widehat{\Phi} = \widehat{\Phi}(\widehat{Y},\widehat{Z}|x) \longrightarrow \text{Weyl tensors and their derivatives} \rightarrow \text{local dof}$$

$$\widehat{V} = dz^\alpha \widehat{V}_\alpha(\widehat{Y},\widehat{Z}|x) + d\bar{z}^{\dot\alpha}\widehat{\bar{V}}_{\dot\alpha}(\widehat{Y},\widehat{Z}|x) \longrightarrow Z\text{-space connection, no extra local dof}$$

- Oscillators $\widehat{Y}_{\underline{\alpha}} = (\widehat{y}_\alpha, \widehat{\bar{y}}_{\dot\alpha})$, $\widehat{Z}_{\underline{\alpha}} = (\widehat{z}_\alpha, -\widehat{\bar{z}}_{\dot\alpha})$ → $\mathfrak{sp}$(4,R) quartets

$$[\widehat{Y}_{\underline{\alpha}},\widehat{Y}_{\underline{\beta}}] = 2iC_{\underline{\alpha\beta}} = 2i\begin{pmatrix} \varepsilon_{\alpha\beta} & 0 \\ 0 & \varepsilon_{\dot\alpha\dot\beta}\end{pmatrix} , \qquad [\widehat{Z}_{\underline{\alpha}},\widehat{Z}_{\underline{\beta}}] = -2iC_{\underline{\alpha\beta}} , \quad [\widehat{Y}_{\underline{\alpha}},\widehat{Z}_{\underline{\beta}}] = 0$$

- $\mathcal{Y}$ → fibre: spin-s generators = degree-2(s-1) totally symmetric monomials

$$(\widehat{T}_s)_{\underline{\alpha}_1\cdots\underline{\alpha}_{2(s-1)}} \longleftrightarrow \widehat{Y}_{(\underline{\alpha}_1}\cdots\widehat{Y}_{\underline{\alpha}_{2(s-1)})}$$

4D bosonic Vasiliev equations

- Full equations:

$$\boxed{\begin{aligned} d\widehat{U} + \widehat{U}\,\widehat{U} &= 0 \\ d\widehat{\Phi} + [\widehat{U}, \widehat{\Phi}]_\pi &= 0 \\ d\widehat{V} + \widehat{q}\widehat{U} + [\widehat{U}, \widehat{V}] &= 0 \\ \widehat{q}\widehat{\Phi} + [\widehat{V}, \widehat{\Phi}]_\pi &= 0 \\ \widehat{q}\widehat{V} + \widehat{V}\,\widehat{V} &= \widehat{\Phi}\,\widehat{J} \end{aligned}}$$

Z-differential

$$\widehat{q}\,\widehat{f}(\widehat{Z}, \widehat{Y}) \;:=\; dZ^{\underline{\alpha}}[\widehat{Z}_{\underline{\alpha}}, \widehat{f}(\widehat{Z}, \widehat{Y})]$$

$$\widehat{J} \;:=\; -\frac{i}{4} dz^2 \widehat{\kappa} - \text{h.c.}$$

- Inner kleinian operator $\hat{\kappa}$: $\widehat{\kappa} \;=\; \widehat{\kappa}_y \widehat{\kappa}_z \;, \qquad \{\widehat{\kappa}_y, \widehat{y}\} = 0 \;, \qquad \widehat{\kappa}_y^2 = 1 \;, \qquad idem \;\; \widehat{\kappa}_z$

$$\widehat{\kappa}_y \;=\; (-1)^{\widehat{N}_y} \;, \qquad \widehat{\kappa}_z \;=\; (-1)^{\widehat{N}_z}$$

- *Z*-oscillators → auxiliary, non-commutative coordinates. Equations solve *Z*-contractions in terms of curvatures of the physical fields, giving rise to an infinite tail of gauge-invariant nonlinear interactions.
  Physical fluctuations contained in initial data (integration constants for Z-eqs)

$$\widehat{W}(x, \widehat{Y}) \;:=\; \mathcal{P}\widehat{U} \;, \qquad \widehat{C}(x, \widehat{Y}) \;=\; \mathcal{P}\widehat{\Phi}$$

Perturbative expansion on AdS

- Simplest vacuum solution → *AdS* spacetime,

$$\widehat{\Phi} \;=\; \widehat{\Phi}^{(0)} \;=\; 0\;, \qquad \widehat{V}_{\underline{\alpha}} \;=\; \widehat{V}^{(0)}_{\underline{\alpha}} \;=\; 0\;, \qquad \widehat{U} \;=\; \widehat{U}^{(0)} \;=\; \widehat{L}^{-1}\, d\widehat{L}$$

with $U^{(0)}$ a flat connection represented via a gauge function $L(x|Y) = AdS_4$ coset element .

- Set up perturbative expansion around AdS : eqs. with at least one component on $\mathcal{Z}$ lend themselves to be integrated with integration constants $\hat{C}(x,\hat{Y})$ and $\hat{W}(x,\hat{Y})$ iteratively in an expansion in curvatures (contained in $\hat{C}$)

$$\widehat{\Phi} = \sum_{n\geq 1} \widehat{\Phi}^{(n)}\;, \quad \widehat{\Phi}^{(1)} = \widehat{C}(x,\widehat{Y})\;; \qquad \widehat{V}_{\alpha} = \sum_{n\geq 1} \widehat{V}^{(n)}_{\alpha}\;; \qquad \widehat{U} = \sum_{n\geq 0} \widehat{U}^{(n)}$$

- Solve *Z*-dependence via equations of the form

$$\widehat{q}\widehat{f} \;=\; \widehat{g}$$

via a resolution operator q^*

$$\boxed{\widehat{f} \;=\; \widehat{q}^{*}\widehat{g} + \underbrace{\widehat{q}\widehat{h}}_{\text{gauge parameter}} + \widehat{c}}$$

integration constant $\in H(\hat{q})$

$\hat{c} = \mathcal{P}\hat{f}$

Orderings

- The perturbative analysis is performed in a given ordering prescription: this facilitates writing down a concrete integral operator *q** and the extraction of physical fields from the generating functions.

- Linear invertible maps between spaces of functions of commuting variables (symbols) and operators (in a given ordering)

$$\widehat{\mathrm{Op}}: \ f \to \ \widehat{\mathrm{Op}}[f] \equiv \widehat{f}$$

$$[\ ]: f = [\widehat{\mathrm{Op}}[f]] \ \leftarrow \ \widehat{\mathrm{Op}}[f]$$

- Operator-symbol correspondence induces algebra isomorphism

$$\left[\widehat{f}_1 \widehat{f}_2\right] = \left[\widehat{f}_1\right] \star \left[\widehat{f}_2\right]$$

 with the star product (concretely realized on symbols via a convolution) implementing the non-commutative operator product on symbols.

Orderings

- Usual choice: normal ordering wrt *Y-Z* and *Y+Z*, in which case

$$F(Y,Z) \star G(Y,Z) \;=\; \int_{\mathcal{R}} \frac{d^4U d^4V}{(2\pi)^4}\, e^{iV^{\underline{\alpha}}U_{\underline{\alpha}}}\, F(Y+U, Z+U)\, G(Y+V, Z-V)$$

  Y and *Z* are maximally entangled (nontrivial contractions from *Y* ★ *Z*) and the total Klein operator is regular,

$$[\widehat{\kappa}]_N \;=\; [\widehat{\kappa}_y \widehat{\kappa}_z]_N \;=\; \kappa_y \star \kappa_z \;=\; e^{iy^\alpha z_\alpha}$$

- Weyl ordering on total space (*Y*, *Z*) disentangles *Y* and *Z*

$$[f(Y) \star g(Z)]_0 \;=\; f(Y) g(Z)$$

  and Klein operator becomes a delta function

$$[\widehat{\kappa}]_0 \;=\; [\widehat{\kappa}_y \widehat{\kappa}_z]_0 \;=\; [\kappa_y]_0 \star_0 [\kappa_z]_0 \;=\; (2\pi)^2\, \delta^2(y)\, \delta^2(z)$$

- Studied extension of the perturbative analysis to a family of orderings interpolating between normal and Weyl

(Didenko, Gelfond, Korybut, Vasiliev;
De Filippi, C.I., Sundell)

Contracting homotopy

- In terms of symbols now

$$q := dZ^{\underline{\alpha}} \frac{\partial}{\partial Z^{\underline{\alpha}}}$$

  and a particular solution to

$$q\, f(x, Z; dx; dZ; Y) \;=\; g(x, Z; dx; dZ; Y)$$

  can be obtained by homotopy-contracting along *Z* (***E*** *:= Z* • ∂/∂*Z*) shifted by any *Z*-independent vector *Ξ*

$$q^{(E+\Xi)*} g(x, Z; dx; dZ; Y) \;=\; \iota_{E+\Xi} \int_0^1 \frac{dt}{t}\, g(x, tZ + (t-1)\Xi; dx; tdZ; Y)$$

$$\iota_{E+\Xi} \;=\; (Z^{\underline{\alpha}} + \Xi^{\underline{\alpha}}) \frac{\partial}{\partial dZ^{\underline{\alpha}}}$$

- *Ξ = 0* is the simplest choice → "standard" perturbative scheme.

- Solutions obtained via two different contracting homotopies differ by gauge choices and field redefinitions.

(Gelfond, Vasiliev)

Standard perturbative analysis

- The eqs. with at least one component on *Z* can be integrated in terms of the original dof in $\Phi_{|Z=0}(x,Y)$ (with no cohomology for *Z*-space 1-forms)

$$\begin{aligned} q\Phi^{(1)} &= 0 & \longrightarrow \quad & \Phi^{(1)} = C^{(1)}(x;Y) \\ D^{(0)}_{\mathrm{tw}}\Phi^{(1)} &= 0 & & \\ qV^{(1)} + \Phi^{(1)} \star J &= 0 & \longrightarrow \quad & V^{(1)} = -q^{(E)*}(\Phi^{(1)} \star J) + q\epsilon \\ qU^{(1)} + D^{(0)}_{\mathrm{ad}} V^{(1)} &= 0 & \longrightarrow \quad & U^{(1)} = q^{(E)*} D^{(0)}_{\mathrm{ad}} q^{(E)*}(\Phi^{(1)} \star J) + W^{(1)}(x;Y) + D^{(0)}_{\mathrm{ad}}\epsilon \\ D^{(0)}_{\mathrm{ad}} U^{(1)} &= 0 & & \end{aligned}$$

- Simplest gauge choice: solving the *Z*-space eqs. with $q\varepsilon = 0$, i.e., Vasiliev gauge

$$i_Z V^{(1)} \equiv Z^{\underline{\alpha}} V^{(1)}_{\underline{\alpha}} = 0$$

- One is then left with the free field equations on $\mathcal{X} \times \mathcal{Y}$:

$$\boxed{D^{(0)}_{\mathrm{ad}} W^{(1)} = -(D^{(0)}_{\mathrm{ad}} q^*)(D^{(0)}_{\mathrm{ad}} q^*)(C^{(1)} \star J) \ , \qquad D^{(0)}_{\mathrm{tw}} C^{(1)} = 0}$$

gauge fields in the 1-form «glued» to local dof in the 0-form

KG + all spin-s Bargmann-Wigner eqs.

Standard perturbative analysis

- Possible in principle to proceed to higher orders and give the *Z*-dependent fields iteratively in terms of non-linear couplings involving the original dof in $\Phi_{|Z=0}$:

$$\Phi^{(n)} = C^{(n)} + q^* \textstyle\sum_{k=1}^{n-1} [V^{(n-k)}, \Phi^{(k)}]_\pi \ , \qquad U^{(n)} = W^{(n)} + q^* \textstyle\sum_{k=0}^{n-1} [W^{(k)}, V^{(n-k)}]_\star$$

$$V^{(n)} = q\epsilon^{(n)} + q^* \left(\Phi^{(n)} \star J + \textstyle\sum_{k=1}^{n-1} V^{(k)} \star V^{(n-k)} \right)$$

On-shell the infinitely many *Z*-contractions turn into an infinite expansion in derivatives of arbitrarily high order → in a generic frame, one has a non-local, Born-Infeld-like tail at every fixed order in weak fields.

- Inserting the perturbative solutions for *U* and Φ of the *Z*-space eqs. into the pure spacetime eqs. and setting *Z*=0, one gets

$$\begin{aligned} dW &= \mathcal{V}_2(W,C) := W \star W + \mathcal{V}_2^{(1)}(W,W,C) + \mathcal{V}_2^{(2)}(W,W,C,C) + \ldots \\ dC &= \mathcal{V}_1(W,C) := [W,C]_\pi + \mathcal{V}_1^{(2)}(W,C,C) + \ldots \end{aligned}$$

Standard perturbative analysis

- A condition for the interpretation of the *W* and *C* as generating functions of gauge fields and Weyl tensors of all spins is that they be *real-analytic* in *Y*.

  In standard perturbative analysis this is ensured by requiring that *all* master fields be (formal) polynomials in *Y* and *Z* *everywhere* on spacetime *.*

  Known interesting solutions force us to considerably soften this condition.

  (Didenko, Vasiliev; C.I., Sundell; Aros, C.I., Sundell, Yin)

- Main disadvantages of standard perturbative scheme:
  - writing down vertices gets harder and harder
  - non-localities already at cubic order *(Boulanger, Kessel, Skvortsov, Taronna)*
  - unclear how to select different solution spaces (imposing b.c.)
  - how to fix gauge ambiguities beyond first order?

- Shifted homotopies studied to improve locality of vertices. Cubic vertices shown to admit local form. Certain types of quartic vertices admit *spin-local* form with shift $\Xi_\alpha = \beta\, \partial/\partial y^\alpha$ in the limit $\beta \rightarrow -\infty$.

  (Didenko, Gelfond, Korybut, Vasiliev)

Alternative schemes

- Worth exploring modifications to the traditional perturbative scheme. Some suggestion, also exploiting shifted contracting homotopies, came independently from the study of exact solutions.

  (C.I., P. Sundell)
  (D. De Filippi, C.I., P. Sundell)

- In order to overcome some of the problems of the simplest perturbative approach, it is worth letting go of some assumptions of the standard scheme and exploring a shift $\Xi_\alpha = \beta\, \partial/\partial y^\alpha$ in the limit $\beta \rightarrow 1$, similar to solving eqs. by separating non-commutative Y and Z variables.

- Price to pay: dealing with some irregular star-product elements in the intermediate steps (requiring some regularization).

- Advantage: perturbative series much easier (can be pushed to all orders) in the new frame. Contact with the "physical" gauge is made by building a gauge transformation order by order, which facilitates imposing boundary conditions.

Factorized perturbative scheme

- A different organization of the perturbative expansion facilitates pushing the solution to higher orders (sometimes to *all orders*).

- New perturbative scheme based on two observations:

  1. At first order, the equations for Φ are

$$
\begin{aligned}
q\Phi^{(1)} &= 0 && \longrightarrow && \boxed{\Phi^{(1)} = C(x,Y)} \\
D^{(0)}_{\mathrm{tw}}\Phi^{(1)} &= 0 && \longrightarrow && C(x,Y) = L^{-1}\star\Phi'(Y)\star\pi(L)
\end{aligned}
$$

  2. The source term that triggers the non-linear corrections can be rewritten as

$$
\Phi\star\kappa \;=\; \Phi\star\kappa_y\star\kappa_z \;=\; \Psi\star\kappa_z\;, \qquad \Psi \;:=\; \Phi\star\kappa_y
$$

→ Organizing the perturbative expansion in powers of Ψ and keeping the *Y* and *Z* dependence factorized, one can solve for the *Z*-dependence *universally*.

- To do that, use a factorized contracting homotopy, s.t.

$$
q^{(F)*}(f(Y)\star g(Z)) \;=\; f(Y)\star q^{(F)*}g(Z) \;=\; f(Y)\star q^{(E)*}g(Z)
$$

- Equivalently: solve with standard homotopy in Weyl order,

$$
q^{(E)*}[\widehat{f}(Y)\widehat{g}(Z)]_0 \;=\; q^{(E)*}(f(Y)g(Z)) \;=\; f(Y)q^{(E)*}g(Z)
$$

Factorized expansion scheme

- The solution at 1st order : $$qV^{(1)} = -\Psi^{(1)} \star j_z - \bar{\Psi}^{(1)} \star \bar{j}_{\bar{z}}\ , \qquad j_z = -\frac{ib}{4}\, dz^2 \kappa_z$$

$$\boxed{V^{(1)} = -\Psi^{(1)} \star \underbrace{q^{(E)*} j_z}_{-v^{(1)}(z)} - \bar{\Psi}^{(1)} \star \underbrace{q^{(E)*} \bar{j}_{\bar{z}}}_{-\bar{v}^{(1)}(\bar{z})}}$$

$$qU^{(1)} = -D^{(0)}_{\text{ad}} V^{(1)} = \Psi^{(1)} \star dq^{(E)*} j_z - \bar{\Psi}^{(1)} \star dq^{(E)*} \bar{j}_{\bar{z}} = 0$$

$$\boxed{U^{(1)} = W^{(1)}(x, Y)}$$

which can be gauge-fixed to 0 (as the 1-form is not glued to the Weyl 0-form).

- κ_z distributional → $q^{(E)*}j_z$ distributional. Requires an integral/limit representation,

$$q^{(E)*}\kappa_z \sim z \int_0^{+\infty} d\tau\, e^{i\tau z^+ z^-} \sim \frac{1}{z} \lim_{\epsilon \to 0} (1 - e^{-\frac{i}{\epsilon} z^+ z^-}) \sim \theta(z^\pm)\delta(z^\mp)$$

- Source term is 1st order in Ψ: higher order for V are $$V^{(n)} = (\Psi^{(1)})^{\star n} \star v^{(n)}(z) - \text{h.c.}$$
  with $$v^{(n)} = -q^{(E)*} \sum_{k=1}^{n} v^{(k)} \star v^{(n-k)}$$

Factorized expansion scheme

- Fixing gauges, one can push the solution to all orders in the form

$$\boxed{\begin{array}{lllll} U &=& U^{(0)} &=& L^{-1}\star dL \\ \Phi &=& C(x;Y) &=& L^{-1}\star\Phi'(Y)\star\pi(L) \;=\; L^{-1}\star\Psi'\star L \\ V_\alpha &=& V_\alpha(x,z;Y) &=& L^{-1}\star V'(z;Y)\star L \;=\; \sum_{k=1}^{\infty}\Psi^{\star k}\star v_\alpha^{(k)}(z) \end{array}}$$

$$V_\alpha = \sum_{k\geq 1} v_\alpha^{(k)}\star\Psi^{\star k} = \int_{-1}^{1}\frac{d\tau}{(\tau+1)^2}\,{}_1F_{1\star}(1/2;2;b\log\tau^2\Psi)\star z_\alpha\, e^{i\frac{\tau-1}{\tau+1}w_z}$$

(C.I., P. Sundell,;
D.De Filippi, C.I., P. Sundell)

- A large space of exact solutions (HS black holes, HSBH + massless scalar, FLRW-like,…) built this way. Symmetries and b.c. encoded in the associative algebra $\mathcal{A}(Y)$ that Φ' belongs to → different solution space are singled out by different basis functions (or distributions) of *Y* on which one expands *C'* (i.e., Ψ') .

- Factorized scheme encodes a (formal) solution space in which Φ is first-order exact, and the *Z*-dependence is solved in a universal way
→ gives a systematic procedure to non-linearly deform solutions of the KG and Bargmann-Wigner eqs. into solutions of the full Vasiliev eqs.

Comments and observations

- Actual solutions must satisfy:
  1. The star-products $(\Psi)^{\star k}$ (and $(\Psi)^{\star k} \star v(z)$) must be finite → conditions on $\mathcal{A}(Y)$
  2. Observables should be finite

Extra conditions on $\mathcal{A}(Y)$ from imposing boundary conditions

- Price for expanding in factorized form (WO): $v^{(k)}(z)$ are distributions, and so is V in WO.

  → However, one only cares that generating functions of physical fields are regular after taking the star products (NO).

- Moreover, Fronsdal fields can't be extracted in this frame as U remains not glued to C. In order to interpret the solution as a configuration of deformed Fronsdal theory → transform to a frame with non-trivial Chevalley-Eilenberg cocycle.

This can be induced by modifying the gauge function (with field-dependent correction)

$$L \ \to \ G \ := \ L \star (1 + \sum_{n \geq 1} H^{(n)})$$

Fronsdal fields and...

- Existence of such H and *regularity* of resulting W impose further restrictions to $\mathcal{A}(Y)$ (restricting admissible class of functions).

- Proposal: perturbative problem moved to building a gauge function $H= H(x,z;Y)$
  - 1^{st} order will be determined via a gauge condition
  - higher orders via imposing asymptotically AdS b.c. on master fields

- Requiring a non-trivial gauge field sector $U^{(n\geq 1)} \neq 0$ that contains a generating function for Fronsdal fields means finding $H^{(1)}$ such that

$$\Phi^{(1,L)} = C^{(1)}(x;Y) = L^{-1}\star C'\star\pi(L) \longrightarrow \Phi^{(1,G)} = (G^{-1}\star C'\star\pi(G))^{(1)} \equiv C^{(1)}(x;Y)$$

$$V^{(1,L)} = L^{-1}\star V'\star L \longrightarrow V^{(1,G)} = (G^{-1}\star V'\star G)^{(1)} = V^{(1,L)}+qH^{(1)}$$

$$U^{(1,L)} = 0 \longrightarrow U^{(1,G)} = (G^{-1}\star dG)^{(1)} = D^{(0)}H^{(1)}$$

with $W := (D^{(0)}H^{(1)})_{Z=0}$ real-analytic in Y .

(D.De Filippi, C.I., P. Sundell)

- Can be obtained by imposing relaxed Vasiliev gauge

$$\boxed{i_Z V^{(1,G)} \equiv Z^{\underline{\alpha}} V^{(1,G)}_{\underline{\alpha}} = O(Z^2)}$$

...AAdS boundary conditions

- Inserting the expression for $V^{(1,G)}$ in the gauge condition and solving for $H^{(1)}$

$$H^{(1)} = -\int_0^1 dt\, Z^{\underline{\alpha}} V^{(1,L)}_{\underline{\alpha}}(x,tZ,Y) + h^{(1)}(x,Y) + H_2^{(1)}(x,Z,Y)$$

- Now the gauge field generating function is non-trivial,

$$W_{\rm phys}(x,Y) \;=\; U^{(1,G)}|_{Z=0} \;=\; \left(D^{(0)}_{\rm ad} H^{(1)}\right)_{Z=0} \;=\; i\Omega^{\underline{\alpha\beta}}\partial^Y_{\underline{\beta}} V^{(1,L)}_{\underline{\alpha}}(x,0,Y) + D^{(0)}_{\rm ad} h^{(1)}(x,Y)$$

regular (singularities in V are absorbed by $D^{(0}h$, i.e. they are pure gauge),
and glued to propagating dof [note that $D^{(0)}(D^{(0)}H^{(1)}(x,Y,Z))_{Z=0} \neq 0$!],

$$D^{(0)}_{\rm ad} W_{\rm phys} \;=\; \Omega^{\underline{\alpha\beta}}\partial^Y_{\underline{\beta}}\Omega^{\underline{\gamma\beta}}\partial^Y_{\underline{\beta}}\left(\partial^Z_{[\underline{\gamma}} V^{(1,L)}_{\underline{\alpha}]}\right)_{Z=0} \;=\; -\frac{ib}{4}\bar{H}^{\dot\alpha\dot\alpha}\partial^{\bar y}_{\dot\alpha}\partial^{\bar y}_{\dot\alpha}\Phi^{(1)}(x;0,\bar y) - \frac{i\bar b}{4} H^{\alpha\alpha}\partial^y_\alpha\partial^y_\alpha\Phi^{(1)}(x;y,0)$$

- H_2 has no impact on gauge fields. However, it can be used to impose b.c. .
  E.g., that all master fields reduce asymptotically to free Fronsdal fields on AdS,
  (to match the usual holographic setup) or, in fact, linearize asymptotically

$$\Phi^{(n,G)} \;=\; \mathcal{O}(1/r)\;, \qquad V^{(n.G)} \;=\; \mathcal{O}(1/r)\;, \qquad U^{(n,G)} \;=\; \mathcal{O}(1/r)\;, \qquad n>1$$

AAdS boundary conditions

- The general idea is to expand the master fields (Φ,*U*,*V*) in powers of 1/*r*, and fix AAdS b.c. by demanding that in the asymptotic limit the bulk master fields solve the unfolded linearized equations (Fronsdal).

  This is nontrivial, because interactions may mix different spins (e.g, nonlinear spin s'< s constructs may contribute to a leading $1/r^{s+1}$).
  [Perturbative expansion and asymptotic expansion do not coincide]

- Simplest version: use the residual gauge freedom in $H^{(n\geq 2)}$ to impose that asymptotically free Fronsdal fields = linearized fields, i.e., in "physical" gauge the asymptotics coincide with the first-order piece of the bulk solution.

$$U^{(n,G)} = D^{(0)}_{\rm ad} H^{(n)} + U^{(n)}_{H;C}\ , \qquad \Phi^{(n,G)} = C^{(n)} + \Phi^{(n)}_{H;C}\ ,$$
$$V^{(n,G)} = V^{(n)} + qH^{(n)} + V^{(n)}_{H;C}\ ,$$

($U_{H;C}^{(n)}$, $\Phi_{H;C}^{(n)}$, $V_{H;C}^{(n)}$ = constructs of $H^{(n')}$ and $C^{(n')}$, n'<n, present for n>1)

$$C^{(n)} = -\Phi^{(n)}_{H;C}\ , \qquad \text{etc.}$$

$$\Phi^{(n,G)} \ = \ \mathcal{O}(1/r)\ , \qquad V^{(n.G)} \ = \ \mathcal{O}(1/r)\ , \qquad U^{(n,G)} \ = \ \mathcal{O}(1/r)\ , \qquad n>1$$

Boundary conditions and observables

- At each order n one uses the freedom in $H^{(n-1)}$ and in the integration constant $C^{(n)}$ to impose AAdS b.c. .

- This means that , while H has no relevance for classical gauge-invariant observables, imposing b.c. leaves nonetheless a trace on them via the higher-order corrections $C^{(n)}$, that will become n-linear functionals of $C^{(1)}$.

- This mechanism can thus induce non-linear corrections to classical observables like

$$K = \oint_{\mathcal{Z}} \mathrm{Tr}_{\mathcal{A}(\mathcal{Y})}(\Phi \star \Phi^\dagger \star J^{\star 2})$$

- Evaluating these observables on bulk-to-boundary propagators → boundary correlation functions. Check finiteness!
  (three-point functions computed by extracting $\Phi^{(2)}$ in standard frame are puzzling)

(Giombi, Yin; Colombo, Sundell; Boulanger, Kessel, Skvortsov, Taronna)

- Recently, this analysis has been extended to a family of M-orderings interpolating between NO and WO, singling out proper homotopy to extract Fronsdal fields.

(D. De Filippi, C.I., P. Sundell, in preparation)

Conclusions and outlook

- Vasiliev eqs are formulated in a language tailored to peculiarities of HS and powerful enough to try and understand pressing issues such as (non-)locality of interactions.
  Several related open questions: criteria to establish allowed class of functions, field redefinitions, role of ordering prescriptions, how to impose b.c. ...

- Several indications that HS gravity requires to go beyond the standard field theoretic interpretation (at the level of component fields), which only makes sense in special regimes.

- Using different homotopy contractions/orderings helps reducing non-locality as well as difficulty of perturbative expansion.

- Imposing b.c. may reduce the ambiguities: AAdS b.c. by adjusting gauge function and integration constants order by order: interactions of bulk master-fields affect leading order in asymptotic expansion → corrections to integration constants → affect observables !
  Computation of observables is the testing ground for this and other perturbative approaches.