$N = 1$ Supersymmetric Higher Spins in Various Dimensions

Mirian Tsulaia

Okinawa Institute of Science and Technology

Integrability, Holography, Higher-Spin Gravity and Strings
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Plan of the Talk

Part I. Free Fields

- Lagrangians for bosons and fermions
- $N = 1$ Supersymmetry in $D = 3, 4, 6, 10$ dimensions
- Lower spin examples

Part II. Cubic Interactions

- Nonsupersymmetric vertices. Supersymmetric vertices
- Conclusions, open problems

Based on

- D. Sorokin, M.T.,
  Nucl. Phys. **B 929**, 216, (2018), arXiv: 1801.04615
- I.L. Buchbinder, V.A. Krykhtin, M.T., D. Weissman,
  Nucl. Phys. **B 967**, 115427, (2021); arXiv: 2103.08231

- Higher Spin theories are already nontrivial at a free level, let alone the interactions
- They are interesting in their own right
- Understanding properties of Higher Spin fields can provide further insights into Holography, String Theory, Quantum Gravity, Cosmology
- Supersymmetric Higher Spin Theories are very interesting but relatively less explored, especially in higher dimensions
- We shall consider massless Higher Spin Fields on a flat background
- Use the “Metric–like” formalism
- Use a particular formulation of Open Superstring Field Theory (and SUGRA’s) as a hint

- Take the second rank tensor field and make the Klein-Gordon equation gauge invariant

$$\Box g_{\mu\nu}(x) = \partial_\mu C_\nu(x) + \partial_\nu C_\mu(x)$$

by introducing an extra field $C_\mu(x)$

$$\delta g_{\mu\nu}(x) = \partial_\mu \lambda_\nu(x) + \partial_\nu \lambda_\mu(x), \quad \delta C_\mu(x) = \Box \lambda_\mu(x)$$

- Similarly the transversality equation

$$\partial^\nu g_{\mu\nu}(x) - \partial_\mu D(x) = C_\mu(x)$$

By introducing one more field $D(x)$, with $\delta D(x) = \partial^\mu \lambda_\mu(x)$

- Finally a gauge invariant field equation for $D(x)$ is

$$\Box D(x) = \partial^\mu C_\mu(x)$$

- The corresponding Lagrangian

$$\mathcal{L} = -\frac{1}{2}(\partial^\mu g^{\nu\rho})(\partial_\mu g_{\nu\rho}) + 2C^\mu \partial^\nu g_{\mu\nu} - C^\mu C_\mu + (\partial^\mu D)(\partial_\mu D) + 2D\partial^\mu C_\mu$$

Describes two physical fields with spins 2 and 0, contained in $g_{\mu\nu}(x)$

- A spin-vector field $\Psi^a_\mu(x)$, where a is a spinorial index.
- Gauge invariant transversality condition

$$\partial^\mu \Psi_\mu(x) + \gamma^\nu \partial_\nu \chi(x) = 0$$

- Introduced an extra field $\chi^a(x)$.

$$\delta\Psi_\mu(x) = \partial_\mu \tilde{\lambda}(x), \quad \delta\chi(x) = -\gamma^\nu \partial_\nu \tilde{\lambda}(x)$$

- The gauge invariant Dirac equation

$$\gamma^\nu \partial_\nu \Psi_\mu(x) + \partial_\mu \chi(x) = 0$$

- The equations are again Lagrangian

$$L_F = -i\bar{\Psi}^\nu \gamma^\mu \partial_\mu \Psi_\nu - i\bar{\Psi}^\mu \partial_\mu \chi + i\bar{\chi}\partial^\mu \Psi_\mu + i\bar{\chi}\gamma^\mu \partial_\mu \chi$$

- Describes spins $\frac{3}{2}$ and $\frac{1}{2}$ - gamma trace

- Always one physical field $\Psi^{(n)}(x)$ and two auxiliary fields $\Sigma^{(n-2)}(x)$ and $\chi^{(n-1)}(x)$
- The Lagrangian

$$\begin{aligned} L_F &= -i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - i\,n\bar{\Psi}\partial\chi + i\,n\bar{\chi}\partial\cdot\Psi + i\,n(n-1)\bar{\Sigma}\gamma^\mu\partial_\mu\Sigma \\ &+ i\,n\bar{\chi}\gamma^\mu\partial_\mu\chi - i\,n(n-1)\bar{\chi}\partial\Sigma + i\,n(n-1)\bar{\Sigma}\partial\cdot\chi\,. \end{aligned}$$

- BRST construction. The fields are of the form

$$|\Phi\rangle = \frac{1}{s!}\Phi_{\mu_1\mu_2,\ldots\mu_s}(x)\alpha^+_{\mu_1}\alpha^+_{\mu_2}\ldots\alpha^+_{\mu_s}|0\rangle, \quad [\alpha_\mu, \alpha^+_\nu] = \eta_{\mu\nu}$$

- Introducing divergence, gradient and Dirac operators

$$l = p\cdot\alpha, \quad l^+ = p\cdot\alpha^+, \quad g_0 = p\cdot\gamma$$

- Expansion in terms of b^+, c^+ ghosts produces the fields $\Sigma^{(n-2)}(x)$ and $\chi^{(n-1)}(x)$.

- Use open superstring field theory as a hint
- It is possible to obtain the free systems above by taking a formal limit $\alpha' \to \infty$ in the free field equations for the Open Superstring
- The fields will become massless, gauge invariance holds in any number of space-time dimensions i.e, $(Q_B)^2 = 0$ and $(Q_F)^2 = 0$
- Superstring (bosonic string) contains different types of oscillators $\alpha_i^{\mu,+}$, $\psi_k^{\mu,+}$, etc. Where i is integer, k is (half)integer
- As a result we have mixed symmetry fields. They are massive in the case of finite α'
- In the limit $\alpha' \to 0$ is possible to truncate i and k to any finite value

- A Lagrangian (schematically)

$$L_{tot.} = \langle \Phi_B | Q_B | \Phi_B \rangle + \langle \Phi_F | Q_F | \Phi_F \rangle$$

- Invariant under supersymmetry transformations

$$\delta \langle \Phi_B | = \langle \Phi_F | \epsilon \mathcal{Q}, \qquad \delta | \Phi_F \rangle = \epsilon \mathcal{Q} | \Phi_B \rangle.$$

  provided the SUSY generator $\mathcal{Q}$ satisfies

$$Q_F \, \mathcal{Q} = \mathcal{Q} \, Q_B$$

- A solution can be found to be

$$\mathcal{Q} \; = {}_B\langle 0| \; \exp \left(\frac{1}{\sqrt{2}} \gamma \cdot \psi + (\beta\gamma \quad ghosts) \right) |0\rangle_F.$$

- SUSY closes on-shell in $D = 3, 4, 6, 10$, in both sectors. No pictures
- Consideration in OSFT: Y.Kazama, A.Neveu, H.Nicolai, P.West, Nucl.Phys. **B 278**, 833 (1986). Contains an infinite number of oscillators and fields. Presence of pictures.

- The fields are of the type $|X^{(n,b)}\rangle$, where n is a number of α_μ^+ oscillators, and $b = 0, 1$ is a number of ψ_μ^+ oscillators.
- The fermionic sector

$$|\Psi^{(n,0)}\rangle + c^+ b^+ |\Sigma^{(n-2,0)}\rangle, \quad b^+ |\chi^{(n-1,0)}\rangle$$

- The bosonic sector contains mixed symmetry fields

$$|\phi^{(n,1)}\rangle + c^+ b^+ |D^{(n-2,1)}\rangle + \gamma^+ b^+ |B^{(n-1,0)}\rangle + c^+ \beta^+ |A^{(n-1,0)}\rangle,$$

$$b^+ |C^{(n-1,1)}\rangle + \beta^+ |E^{(n,0)}\rangle + c^+ b^+ \beta^+ |F^{(n-2,0)}\rangle.$$

- The Lagrangians, gauge and SUSY transformations can be written in terms of these fields using explicit forms of Q_B, Q_F and $\mathcal{Q}$.
- For $n = 0$: Super-Maxwell.
  Bosons: $\phi_\mu(x)$, $E(x)$-auxiliary. Fermion $\Psi^a(x)$

- For $n = 1$: Linearized $N = 1$ SUGRA's in $D = 4, 6$ and 10
- The Lagrangian in the bosonic sector ($\phi_{\nu,\mu}(x)$ is physical)

$$
\begin{aligned}
L_B \quad &= -\phi^{\nu,\mu}\Box\phi_{\nu,\mu} + B\Box A + A\Box B \\
&+E^{\mu}\partial_{\mu}B + C^{\nu}\partial^{\mu}\phi_{\nu,\mu} + C^{\nu}\partial_{\nu}A + E^{\mu}\partial^{\nu}\phi_{\nu,\mu} \\
&-B\partial_{\alpha}E^{\mu} - \phi^{\nu,\mu}\partial_{\mu}C_{\nu} - A\partial_{\nu}C^{\nu} - \phi^{\nu\mu}\partial_{\nu}E_{\mu} \\
&+C^{\nu}C_{\nu} + E^{\mu}E_{\mu}\,.
\end{aligned}
$$

- The Lagrangian in the fermionic sector ($\Psi^{a}_{\mu}(x)$ is physical)

$$
L_F = -i\bar{\Psi}^{\mu}\gamma^{\nu}\partial_{\nu}\Psi_{\mu} - i\bar{\Psi}^{\mu}\partial_{\mu}\chi + i\bar{\chi}\partial_{\mu}\Psi^{\mu} + i\bar{\chi}\gamma^{\nu}\partial_{\nu}\chi,
$$

- SUSY transformations

$$
\delta\phi_{\nu,\mu}(x) = i\bar{\Psi}_{\mu}(x)\gamma_{\nu}\,\epsilon, \quad \delta C_{\nu}(x) = -i(\partial_{\mu}\bar{\chi}(x))\gamma^{\mu}\gamma_{\nu}\,\epsilon, \quad \delta B(x) = -i\bar{\chi}(x)\,\epsilon,
$$

$$
\delta\Psi_{\mu}(x) = -\gamma^{\nu}\gamma^{\rho}\epsilon\,\partial_{\nu}\phi_{\rho,\mu}(x) - \epsilon E_{\mu}(x), \quad \delta\chi(x) = -\gamma^{\nu}\epsilon\, C_{\nu}(x)\,.
$$

- Taking three copies of these fields we get for the Lagrangian

$$\mathcal{L}_{3B} \sim \sum_{i=1}^{3} \langle \Phi_{(i)} | Q_{(i)} | \Phi_{(i)} \rangle + g \langle \Phi_{(3)} | \langle \Phi_{(2)} | \langle \Phi_{(1)} || V \rangle$$

- Nonlinear gauge transformations

$$\delta_{cub.} | \Phi_{(1)} \rangle \sim Q_{(1)} | \Lambda_{(1)} \rangle - g (\langle \Phi_{(2)} | \langle \Lambda_{(3)} | + \langle \Phi_{(3)} | \langle \Lambda_{(2)} |) | V \rangle)$$

- The invariance of $\mathcal{L}_{3B}$:

$$\begin{aligned} g^0 : \qquad & Q_{(1)}^2 = Q_{(2)}^2 = Q_{(3)}^2 = 0 \\ g^1 : \qquad & (Q_{(1)} + Q_{(2)} + Q_{(3)}) | V \rangle = 0 \end{aligned}$$

- In the light cone gauge R.R.Metsaev (arXiv: 0712.3526)
- Elimination of the bosonic ghost zero modes "breaks" the BRST charge in the Fermionic sector into pieces

$$g_0 = p \cdot \gamma, \quad \tilde{Q}_F = c^+ \, \alpha \cdot p + c \, \alpha^+ \cdot p, \quad M_F = c^+ c$$

- It is easier to consider only physical components

$$|\Phi_B\rangle \equiv |\phi\rangle \quad and \quad |\Phi_F\rangle^a \equiv |\Psi\rangle^a$$

- Impose an off-shell gauge fixing condition

$$p \cdot \alpha|\phi\rangle = p \cdot \psi|\phi\rangle = 0 \quad and \quad p \cdot \alpha|\Psi\rangle = 0$$

- Take two copies of the fermionic field and one copy of the bosonic field

$$\mathcal{L}_{FFB} \sim \sum_{i=1}^{2} \langle \Psi_{(i)}| p_{(i)} \cdot \gamma |\Psi_{(i)}\rangle + \langle \phi_{(3)}| p_{(3)} \cdot p_{(3)} |\phi_{(3)}\rangle + g \langle \phi_{(3)}| \langle \Psi_{(2)}| \langle \Psi_{(1)}| |\mathcal{V}\rangle$$

- Nonlinear gauge transformations

$$
\begin{aligned}
\delta_{cub.}|\Psi_{(1)}\rangle &\sim Q_{(1)}|\Lambda_{(1)}\rangle - \\
&- g(\langle\Psi_{(2)}|\langle\Lambda_{(3)}||\mathcal{W}^{1}_{2,3}\rangle + \langle\phi_{(3)}|\langle\Lambda_{(2)}|)|\mathcal{W}^{1}_{3,2}\rangle) \\
\delta_{cub.}|\phi_{(3)}\rangle &\sim Q_{(3)}|\Lambda_{(3)}\rangle - \\
&- g(\langle\Psi_{(1)}|\langle\Lambda_{(2)}||\mathcal{W}^{3}_{1,2}\rangle + \langle\Psi_{(2)}|\langle\Lambda_{(1)}|)|\mathcal{W}^{3}_{2,1}\rangle)
\end{aligned}
$$

- Requirement of invariance of the Lagrangian $\mathcal{L}_{FFB}$ imposes conditions on the vertices $|\mathcal{V}\rangle$ and $|\mathcal{W}^{k}_{ij}\rangle$
- One has also to require preservation of the group structure
- One can add internal indices to the fields and to the vertices
- One can promote these systems to unconstrained ones

- Super Yang-Mills type

$$|\mathcal{V}\rangle_{ABC} = f_{ABC}(\gamma\cdot\psi^{+}_{(3)})\mathcal{F}(\mathcal{K},\mathcal{Z}_\alpha)|0\rangle_{FFB}, \quad |V\rangle_{ABC} = f_{ABC}\mathcal{Z}_\psi\mathcal{F}(\mathcal{K},\mathcal{Z}_\alpha)|0\rangle_{3B}$$

- Supergravity type

$$|\mathcal{V}\rangle = \gamma\cdot\psi^{+}_{(3)}\mathcal{Z}_\alpha\mathcal{F}(\mathcal{K},\mathcal{Z}_\alpha)|0\rangle_{FFB}, \quad |V\rangle = \mathcal{Z}_\psi\mathcal{Z}_\alpha\mathcal{F}(\mathcal{K},\mathcal{Z}_\alpha)|0\rangle_{3B}$$

where

$$\mathcal{Z}_\psi = (\psi^{+}_{(1)}\cdot\psi^{+}_{(2)})(\psi^{+}_{(3)}\cdot(p_{(1)}-p_{(2)})) + cyclic$$

$$\mathcal{Z}_\alpha = (\alpha^{+}_{(1)}\cdot\alpha^{+}_{(2)})(\alpha^{+}_{(3)}\cdot(p_{(1)}-p_{(2)})) + cyclic$$

$$\mathcal{K} = (\alpha^{+}_{(3)}\cdot(p_{(1)}-p_{(2)})) + cyclic$$

- In this gauge the supersymmetry transformations

$$\delta|\phi\rangle = \bar\epsilon\,(\psi^{+}\cdot\gamma)|\Psi\rangle, \quad \delta|\Psi\rangle = -2(p\cdot\gamma)\,(\psi\cdot\gamma)\,\epsilon\,|\phi\rangle$$

put the fields completely on-shell

- Construction of massive SUSY theories for the dimensions $D \geq 3$
- Consideration of higher order interactions and possible applications for Quantum Higher Spin thoeries
  (E.D. Skvortsov, T.Tran. M.T., (2018, 2020))
- Deformation to (anti)de Sitter spaces
- Further connection with the String Theory
- Many other questions

THANK YOU!!!