Higher Spins Down Under

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Integrability, Holography, Higher-Spin Gravity and Strings

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Based on:

S. M. Kuzenko and E. S. N. Raptakis
$\mathcal{N} = 2$ superconformal higher-spin gauge theories in four dimensions
[arXiv:2104.10416 [hep-th]]

S. M. Kuzenko and M. Ponds
Spin projection operators in (A)dS and partial masslessness
Phys. Lett. B **800**, 135128 (2020) [arXiv:1910.10440 [hep-th]].

Fortress Australia and higher spins

- Australia $\iff$ Down Under.
- In March 2020, the Australian government closed the borders.
- Australia is not expected to re-open its borders until mid-2022.
- Since we are not able to travel overseas to attend conferences etc., we just keep working ... and enjoying a COVID-free environment.
- This talk will give an overview of two projects completed over the period of one year and a half.

Fortress Australia and higher spins

HS projects completed at UWA during the last 1.5 years:

1. Spin projector operators in (A)dS_4 and partial masslessness
   SMK & M. Ponds, arXiv:1910.10440
2. (Super)conformal HS gauge models in Bach-flat backgrounds
   SMK & M. Ponds, arXiv:1912.00652;
   SMK, M. Ponds & E. S. N. Raptakis, arXiv:2005.08657; arXiv:2011.11300
3. Higher-spin gauge models with (p, q) supersymmetry in AdS_3
   D. Hutchings, J. Hutomo & SMK, [arXiv:2011.14294 [hep-th]].
4. AdS superprojectors and partial masslessness
   E. I. Buchbinder, D. Hutchings, SMK & M. Ponds, arXiv:2101.05524
5. Higher-spin (super) Cotton tensors in AdS_3 & gauge-invariant models
   SMK & M. Ponds, arXiv:2103.11673
6. $\mathcal{N} = 2$ superconformal HS gauge theories in four dimensions
   SMK & E. S. N. Raptakis, arXiv:2104.10416

This talk will review the projects **1** and **6**.

CHS actions & spin projection projectors

- Fradkin-Tseytlin action for a conformal integer spin-s field (1985)

$$S^{(s)}_{\rm CHS} \propto \int {\rm d}^4 x \, h^{a(s)} \Box^s \Pi^{(s)} h_{a(s)}$$

Gauge field $h_{a(s)} = h_{a_1 \dots a_s}$ is symmetric and traceless.

- $\Pi^{(s)}$ is Behrends-Fronsdal transverse & traceless projector (1957)

$$\partial^b h^{\rm T}_{ba(s-1)} = 0 \, , \qquad h^{\rm T}_{a(s)} := \Pi^{(s)} h_{a(s)}$$

In the vector notation, the expression for $\Pi^{(s)}$ is horribly complicated.

- Switching to spinor notation

$$h_{a(s)} \ \to \ h_{\alpha_1 \dots \alpha_s \dot\alpha_1 \dots \dot\alpha_s} := (\sigma^{a_1})_{\alpha_1 \dot\alpha_1} \dots (\sigma^{a_s})_{\alpha_s \dot\alpha_s} h_{a_1 \dots a_s} = h_{(\alpha_1 \dots \alpha_s)(\dot\alpha_1 \dots \dot\alpha_s)}$$

makes $\Pi^{(s)}$ remarkably simple: J. Gates et al., *Superspace* (1983)

$$\Pi^{(s)} h_{\alpha(s)\dot\alpha(s)} = \Box^{-s} \partial^{\beta_1}{}_{\dot\alpha_1} \dots \partial^{\beta_s}{}_{\dot\alpha_s} C_{\alpha_1 \dots \alpha_s \beta_1 \dots \beta_s}$$

$$C_{\alpha(2s)} := \partial_{(\alpha_1}{}^{\dot\beta_1} \dots \partial_{\alpha_s}{}^{\dot\beta_s} h_{\alpha_{s+1} \dots \alpha_{2s})\dot\beta_1 \dots \dot\beta_s}$$

CHS actions & linearised HS Weyl tensors

- Fradkin-Linetsky action for a conformal integer spin-s field (1989)

$$S^{(s)}_{\rm CHS} \propto \int {\rm d}^4 x\, C^{\alpha(2s)} C_{\alpha(2s)} + {\rm c.c.}$$

- Conformal higher-spin actions in diverse dimensions
  E. Fradkin & V. Linetsky
  C. Pope & P. Townsend
  A. Segal
  M. Vasiliev
  R. Metsaev
  ....

Transverse projectors in AdS_4

- AdS_4 Lorentz covariant derivative $\mathcal{D}_a = e_a{}^m \partial_m + \frac{1}{2}\omega_a{}^{bc} M_{bc}$

$$\big[\mathcal{D}_a, \mathcal{D}_b\big] = -\mu\bar{\mu} M_{ab} \quad \Leftrightarrow \quad \big[\mathcal{D}_{\alpha\dot{\alpha}}, \mathcal{D}_{\beta\dot{\beta}}\big] = -2\mu\bar{\mu}\Big(\varepsilon_{\alpha\beta}\bar{M}_{\dot{\alpha}\dot{\beta}} + \varepsilon_{\dot{\alpha}\dot{\beta}} M_{\alpha\beta}\Big)$$

scalar curvature $\mathcal{R} = -12\mu\bar{\mu}$ (Useful parametrisation in a SUSY framework)

- $V_{(m,n)}$ = linear space of unconstrained fields $\phi_{\alpha(m)\dot{\alpha}(n)}$ of Lorentz type $(\frac{m}{2}, \frac{n}{2})$
- $V^{\mathrm{T}}_{(m,n)}$ = linear space of transverse fields $\phi^{\mathrm{T}}_{\alpha(m)\dot{\alpha}(n)}$ of Lorentz type $(\frac{m}{2}, \frac{n}{2})$

$$\mathcal{D}^{\beta\dot{\beta}} \phi^{\mathrm{T}}_{\beta\alpha(m-1)\dot{\beta}\dot{\alpha}(n-1)} = 0$$

- Transverse projector is a surjective map $\Pi^{(m,n)} : V_{(m,n)} \to V^{\mathrm{T}}_{(m,n)}$

$$\phi_{\alpha(m)\dot{\alpha}(n)} \quad \mapsto \quad \Pi^{(m,n)} \phi_{\alpha(m)\dot{\alpha}(n)} \equiv \phi^{\mathrm{T}}_{\alpha(m)\dot{\alpha}(n)}$$

- Transverse projectors $\Pi^{(m,n)}$ square to themselves

$$\Pi^{(m,n)} \Pi^{(m,n)} = \Pi^{(m,n)}$$

Transverse projectors in AdS_4

Lessons from conformal higher-spin theory:

- Conformal gauge prepotential $\phi_{\alpha(m)\dot\alpha(n)}$ and its conjugate $\bar\phi_{\alpha(n)\dot\alpha(m)}$

$$\delta_\zeta \phi_{\alpha(m)\dot\alpha(n)} = \mathcal{D}_{(\alpha_1(\dot\alpha_1}\zeta_{\alpha_2\dots\alpha_m)\dot\alpha_2\dots\dot\alpha_n)}$$

- Linearised conformal higher-spin (CHS) Weyl tensors

$$\hat{\mathfrak{C}}_{\alpha(m+n)}(\phi) = \mathcal{D}_{(\alpha_1}{}^{\dot\beta_1}\cdots\mathcal{D}_{\alpha_n}{}^{\dot\beta_n}\phi_{\alpha_{n+1}\dots\alpha_{m+n})\dot\beta(n)}$$
$$\check{\mathfrak{C}}_{\alpha(m+n)}(\bar\phi) = \mathcal{D}_{(\alpha_1}{}^{\dot\beta_1}\cdots\mathcal{D}_{\alpha_m}{}^{\dot\beta_m}\bar\phi_{\alpha_{m+1}\dots\alpha_{m+n})\dot\beta(m)}$$

- They are gauge invariant

$$\delta_\zeta\hat{\mathfrak{C}}_{\alpha(m+n)}(\phi) = 0 \qquad\qquad \delta_\zeta\check{\mathfrak{C}}_{\alpha(m+n)}(\bar\phi) = 0$$

- Conformal higher-spin action in AdS_4

$$S^{(m,n)}_{\mathrm{CHS}}[\phi,\bar\phi] = \mathrm{i}^{m+n}\int \mathrm{d}^4x\, e\, \hat{\mathfrak{C}}^{\alpha(m+n)}(\phi)\check{\mathfrak{C}}_{\alpha(m+n)}(\bar\phi) + \mathrm{c.c.}$$

Transverse projectors in AdS$_4$

- Integrate CHS action by parts

$$S^{(m,n)}_{\text{CHS}}[\phi,\bar\phi] = \mathrm{i}^{m+n}\int \mathrm{d}^4x\, e\, \bar\phi^{\alpha(n)\dot\alpha(m)}\mathbb{B}_{\alpha(n)\dot\alpha(m)}(\phi) + \text{c.c.}$$

- Linearised higher-spin Bach tensor

$$\mathbb{B}_{\alpha(n)\dot\alpha(m)}(\phi) = \mathcal{D}_{(\dot\alpha_1}{}^{\beta_1}\cdots\mathcal{D}_{\dot\alpha_m)}{}^{\beta_m}\hat{\mathfrak{C}}_{\alpha(n)\beta(m)}(\phi)$$

- Transverse and gauge invariant

$$\mathcal{D}^{\beta\dot\beta}\mathbb{B}_{\beta\alpha(n-1)\dot\beta\dot\alpha(m-1)}(\phi) = 0\ , \qquad \delta_\zeta\mathbb{B}_{\alpha(n)\dot\alpha(m)}(\phi) = 0$$

- Rank-(m,n) Bach operator $\mathbb{B}^{(m,n)} : V_{(m,n)} \to V^{\text{T}}_{(n,m)}$
- Assume $m \geq n$ and introduce operator $\mathbb{P}^{(m,n)} : V_{(m,n)} \to V^{\text{T}}_{(n,m)} \to V^{\text{T}}_{(m,n)}$

$$\mathbb{P}_{\alpha(m)\dot\alpha(n)}(\phi) = \mathcal{D}_{(\alpha_1}{}^{\dot\beta_1}\cdots\mathcal{D}_{\alpha_{m-n}}{}^{\dot\beta_{m-n}}\mathbb{B}_{\alpha_{m-n+1}\dots\alpha_m)\dot\alpha(n)\dot\beta(m-n)}(\phi)$$

- $\mathbb{P}^{(m,n)}$ preserves rank of $\phi_{\alpha(m)\dot\alpha(n)}$ and is transverse

$$\mathcal{D}^{\beta\dot\beta}\mathbb{P}_{\beta\alpha(m-1)\dot\beta\dot\alpha(n-1)}(\phi) = 0$$

Transverse projectors in AdS_4

- However $\mathbb{P}^{(m,n)}$ does not square to itself

$$\mathbb{P}^{(m,n)}\mathbb{P}^{(m,n)}\phi_{\alpha(m)\dot\alpha(n)} = \prod_{t=1}^{n}\Big(\mathcal{Q} - \lambda_{(t,m,n)}\mu\bar\mu\Big)\mathbb{P}^{(m,n)}\phi_{\alpha(m)\dot\alpha(n)}$$

- Here $\mathcal{Q}$ is the quadratic Casimir of the AdS_4 isometry group

$$\mathcal{Q} := \Box - \mu\bar\mu\Big(M^{\gamma\delta}M_{\gamma\delta} + \bar M^{\dot\gamma\dot\delta}\bar M_{\dot\gamma\dot\delta}\Big)~, \qquad \Big[\mathcal{Q}, \mathcal{D}_a\Big] = 0$$

  and $\lambda_{(t,m,n)}$ are dimensionless constants

$$\lambda_{(t,m,n)} := \frac{1}{2}\Big[(m+n-t+3)(m+n-t-1) + (t-1)(t+1)\Big]$$

- Define the transverse operator $\Pi^{(m,n)} : V_{(m,n)} \to V^{\mathrm{T}}_{(n,m)}$

$$\Pi^{(m,n)} := \Big[\prod_{t=1}^{n}\Big(\mathcal{Q}-\lambda_{(t,m,n)}\mu\bar\mu\Big)\Big]^{-1}\mathbb{P}^{(m,n)}$$

- $\Pi^{(m,n)}$ squares to itself: $\qquad \Pi^{(m,n)}\Pi^{(m,n)} = \Pi^{(m,n)}$

Transverse projectors in AdS$_4$

- $\Pi^{(m,n)}$ projects onto the transverse subspace $V^{\rm T}_{(m,n)}$:

$$\mathcal{D}^{\beta\dot\beta}\phi^{\rm T}_{\beta\alpha(m-1)\dot\beta\dot\alpha(n-1)} = 0 \qquad \Pi^{(m,n)}\phi_{\alpha(m)\dot\alpha(n)} \equiv \phi^{\rm T}_{\alpha(m)\dot\alpha(n)}$$

- Orthogonal complement $\Pi^{(m,n)}_{\perp}$

$$\Pi^{(m,n)}_{\perp} : V_{(m,n)} \to V^{\rm L}_{(m,n)} \qquad \Pi^{(m,n)}_{\perp} := \mathbb{1} - \Pi^{(m,n)}$$

projects onto the space of longitudinal fields $V^{\rm L}_{(m,n)}$

$$\Pi^{(m,n)}_{\perp}\phi_{\alpha(m)\dot\alpha(n)} = \mathcal{D}_{\alpha\dot\alpha}\phi_{\alpha(m-1)\dot\alpha(n-1)}$$

- Can decompose any unconstrained field $\phi_{\alpha(m)\dot\alpha(n)}$ as follows

$$\phi_{\alpha(m)\dot\alpha(n)} = \phi^{\rm T}_{\alpha(m)\dot\alpha(n)} + \mathcal{D}_{\alpha\dot\alpha}\phi^{\rm T}_{\alpha(m-1)\dot\alpha(n-1)} + \cdots + \underbrace{\mathcal{D}_{\alpha\dot\alpha}\dots\mathcal{D}_{\alpha\dot\alpha}}_{n-\text{times}}\phi_{\alpha(m-n)}$$

- $\Pi^{(m,n)}$ selects subspace $V^{\rm T}_{(m,n)}$ from decomposition (Behrends-Fronsdal)

$$V_{(m,n)} = V^{\rm T}_{(m,n)} \oplus V^{\rm T}_{(m-1,n-1)} \oplus \cdots \oplus V^{\rm T}_{(m-n+1,1)} \oplus V_{(m-n,0)}$$

Transverse projectors in AdS_4

- Transverse projectors have poles at special mass values

$$\Pi^{(m,n)} \quad \propto \quad \frac{1}{\big(\mathcal{Q} - \lambda_{(1,m,n)}\mu\bar{\mu}\big)\big(\mathcal{Q} - \lambda_{(2,m,n)}\mu\bar{\mu}\big)\cdots\big(\mathcal{Q} - \lambda_{(n,m,n)}\mu\bar{\mu}\big)}$$

- What is the significance of the constants $\lambda_{(t,m,n)}$?
- Consider an on-shell field $\phi^{\mathrm{T}}_{\alpha(m)\dot{\alpha}(n)}$

$$\big(\mathcal{Q} - \rho^2\big)\phi^{\mathrm{T}}_{\alpha(m)\dot{\alpha}(n)} = 0 \ , \qquad \mathcal{D}^{\beta\dot{\beta}}\phi^{\mathrm{T}}_{\beta\alpha(m-1)\dot{\beta}\dot{\alpha}(n-1)} = 0$$

- When $\rho^2 = \lambda_{(t,m,n)}\mu\bar{\mu}$ this system of equations admits the (restricted) depth-t partially-massless gauge symmetry

$$\delta_\zeta\phi_{\alpha(m)\dot{\alpha}(n)} = \underbrace{\mathcal{D}_{\alpha\dot{\alpha}}\cdots\mathcal{D}_{\alpha\dot{\alpha}}}_{t-\text{times}}\zeta_{\alpha(n-t)\dot{\alpha}(m-t)} \qquad 1 \leq t \leq n$$

- $\Pi^{(m,n)}$ contains information about massless ($t = 1$) and partially-massless ($t > 1$) fields of all possible depths

Transverse projectors in AdS_4

- Come back to the spin-s conformal higher-spin action with $m = n = s$

$$S^{(s,s)}_{\mathrm{CHS}}[\phi, \bar{\phi}] = \int \mathrm{d}^4x \, e \, \phi^{\alpha(s)\dot{\alpha}(s)} \mathbb{B}^{(s,s)} \phi_{\alpha(s)\dot{\alpha}(s)}$$

- Projectors $\Pi^{(s,s)}$ are related to the Bach operator $\mathbb{B}^{(s,s)}$ via

$$\mathbb{B}^{(s,s)} = \prod_{t=1}^{s} \Big(\mathcal{Q} - \lambda_{(t,s,s)} \mu \bar{\mu} \Big) \Pi^{(s,s)}$$

- Action is gauge invariant $\implies$ can fix transverse gauge $\phi_{\alpha(s)\dot{\alpha}(s)} \equiv \phi^{\mathrm{T}}_{\alpha(s)\dot{\alpha}(s)}$
- Transverse projector $\Pi^{(s,s)}$ acts as identity on $V^{\mathrm{T}}_{(s,s)}$
- CHS kinetic operator factorises into (minimal) second order wave operators for partially-massless fields of all depths $1 \leq t \leq s$

$$S^{(s,s)}_{\mathrm{CHS}}[\phi, \bar{\phi}] = \int \mathrm{d}^4x \, e \, \phi_{\mathrm{T}}^{\alpha(s)\dot{\alpha}(s)} \prod_{t=1}^{s} \Big(\mathcal{Q} - \lambda_{(t,s,s)} \mu \bar{\mu} \Big) \phi^{\mathrm{T}}_{\alpha(s)\dot{\alpha}(s)}$$

- Similar results for fermionic case $m = n + 1 = s + 1$ and general mixed symmetry case $m \neq n$

Supercurrents and off-shell gauge supermultiplets

- To determine the structure of (superconformal) higher-spin gauge prepotentials, we make use of the method of supercurrent multiplets.

  V. Ogievetsky & E. Sokatchev (1977)
  E. Bergshoeff, M. de Roo & B. de Wit (1981)
  P. Howe, K. Stelle & P. Townsend (1981)

- In supersymmetric field theory, all multiplets of (conformal) currents furnish off-shell representations of (conformal) supersymmetry.
- Once a (conformal) (higher-spin) supercurrent $J = \{J^i\}$ is known, associated gauge prepotential(s) $\Upsilon = \{\varphi_i\}$ are determined via Noether's coupling

$$\begin{aligned} S_{\rm NC} &= \int {\rm d}^d x {\rm d}^\delta \theta \, E \, J \cdot \Upsilon \ , \qquad E = {\rm Ber}(E_M{}^A) \\ &= \int {\rm d}^d x \, e \, J^i \varphi_i \end{aligned}$$

  The resulting gauge supermultiplet is automatically off-shell.

- This procedure is concisely described by Bergshoeff *et al.*: "One first assigns a field to each component of the current multiplet, and forms a generalized inner product of field and current components."

Supercurrents and off-shell gauge supermultiplets

Example:

- (Practically all) Weyl multiplets of conformal supergravity in dimensions $d \leq 6$.

Example:

- Massless higher-spin gauge $\mathcal{N} = 1$ supermultiplets in AdS_4

  SMK & A. Sibiryakov (1994)

- Non-conformal higher-spin $\mathcal{N} = 1$ supercurrents in AdS_4

  E. Buchbinder, J. Hutomo & SMK (2018)

Example: $\mathcal{N} = 1$ conformal supercurrents

- $\mathcal{N} = 1$ Minkowski superspace $\mathbb{M}^{4|4}$
  Covariant derivatives $\quad D_A = (\partial_a, D_\alpha, \bar{D}^{\dot\alpha})~, \quad \{D_\alpha, \bar{D}_{\dot\alpha}\} = -2\mathrm{i}\partial_{\alpha\dot\alpha}$
- **Goal:** Identify $\mathcal{N} = 1$ conformal supercurrents $\longrightarrow$ SCHS prepotentials
- **Type 1:** Described by $J^{\alpha(m)\dot\alpha(n)}$, $m, n \neq 0$, subject to:

$$D_\beta J^{\alpha(m-1)\beta\dot\alpha(n)} = 0~, \qquad \bar{D}_{\dot\beta} J^{\alpha(m)\dot\alpha(n-1)\dot\beta} = 0$$

  $J^{\alpha\dot\alpha}$ Ferrara-Zumino supercurrent (1975)
- **Type 2:** Rank-m spinor supercurrent $J^{\alpha(m)}$, $m \neq 0$, constrained by:

$$D_\beta J^{\alpha(m-1)\beta} = 0~, \qquad \bar{D}^2 J^{\alpha(m)} = 0$$

- **Type 3:** Flavour current supermultiplet J:

$$D^2 J = 0~, \qquad \bar{D}^2 J = 0$$

  J^α contains a conserved spinor current
- Identify prepotentials by requiring Noether's coupling to be gauge invariant

$$\delta S_{\rm NC} = \int \mathrm{d}^{4|4}z\, J \cdot \delta\Upsilon = 0$$

$\mathcal{N} = 1$ superconformal gauge prepotentials

- Arguments of the previous page lead to the three families of SCHS prepotentials.
- **Type 1:** Described by $\Upsilon_{\alpha(m)\dot{\alpha}(n)}$, $m, n \neq 0$, defined modulo:

$$\delta_{\zeta,\xi}\Upsilon_{\alpha(m)\dot{\alpha}(n)} = D_{(\alpha_1}\zeta_{\alpha_2\dots\alpha_m)\dot{\alpha}(n)} + \bar{D}_{(\dot{\alpha}_1}\xi_{\alpha(m)\dot{\alpha}_2\dots\dot{\alpha}_n)}$$

$H_{\alpha\dot{\alpha}} \equiv \Upsilon_{\alpha\dot{\alpha}} = \bar{\Upsilon}_{\alpha\dot{\alpha}}$ conformal supergravity prepotential
V. Ogievetsky & E. Sokatchev (1977); W. Siegel (1977); S. Ferrara & B. Zumino (1978)

- **Type 2:** Encoded in $\Upsilon_{\alpha(m)}$, $m \neq 0$, defined modulo:

$$\delta_{\zeta,\lambda}\Upsilon_{\alpha(m)} = D_{(\alpha_1}\zeta_{\alpha_2\dots\alpha_m)} + \lambda_{\alpha(m)} , \quad \bar{D}_{\dot{\alpha}}\lambda_{\alpha(m)} = 0$$

Υ_α Conformal gravitino multiplet

- **Type 3:** Vector multiplet Υ:

$$\delta_\lambda\Upsilon = \lambda + \bar{\lambda} , \quad \bar{D}_{\dot{\alpha}}\lambda = 0$$

$\mathcal{N} = 2$ conformal superspace

D. Butter (2011)

- Gauge $\mathcal{N} = 2$ superconformal algebra $\longrightarrow$ $\mathcal{N} = 2$ conformal superspace

$$\nabla_A = E_A{}^M \partial_M + \frac{1}{2}\Omega_A{}^{bc} M_{bc} + {\rm i}\Phi_A Y + {\rm i}\Phi_A{}^{ij} J_{ij} + B_A \mathbb{D} + \mathfrak{F}_A{}^B K_B$$

- Geometry has SYM structure and encoded in $\mathcal{N} = 2$ super-Weyl tensor $W_{\alpha\beta}$

$$\{\nabla_\alpha^i \,, \nabla_\beta^i\} = \varepsilon^{ij}\varepsilon_{\alpha\beta}\Big\{2\bar{W}_{\dot\gamma\dot\delta}\bar{M}^{\dot\gamma\dot\delta} + \frac{1}{2}\bar\nabla_{\dot\gamma k}\bar{W}^{\dot\gamma\dot\delta}\bar{S}^k_{\dot\delta} - \frac{1}{2}\nabla_{\gamma\dot\delta}\bar{W}^{\dot\delta}{}_{\dot\gamma}K^{\gamma\dot\gamma}\Big\}$$

$$\{\nabla_\alpha^i \,, \bar\nabla^{\dot\beta}_j\} = -2{\rm i}\delta^j_i \nabla_\alpha{}^{\dot\beta}$$

- $\mathcal{N} = 2$ super-Bach tensor

$$B = \nabla_{\alpha\beta}W^{\alpha\beta} = \bar\nabla^{\dot\alpha\dot\beta}\bar{W}_{\dot\alpha\dot\beta} = \bar{B} \ .$$

- Some useful shorthand:

$$\nabla_{\alpha\beta} = \nabla^i_{(\alpha}\nabla_{\beta)i} \ , \quad \bar\nabla_{\dot\alpha\dot\beta} = \bar\nabla_{(\dot\alpha i}\bar\nabla^i_{\dot\beta)} \ , \quad \nabla^{ij} = \nabla^{\alpha(i}\nabla^{j)}_\alpha \ , \quad \bar\nabla^{ij} = \bar\nabla^{(i}_{\dot\alpha}\bar\nabla^{\dot\alpha j)} \ .$$

$\mathcal{N}=2$ conserved current supermultiplets

- **Goal:** Identify $\mathcal{N}=2$ conformal supercurrents $\longrightarrow$ SCHS prepotentials
- **Type 1:** Described by $J^{\alpha(m)\dot\alpha(n)}$, $m,n\neq 0$, subject to:

$$\nabla^i_\beta J^{\alpha(m-1)\beta\dot\alpha(n)}=0~, \qquad \bar\nabla^i_{\dot\beta} J^{\alpha(m)\dot\alpha(n-1)\dot\beta}=0$$

  Special case $m=n$ Reality condition $J^{\alpha(n)\dot\alpha(n)}=\bar J^{\alpha(n)\dot\alpha(n)}$
  $\mathcal{N}=2$ Minkowski superspace $\mathbb{M}^{4|8}$: P. Howe, K Stelle & P. Townsend (1981)

- **Type 2:** Spinor supercurrents $J^{\alpha(m)}$, $m\neq 0$, constrained by:

$$\nabla^i_\beta J^{\alpha(m-1)\beta}=0~, \qquad \bar\nabla^{ij} J^{\alpha(m)}=0$$

- **Type 3:** Supergravity supercurrent J: M. Sohnius (1978)

$$\nabla^{ij}J=0~, \qquad \bar\nabla^{ij}J=0~.$$

- Identify prepotentials by requiring that the Noether coupling is gauge invariant

$$\delta S_{\rm NC}=\int {\rm d}^{4|8}z\, J\cdot\delta\Upsilon=0~.$$

$\mathcal{N} = 2$ SCHS prepotentials

- Arguments of the previous page imply three families of SCHS prepotentials
- **Type 1:** Described by $\Upsilon_{\alpha(m)\dot\alpha(n)}$, $m, n \neq 0$, defined modulo:

$$\delta_{\zeta,\xi}\Upsilon_{\alpha(m)\dot\alpha(n)} = \nabla^i_{(\alpha_1}\zeta_{\alpha_2\dots\alpha_m)\dot\alpha(n)i} + \bar\nabla^i_{(\dot\alpha_1}\xi_{\alpha(m)\dot\alpha_2\dots\dot\alpha_n)i}$$

  Special case $m = n$ Reality condition $\Upsilon_{\alpha(n)\dot\alpha(n)} = \bar\Upsilon\alpha(n)\dot\alpha(n)$
  $\mathcal{N} = 2$ Minkowski superspace $\mathbb{M}^{4|8}$: P. Howe, K Stelle & P. Townsend (1981)
- **Type 2:** Encoded in $\Upsilon_{\alpha(m)}$, $m \neq 0$, defined modulo:

$$\delta_{\zeta,\omega}\Upsilon_{\alpha(m)} = \nabla^i_{(\alpha_1}\zeta_{\alpha_2\dots\alpha_m)i} + \bar\nabla^{ij}\omega_{\alpha(m)ij}$$

- **Type 3:** Conformal supergravity multiplet $H = \Upsilon = \bar\Upsilon$:

$$\delta_\omega K = \bar\nabla^{ij}\omega_{ij} + \nabla^{ij}\bar\omega_{ij}$$

  $\mathcal{N} = 2$ Minkowski superspace $\mathbb{M}^{4|8}$: P. Howe, K Stelle & P. Townsend (1981)
  This transformation describes linearised $\mathcal{N} = 2$ Weyl multiplet
  B. de Wit, J. van Holten & A. Van Proeyen (1980)

$\mathcal{N} = 2$ SCHS theories

- Consider the higher-derivative chiral descendants of $\Upsilon_{\alpha(m)\dot\alpha(n)}$, $m, n \geq 0$

$$\hat{\mathfrak{W}}_{\alpha(m+n+2)}(\Upsilon) = \frac{1}{48}\bar\nabla^{ij}\bar\nabla_{ij}\nabla_{(\alpha_1}{}^{\dot\beta_1}\dots\nabla_{\alpha_n}{}^{\dot\beta_n}\nabla_{\alpha_{n+1}\alpha_{n+2}}\Upsilon_{\alpha_{n+3}\dots\alpha_{m+n+2})\dot\beta(n)}$$

$$\check{\mathfrak{W}}_{\alpha(m+n+2)}(\bar\Upsilon) = \frac{1}{48}\bar\nabla^{ij}\bar\nabla_{ij}\nabla_{(\alpha_1}{}^{\dot\beta_1}\dots\nabla_{\alpha_m}{}^{\dot\beta_m}\nabla_{\alpha_{m+1}\alpha_{m+2}}\bar\Upsilon_{\alpha_{m+3}\dots\alpha_{m+n+2})\dot\beta(m)}$$

- The following actions

$$S^{(m,n)} = \mathrm{i}^{m+n}\int \mathrm{d}^4x\mathrm{d}^4\theta\,\mathcal{E}\,\hat{\mathfrak{W}}^{\alpha(m+n+2)}(\Upsilon)\,\check{\mathfrak{W}}_{\alpha(m+n+2)}(\bar\Upsilon) + \mathrm{c.c.}$$

  are locally superconformal and gauge invariant in any conformally-flat background, $W^{\alpha\beta} = 0$.

- **Open problem:** Generalisation to Bach-flat backgrounds

$\mathcal{N}=2$ SCHS theory à la Tseytlin and Segal

- Couple SCHS prepotentials to hypermultiplet q^i; $\nabla_\alpha^{(i} q^{j)} = 0$ and $\bar\nabla_{\dot\alpha}^{(i} q^{j)} = 0$

$$S[q,\bar q;\Upsilon] = S_{\text{hyper}}[q,\bar q] + \sum_{s=0}^{\infty} \int \mathrm{d}^4x \mathrm{d}^4\theta \mathrm{d}^4\bar\theta \, E \, \Upsilon_{\alpha(s)\dot\alpha(s)} J^{\alpha(s)\dot\alpha(s)}$$

- Supercurrents in conformally-flat backgrounds:

$$J^{\alpha(s)\dot\alpha(s)} = \mathrm{i}^{s+1} \sum_{k=0}^{s} (-1)^k \binom{s}{k}^2 \nabla^{(\alpha_1(\dot\alpha_1} \dots \nabla^{\alpha_k\dot\alpha_k} q^i \nabla^{\alpha_{k+1}\dot\alpha_{k+1}} \dots \nabla^{\alpha_s)\dot\alpha_s)} \bar q_i$$

$$+ \frac{\mathrm{i}^s}{8} \sum_{k=0}^{s-1} (-1)^k \binom{s}{k} \binom{s}{k+1} \Big\{ \nabla^{(\alpha_1(\dot\alpha_1} \dots \nabla^{\alpha_k\dot\alpha_k} \nabla^{\alpha_{k+1} i} q_i \nabla^{\alpha_{k+2}\dot\alpha_{k+1}} \dots \nabla^{\alpha_s)\dot\alpha_{s-1}} \bar\nabla^{\dot\alpha_s) j} \bar q_j$$

$$- \nabla^{(\alpha_1(\dot\alpha_1} \dots \nabla^{\alpha_k\dot\alpha_k} \bar\nabla^{\dot\alpha_{k+1} i} q_i \nabla^{\alpha_{k+1}\dot\alpha_{k+2}} \dots \nabla^{\alpha_{s-1}\dot\alpha_s)} \nabla^{\alpha_s) j} \bar q_j \Big\}$$

- Compute effective action for $\Upsilon_{\alpha(s)\dot\alpha(s)}$:

$$\mathrm{e}^{\mathrm{i}\,\Gamma[\Upsilon]} = \int \mathcal{D}q \mathcal{D}\bar q \, \mathrm{e}^{\mathrm{i}\, S[q,\bar q;\Upsilon]}$$

- Logarithmically divergent sector of $\Gamma[\Upsilon] \longrightarrow$ SCHS models

Special case: $\mathcal{N}=2$ SCHS theories in AdS superspace

- AdS$^{4|8}$ covariant derivative $\mathcal{D}_A = E_A + \frac{1}{2}\Omega_A{}^{bc}M_{bc} + \Phi_A{}^{ij}J_{ij}$

$$\{\mathcal{D}^i_\alpha, \mathcal{D}^j_\beta\} = 4S^{ij}M_{\alpha\beta} + 2\varepsilon_{\alpha\beta}\varepsilon^{ij}S^{kl}J_{kl}\ , \quad \{\mathcal{D}^i_\alpha, \bar{\mathcal{D}}^{\dot\beta}_j\} = -2\mathrm{i}\delta^i_j\mathcal{D}_\alpha{}^{\dot\beta}$$

- **SCHS prepotentials:**

$$\delta_{\zeta,\lambda}\Upsilon_{\alpha(m)\dot\alpha(n)} = \mathcal{D}^i_{(\alpha_1}\zeta_{\alpha_2\dots\alpha_m)\dot\alpha(n)i} + \bar{\mathcal{D}}^i_{(\dot\alpha_1}\lambda_{\alpha(m)\dot\alpha_2\dots\dot\alpha_n)i} \quad (m,n\neq 0)$$

$$\delta_{\zeta,\omega}\Upsilon_{\alpha(m)} = \mathcal{D}^i_{(\alpha_1}\zeta_{\alpha_2\dots\alpha_m)i} + (\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij})\omega_{\alpha(m)ij} \quad (m\neq 0)$$

$$\delta_\omega\Upsilon = (\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij})\omega_{ij} + (\mathcal{D}^{ij} + 4S^{ij})\bar\omega_{ij}$$

- **Chiral and gauge invariant field strengths:**

$$\hat{\mathfrak{W}}_{\alpha(m+n+2)}(\Upsilon) = \frac{1}{48}(\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij})\bar{\mathcal{D}}_{ij}\mathcal{D}_{(\alpha_1}{}^{\dot\beta_1}\dots\mathcal{D}_{\alpha_n}{}^{\dot\beta_n}\mathcal{D}_{\alpha_{n+1}\alpha_{n+2}}\Upsilon_{\alpha_{n+3}\dots\alpha_{m+n+2})\dot\beta(n)}$$

$$\check{\mathfrak{W}}_{\alpha(m+n+2)}(\bar\Upsilon) = \frac{1}{48}(\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij})\bar{\mathcal{D}}_{ij}\mathcal{D}_{(\alpha_1}{}^{\dot\beta_1}\dots\mathcal{D}_{\alpha_m}{}^{\dot\beta_m}\mathcal{D}_{\alpha_{m+1}\alpha_{m+2}}\bar\Upsilon_{\alpha_{m+3}\dots\alpha_{m+n+2})\dot\beta(m)}$$

- **Locally superconformal and gauge invariant actions:**

$$S^{(m,n)} = \mathrm{i}^{m+n}\int \mathrm{d}^4x\mathrm{d}^4\theta\,\mathcal{E}\,\hat{\mathfrak{W}}^{\alpha(m+n+2)}(\Upsilon)\,\check{\mathfrak{W}}_{\alpha(m+n+2)}(\bar\Upsilon) + \mathrm{c.c.}$$

Thank you!