Acymptotes of multiplicity and transverse momentum correlation coefficients at large string density

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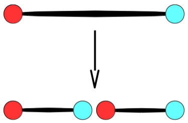
Quarks-2018
Valday, May 27- June 2

The string model

A. Capella, U. Sukhatme, Chung-I Tan, J. Tran Thanh Van, Phys. Lett. B 81, 68 (1979).
A.B. Kaidalov, Phys. Lett. B 116, 459 (1982).
A.B. Kaidalov, K.A.Ter-Martirosyan, Phys. Lett., 117B (1982) 247.
A. Capella, U. Sukhatme, Chung-I Tan, J. Tran Thanh Van, Phys. Rep. 236 (1994) 225.

First stage: colour quark-gluon strings (colour flux tubes) are formed
Second stage: hadronization of these strings produces the observed hadrons

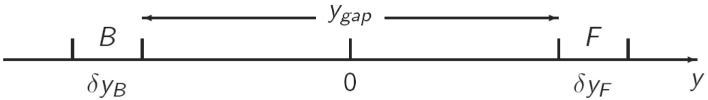


A. Capella and A. Krzywicki, Phys.Rev.D**18**, 4120 (1978)
Event-by-event variance in the number of strings ⇒
Long-Range FB Correlations (LRC) at large rapidity gap y_{gap}

Forward-Backward Rapidity Correlations

The definition of the rapidity:

$$y = \frac{1}{2} \ln \frac{p_0 + p_z}{p_0 - p_z}.$$



The correlation coefficient:

$$b_{FB} = \frac{\langle FB \rangle - \langle F \rangle \langle B \rangle}{\langle F^2 \rangle - \langle F \rangle^2} = \frac{cov(F,B)}{D_F}\ , \qquad b_{FB} = \left. \frac{d\langle B \rangle_F}{dF} \right|_{F=\langle F \rangle}$$

$\langle B \rangle_F = f(F)$ - the FB correlation function
$\langle B \rangle_F = a + b_{BF} F$ - the linear regression
The definitions are equivalent for linear regression function.

ALICE collaboration et al., J. Phys. G 32 1295 (2006), [Sect. 6.5.15]

Three types of correlations:

— $n - n$ -correlations;

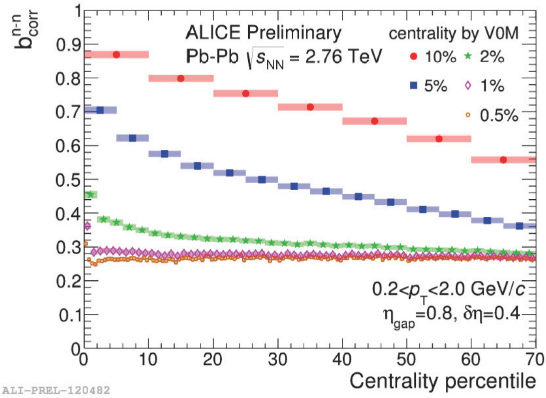
— $p_t - n$ -correlations;

— $p_t - p_t$ -correlations

$$p_{tB} \equiv \frac{1}{n_B} \sum_{i=1}^{n_B} |\mathbf{p}^i_{tB}| \qquad p_{tF} \equiv \frac{1}{n_F} \sum_{i=1}^{n_F} |\mathbf{p}^i_{tF}|$$

$n - n$ correlations in ALICE

B, $F \Rightarrow n_B$, n_F - the extensive variables $\Rightarrow b_{nn}$
Strongly influenced by "volume"fluctuations.



I. Altsybeev for the ALICE Collaboration.
Quark Matter 2017, 5-11 February 2017, Chicago, IL.

arXiv: 1711.04844v1

Intensive Observables

FB correlation between event-mean p_t of particles in $\delta\eta_F$-$\delta\eta_B$ intervals:

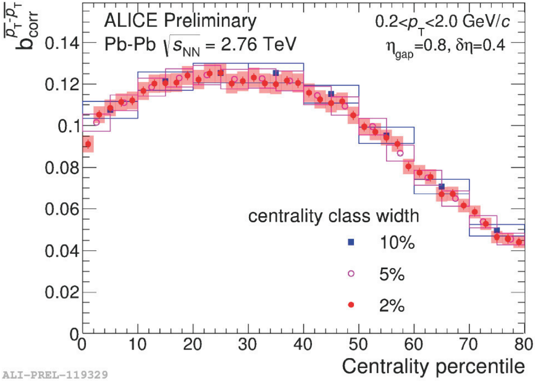
$$p_{tB} \equiv \frac{1}{n_B} \sum_{i=1}^{n_B} |\mathbf{p}_{tB}^i| \qquad p_{tF} \equiv \frac{1}{n_F} \sum_{i=1}^{n_F} |\mathbf{p}_{tF}^i|$$

$B,\ F \Rightarrow p_{tB},\ p_{tF}$ - the intensive variables $\Rightarrow b_{p_t p_t}$

LR event-mean p_t correlation is not sensitive to the fluctuation in the number of sources, but is sensitive to the fluctuation in the quality of sources.

$p_t - p_t$ correlations in ALICE

Resistant to "volume"fluctuations.



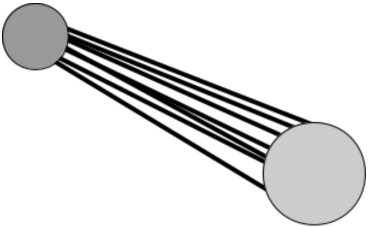
I. Altsybeev for the ALICE Collaboration.
Quark Matter 2017, 5-11 February 2017, Chicago, IL.

arXiv: 1711.04844v1

The model with string fusion

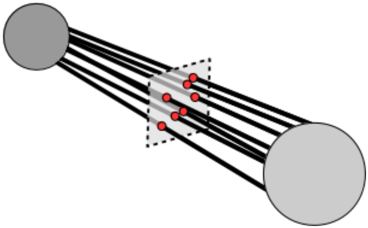
pp $\rightarrow$ pA $\rightarrow$ AA - the increase of the string density in transverse plain
M.A. Braun,C. Pajares, Phys.Lett. **B287**, 154 (1992);
Nucl. Phys. **B390**, 542 (1993).
$\Rightarrow$ Reduction of multiplicity, increase of transverse momenta.
N.S. Amelin, N. Armesto, M.A. Braun, E.G. Ferreiro, C. Pajares,
Phys.Rev.Lett. **73**, 2813 (1994).

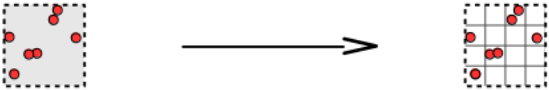
$\Rightarrow$ The influence on the Long-Range FB Correlations (LRC).



The version of SFM with the finite lattice in transverse plane

Vechernin V., Kolevatov R.S., hep-ph/0304295; hep-ph/0305136
Braun M.A., Kolevatov R.S., Pajares C., V.Vechernin Eur.Phys.J. C32 (2004) 535.
V.Vechernin, Kolevatov R.S. Phys.of Atom.Nucl. **70** (2007) 1797; 1858.





Model

M is the number of cells.
Event is characterized by the set of numbers:

$$C = \left\{C_\eta,\ C_n^B,\ C_n^F,\ C_p^B,\ C_p^F\right\},$$
$$C_\eta = \{\eta_1, \dots \eta_M\}\ ,$$
$$C_n^F = \left\{n_1^F,\ \dots,\ n_M^F\right\},$$
$$C_p^F = \left\{p_1^{1F},\ \dots,\ p_1^{n_1 F};\ \dots;\ p_M^{1F},\ \dots,\ p_M^{n_M F}\right\}$$
$$C_n^B = \left\{n_1^B,\ \dots,\ n_M^B\right\},$$
$$C_p^B = \left\{p_1^{1B},\ \dots,\ p_1^{n_1 B};\ \dots;\ p_M^{1B},\ \dots,\ p_M^{n_M B}\right\}.$$

$$n_F = \sum_{i=1}^{M} n_i^F,\ \ n_B = \sum_{i=1}^{M} n_i^B$$

$$p_t^F = \frac{1}{n_F} \sum_{i=1}^{M} \sum_{j=1}^{n_i^F} p_i^{j\,F},\ \ p_t^B = \frac{1}{n_B} \sum_{i=1}^{M} \sum_{j=1}^{n_i^B} p_i^{j\,B}.$$

The Gaussian approximation

Fluctuations in the number of strings in the cell are assumed independent and Gaussian distributed with variance proportional to the average number of strings in the cell:

$$P(\eta_i) = \frac{1}{\sqrt{2\pi d_{\eta_i}}} e^{-\frac{(\eta_i - \overline{\eta}_i)^2}{2 d_{\eta_i}}},$$

$$d_{\eta_i} = \omega_\eta \overline{\eta}_i,$$

and similarly for the number of particles in forward and backward windows:

$$P(n_i^F) = \frac{1}{\sqrt{2\pi d_{n_i^F}}} e^{-\frac{\left(n_i^F - \overline{n}_i^F\right)^2}{2 d_{n_i^F}}}, \; P(n_i^B) = \frac{1}{\sqrt{2\pi d_{n_i^B}}} e^{-\frac{\left(n_i^B - \overline{n}_i^B\right)^2}{2 d_{n_i^B}}},$$

$$d_{n_i^F} = \omega_\mu \overline{n}_i^F, \; d_{n_i^B} = \omega_\mu \overline{n}_i^B.$$

The average transverse momentum of the particles prodused from the hadronization of the strings in the cells is assumed independent on the numbers of particles and depend on the numbers of strings in the cell. The variance of the transverse momentum of the one particle is assumed proportional to the mean transverse mometnum sqwared.

$$d_{p_i}(\eta_i) = \overline{p^2}(\eta_i) - \overline{p}^2(\eta_i) = \gamma\overline{p}^2(\eta_i).$$

Model

The dependence of the average number of particles formed by hadronization of the string in the cell and the transverse momentum of these particles of the number of strings η_i in the cell:

$$\overline{n}\left(\eta_i\right) = \sqrt{\eta_i},\ \overline{p}\left(\eta_i\right) = p_0\sqrt[4]{\eta_i}.$$

The numbers of particles formed from the hadronizations of the strings in i-th cell in forward rapidity window

$$n_i^F = \mu_F \overline{n}(\eta_i).$$

And the same for the backward window

$$n_i^B = \mu_B \overline{n}(\eta_i).$$

The small paremeters:

$$\frac{1}{\overline{\eta}_i} \ll 1,\ \frac{1}{M} \ll 1.$$

Definitions

$$b^{mean}_{n\,n} = \frac{\langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle}{\langle (n_F)^2 \rangle - \langle n_F \rangle^2}, \;\; b^{corr\,f}_{n\,n} = \left.\frac{d\langle n_B \rangle_{n_F}}{dn_F}\right|_{n_F = \langle n_F \rangle}.$$

$$b^{mean}_{p_t\,n} = \frac{\langle n_F p_t^B \rangle - \langle p_t^F \rangle \langle n_B \rangle}{\langle (n_F)^2 \rangle - \langle p_t^F \rangle^2}, \;\; b^{corr\,f}_{p_t\,n} = \left.\frac{d\langle p_t^B \rangle_{n_F}}{dn_F}\right|_{n_F = \langle n_F \rangle}.$$

$$b^{mean}_{p_t\,p_t} = \frac{\langle p_t^F p_t^B \rangle - \langle p_t^F \rangle \langle p_t^B \rangle}{\langle (p_t^F)^2 \rangle - \langle p_t^F \rangle^2}, \;\; b^{corr\,f}_{p_t\,p_t} = \left.\frac{d\langle p_B \rangle_{p_F}}{dp_F}\right|_{p_F = \langle p_F \rangle}$$

Calcutation of the mean values

$$b_{BF}^{mean} \equiv \frac{\langle FB \rangle - \langle F \rangle \langle B \rangle}{\langle F^2 \rangle - \langle F \rangle^2}.$$

$$\langle F \rangle = \left\langle \left\langle \langle F \rangle^{C_p^F} \right\rangle^{C_n^F} \right\rangle^{C_\eta},$$

where $\langle \cdots \rangle^C$ is averaging over the configuration C,

$$\langle F \rangle^{C_n^F} = \prod_{i=1}^{M} \int dn_i^F P(n_i^F) F.$$

$$\langle F \rangle^{C_\eta} = \prod_{i=1}^{M} \int d\eta_i P(\eta_i) F,$$

Calculating of the correlation function

$$b_{BF}^{corr\ f} \equiv \left. \frac{d\langle B\rangle_F}{dF} \right|_{F=\langle F\rangle} .$$

$$\langle B\rangle_F = \frac{\sum_{C'} \langle B\rangle_{C'} P(C')P_{C'}(F)}{\sum_{C'} P(C')P_{C'}(F)}$$

In the case of great string density the configuration sum can be approximately rewritten as:

$$\sum_{C_\eta} \dots = \prod_{j=1}^{M} \sum_{\eta_j=0}^{\infty} \dots \longrightarrow \prod_{j=1}^{M} \int_0^\infty d\eta_j \dots ,$$

$$\langle B\rangle_F = \frac{1}{P(F)} \prod_{j=1}^{M} \int_0^\infty d\eta_j P(C_{\eta_j}) \int_0^\infty dn_j^B P_{C_\eta}(C_n^B) \langle B\rangle_{C_\eta C_n^B} \int_0^\infty dn_j^F P_{C_\eta}(C_n^F) P_{C_\eta C_n^F}(F),$$

$$P(F) = \prod_{j=1}^{M} \int_0^\infty d\eta_j P(C_{\eta_j}) \int_0^\infty dn_j^B P_{C_\eta}(C_n^B) \int_0^\infty dn_j^F P_{C_\eta}(C_n^F) P_{C_\eta C_n^F}(F).$$

$$S_\nu = \sum_{i=1}^{M} \overline{\eta}_i^\nu$$

$$b_{n\,n} = \frac{\omega_\eta \mu_B M}{4\omega_\mu S_{1/2} + \mu_F \omega_\eta M},$$

$$b_{p_t\,n} = \frac{p_0 \omega_\eta \left(\frac{3}{2} \frac{S_{1/4}}{S_{1/2}} - \frac{M S_{3/4}}{\left(S_{1/2}\right)^2} \right)}{4\omega_\mu S_{1/2} + M \mu_F \omega_\eta},$$

$$b_{p_t\,p_t} = \frac{\omega_\eta \mu_F \left(9S_{1/2}^3 - 12 S_{1/4} S_{3/4} S_{1/2} + 4 M S_{3/4}^2 \right)}{16\gamma S_1 S_{1/2}^2 + \omega_\eta \mu_F \left(9S_{1/2}^3 - 12 S_{1/4} S_{3/4} S_{1/2} + 4 M S_{3/4}^2 \right) + 16\omega_\mu S_{1/2} \left(S_1 S_{1/2} - S_{3/4}^2 \right)}.$$

Non-uniform string distribution in transverse plane:

$$\begin{array}{rcl} M_+ = mM & \qquad & \overline{\eta}_+ = a\overline{\eta} \\ M - M_+ = (1-m)M & \qquad & \overline{\eta} \end{array}$$

$a > 1,\ 0 \leqslant m \leqslant 1$

$$S_\nu = \sum_{i=1}^{M} \overline{\eta}_i^\nu = mM(a\overline{\eta})^\nu + (1-m)M\overline{\eta}^\nu$$

$$b_{n\,n} = \frac{\omega_\eta \mu_B M}{4\omega_\mu S_{1/2} + \mu_F \omega_\eta M},$$

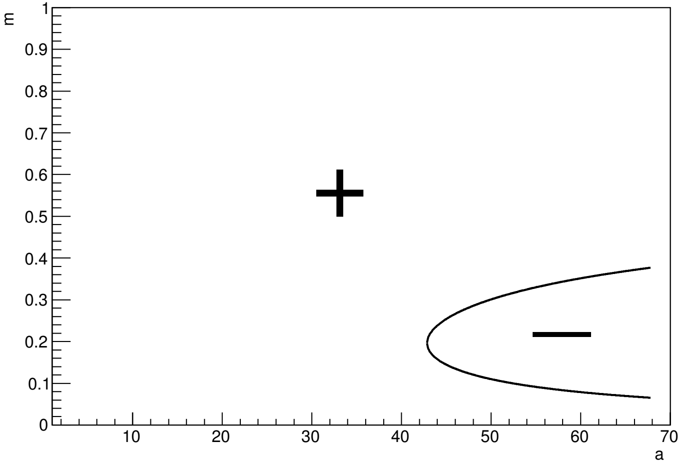
$$b_{p_t\,n} = \frac{p_0 \omega_\eta \left(\frac{3}{2} \frac{S_{1/4}}{S_{1/2}} - \frac{M S_{3/4}}{\left(S_{1/2}\right)^2} \right)}{4\omega_\mu S_{1/2} + M \mu_F \omega_\eta} = b_{n\,n} \frac{p_0 S_{3/4}}{\mu_B \left(S_{1/2}\right)^2} \left(\frac{3}{2} \frac{S_{1/4} S_{1/2}}{M S_{3/4}} - 1 \right)$$

$$\frac{3}{2} \frac{S_{1/4} S_{1/2}}{M S_{3/4}} - 1 < 0,$$

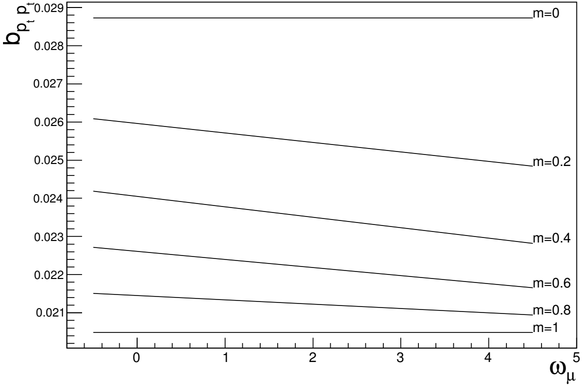
$$m^2(3a^{3/4} - 3a^{1/2} - 3a^{1/4} + 3) + m(-2a^{3/4} + 3a^{1/2} + 3a^{1/4} - 4) + 1 < 0$$

$$F(a,m) \equiv m^2(3a^{3/4} - 3a^{1/2} - 3a^{1/4} + 3) + m(-2a^{3/4} + 3a^{1/2} + 3a^{1/4} - 4) + 1.$$

F(a,m)=0

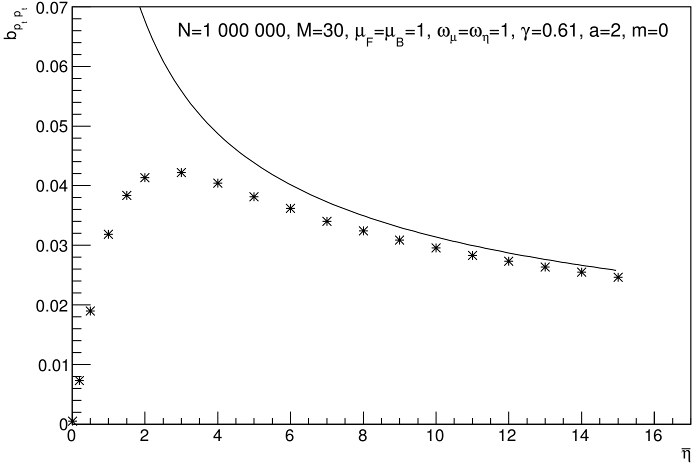


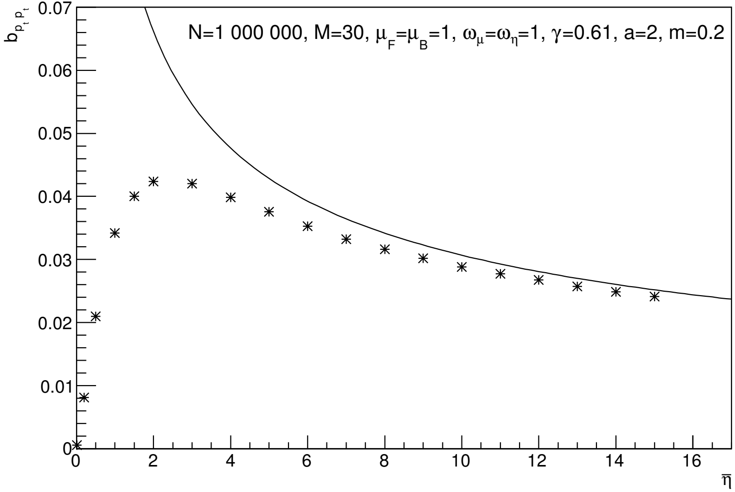
Studing dependence of $b_{p_t\,p_t}$ on ω_μ

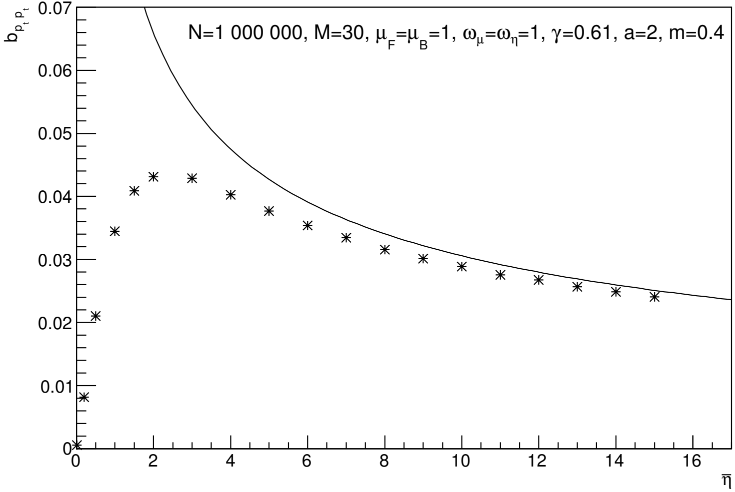


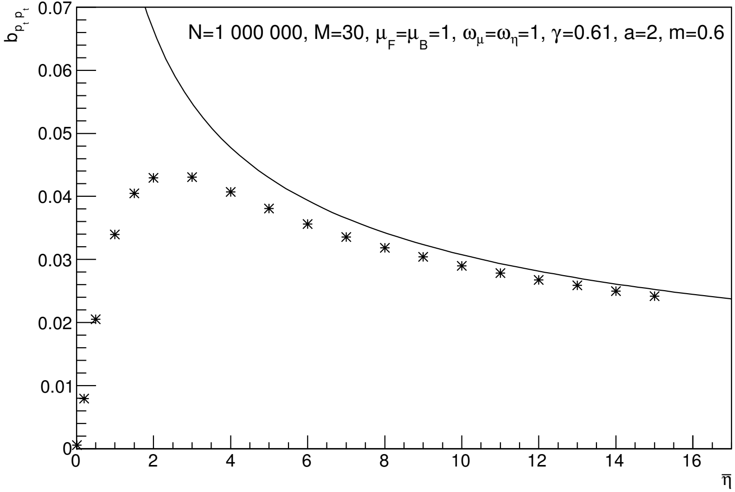
$$M = 450,\ a = 2,\ \eta = 12,\ \omega_\eta = 1,\ \mu_F = \mu_B = 1,\ \gamma = 0.61$$

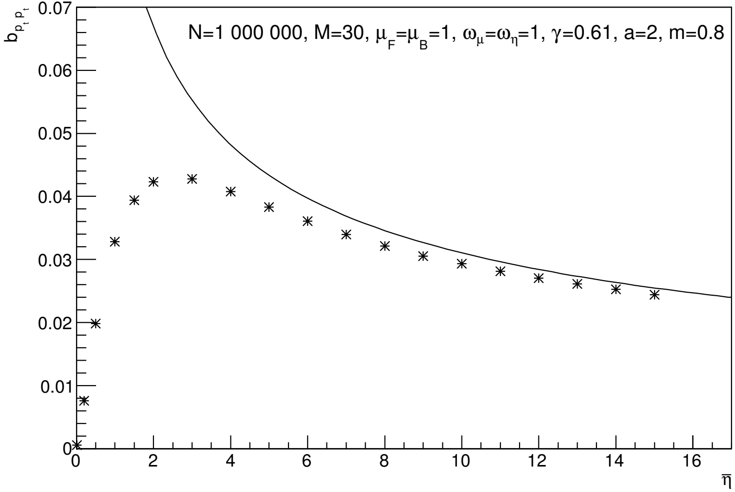
MC numerical calculations of the coefficient $b_{p_t\,p_t}$ in the case of random string distribution




$b_{p_t p_t}$
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
2
4
6
8
10
12
14
16
$\bar{\eta}$
N=1 000 000, M=30, $\mu_F=\mu_B=1$, $\omega_\mu=\omega_\eta=1$, $\gamma=0.61$, a=2, m=0.2


N=1 000 000, M=30, μ_F=μ_B=1, ω_μ=ω_η=1, γ=0.61, a=2, m=0.4
b_{p_t p_t}
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
2
4
6
8
10
12
14
16
η̄


N=1 000 000, M=30, μ_F=μ_B=1, ω_μ=ω_η=1, γ=0.61, a=2, m=0.6
$b_{p_t\,p_t}$
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
2
4
6
8
10
12
14
16
$\bar{\eta}$



N=1 000 000, M=30, μ_F=μ_B=1, ω_μ=ω_η=1, γ=0.61, a=2, m=0.8
b_{p_t p_t}
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
2
4
6
8
10
12
14
16
η̄

Conclusion

- $b_{n\,n}^{mean} = b_{n\,n}^{corr\,f}$;
- $b_{p_t\,n}^{mean} = b_{p_t\,n}^{corr\,f}$;
- $b_{p_t\,p_t}^{mean} = b_{p_t\,p_t}^{corr\,f}$;
- The examples with different cases of non-uniform string distribution in transverse plane were considered;
- It is shown that there are distributions of strings for which coefficient $b_{p_t\,n}$ becomes negative;
- The dependence of the correlation coefficient $b_{p_t\,p_t}$ on ω_μ disappears in the case of a homogeneous distribution of strings;
- The received asymptotes for the correlation coefficient between transverse momenta were compared are compared with the results of the MC numerical calculations of this coefficient.