Goldsone theorem for the spontaneous breakdown of spacetime symmetries

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Outline

1. Introduction: known peculiarities of the theories resulting from the spontaneous breakdown of spacetime symmetries

2. New results:
   - New massive Nambu-Goldstone bosons
   - Understanding the inverse Higgs phenomenon
   - Goldstone’s theorem

3. Conclusion
Known peculiarities

Redundant Nambu-Goldstone fields (picture from hep-th/0110285)

1. Introduce coset $G/H$:

$$gH = e^{iP_\mu x^\mu} e^{iP_z \xi} e^{iM_z \mu \omega_\mu}$$

2. Calculate Maurer-Cartan forms:

$$g^{-1}H dg_H = iP_\mu \Omega_\mu + iP_z \Omega_z + iM_z \mu \Omega_\mu M + iM_\mu\nu \Omega_\mu\nu M$$

3. Impose inverse Higgs constraints:

$$\Omega_z P^z(\partial_\mu \xi, \omega_\mu) = 0 \implies \omega_\mu = \omega_\mu(\partial_\mu \xi)$$
Known peculiarities

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Open questions

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→ Inverse Higgs effect - a trick, an effect, a gauge choice, ... ?
New massive Nambu-Goldstone bosons

SSB pattern: \( \text{ISO}(d)_{ST} \times \text{ISO}(d)_{int} \rightarrow \text{ISO}(d)_{V} \)

The Lagrangian of the theory:

\[
\mathcal{L} = -\frac{1}{2} (\partial_i \varphi^a)^2 + \frac{1}{4} (\partial_{[i} V^a_{j]} )^2 + \kappa V^i_a \partial_i \varphi^a + \frac{\lambda}{4d} (V^i_a V^a_i - dM^2_V )^2
\]

Vacuum solution:

\[
\varphi^a = \mu^2 x^a , \quad V^i_a = M \delta^i_a , \quad M = \sqrt{M^2_V - \frac{\kappa^2}{\lambda}} , \quad \mu^2 = \kappa M
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Parametrizing Nambu-Goldstone modes:

\[
\varphi^a(x) = \mu^2 x^a + \psi^a(x), \quad V^i_a(x) = \Omega^i_a(\omega) M, \quad \Omega^i_a = \delta^i_a + \omega^i_a - \frac{1}{2} \omega^i_b \omega^b_a + ...
\]

Effective Lagrangian(s):

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\mathcal{L}_{\psi,A} = -\frac{1}{2} (\partial_i \psi^a)^2 + \frac{1}{4} (\partial_{[i} A^a_{j]} )^2 - \frac{1}{2} \kappa^2 A^i_j A^j_i + \kappa A^i_a \partial_i \psi^a
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\]

\( A^i_j \) integrated out: \( \mathcal{L}_\psi = -\frac{1}{4} \left( (\partial_i \psi^a)^2 + (\partial_a \psi^a)^2 \right) \)
Applying the coset space construction

The corresponding coset space: \( g_H = e^{i\bar{P}_\mu \xi^\mu} e^{i\bar{P}_a \psi^a} e^{i\bar{M}_{ab} \omega^{ab}} \)

Covariant derivatives: \( D_\mu \psi^a = \partial_\mu \psi^a - \mu^2 \omega_\mu^a, \quad D_\mu \omega^{\lambda \sigma} \simeq \partial_\mu \omega^{\lambda \sigma} \)

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Imposing inverse Higgs constraints:

\[ \mathcal{L}_\psi = -\frac{1}{8} (D_{\{i} \psi_{a\}})^2 = -\frac{1}{4} \left( (\partial_i \psi^a)^2 + (\partial_a \psi^a)^2 \right) \]
Understanding the inverse Higgs phenomenon

What is the physical meaning of the inverse Higgs phenomenon?
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The same SSB pattern, but with redundant fields:

\[ ISO(d)_{ST} \times ISO(d)_{int} \rightarrow ISO(d)_V \]

The Lagrangian of the theory:

\[ \mathcal{L} = -\frac{1}{2}(\Box \varphi^a)^2 - \frac{1}{2}(\partial_i \theta)^2 + \frac{1}{4}(\partial_{[i} V_{j]}^a)^2 + \lambda \theta V_i^a \partial_i \varphi^a \]

Vacuum solution: \( \varphi^a = \mu^2 x^a, \quad \theta = 0, \quad V^i_a = 0. \)
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How to obtain a theory including fields charged only under \( SO_V \)?
Understanding the inverse Higgs phenomenon

*Which coset should be used within the coset space technique?*

Polar decomposition: \( \chi(x) = \gamma(x)\tilde{\chi}(x) \), \( \tilde{\chi}^T(x)(\hat{Z}_a\chi_{\text{vac}}(x)) = 0 \)

Introduce \( \chi(x), \tilde{\chi}(x) \) as:

\[
\chi(x) = (\phi^1, \ldots, \phi^d, V_1^1, \ldots, V_d^d, \theta), \quad \tilde{\chi}(x) = (\tilde{\phi}^1, \ldots, \tilde{\phi}^d, \tilde{V}_1^1, \ldots, \tilde{V}_d^d, \tilde{\theta})
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Understanding the inverse Higgs phenomenon

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\[ \mathbf{Z}_a \rightarrow \bar{P}_a \implies \tilde{\phi}^a = 0 \]

\[ \mathbf{Z}_a \rightarrow \bar{M}_{ab} \implies \tilde{\phi}^a = 0 \]
Understanding the inverse Higgs phenomenon

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- \( Z_a \rightarrow \bar{P}_a \Rightarrow \tilde{\phi}^a = 0 \)
- \( Z_a \rightarrow \bar{M}_{ab} \Rightarrow \tilde{\phi}^a = 0 \)
- Hence,  
  \[ \tilde{\chi}(x) = (0, \ldots, 0, V_1^1, \ldots, V_d^d, \theta), \quad \gamma(x) = e^{i\bar{P}_a\xi^a} \]

Since homogeneously transforming quantities are obtained from \( \gamma^{-1}d\gamma \), one should not introduce \( \omega^{ab} \) at all!
Understanding the inverse Higgs phenomenon

How to obtain a theory including fields charged only under $SO_V$?

Redefine degrees of freedom: $V_a^i \rightarrow \Omega_a^b(\psi)\tilde{V}_b^i$
Understanding the inverse Higgs phenomenon

*How to obtain a theory including fields charged only under $SO_N$?*

Redefine degrees of freedom: $V^i_a \rightarrow \Omega^b_a(\psi) \tilde{V}^i_b$

*Does suitable $\Omega^a_b(\psi)$ exist?*
Understanding the inverse Higgs phenomenon

How to obtain a theory including fields charged only under $SO_V$?

Redefine degrees of freedom: $V_a^i \rightarrow \Omega^b_a(\psi) \tilde{V}_b^i$

Does suitable $\Omega^a_b(\psi)$ exist?

Yes, if one can find any suitable coset:

consider $g_H = e^{i\tilde{P}_\mu x^\mu} e^{i\tilde{P}_a \psi^a} e^{i\tilde{M}_{ab} \omega^{ab}}$ and find the searched for expression.

Via polar decomposition:

$$\gamma(x) = e^{i\tilde{P}_a \psi^a} e^{\frac{i}{2} \tilde{M}_{ab} \omega^{ab}}, \quad \omega^{ab} = \omega^{ab}(\psi^a)$$
Goldstone’s theorem

Let one be given an SSB pattern:

\[ G \rightarrow H, \]

and let \( Z_a \) be broken generators and \( B_n \in Z_a : \hat{B}_n \Phi|_0 \neq 0 \), then:

- \( n_{NG} = \text{nuber of } B_n \)
- Nambu-Goldstone fields corresponding to \( B_\alpha \) such that \([P_\mu, B_\alpha] \sim B_n \) are massive
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such that \([P_\mu, B_\alpha] \sim B_n\) are massive

If some of the generators always act trivially at the origin, they never give rise to Nambu-Goldstone fields.

The conformal group: \( \forall \Phi \, \hat{K}_n \Phi = 0 \)
Conclusion

- The action of the generators on the vacuum at the origin uniquely fixes the number of Nambu-Goldstone fields
- Some of the Nambu-Goldstone fields are necessarily gapped
- Inverse Higgs constraints is a trick used to uncharge fields under the action of broken but acting trivially at the origin generators