Lipatov's effective action in high energy QCD

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Plan of the report

1. The Lipatov's effective action.
2. The search of classical solutions for gluon fields: connection with CGC approach.
3. The construction of RFT action of the approach.
4. One loop expression: the NLO kernel $A_+ A_-$.
5. The description of gluon and quark production amplitudes.
6. The inclusion of quarks in action.

Effective action

The Lipatov effective action is gauge invariant and written in the covariant form in terms of gluon field v as

$$S_{eff} \,=\, -\int d^4 x \left(\frac{1}{4}\, G^a_{\mu\nu}\, G_a^{\mu\nu} \right.$$
$$\left. +\, \frac{1}{N} tr\, \left[\, (\, A_+(v_+) \,-\, A_+\,)\, \partial_i^2\, A_a^+ \,+\, (\, A_-(v_-) \,-\, A_-\,)\, \partial_i^2\, A_a^- \,\right] \,-\, L_{quark} \right),$$

where

$$A_\pm(v_\pm) \,=\, \frac{1}{g}\,\partial_\pm\, O(x^\pm, v_\pm);\;\; O(x^\pm, v_\pm) \,=\, P\, e^{g \int_{-\infty}^{x^\pm} dx'^{\pm}\, v_\pm}$$

and

$$L_{quark} = \bar{\psi}(i\gamma^\nu \partial_\nu - m - ig\gamma^\nu T^a v_{a\nu})\psi\,.$$

There are additional kinematical constraints for the reggeon fields

$$\partial_-\, A_+ \,=\, \partial_+\, A_- \,=\, 0\,.$$

[1] L. N. Lipatov, Nucl. Phys. B **452**, 369 (1995); Phys. Rept. **286**, (1997) 131;

We consider the inclusion of quarks separately later. In the framework with an external source of the color charge introduced, keeping only gluon field depending terms in the action, we rewrite it as

$$S_{eff} = -\int d^4 x \left(\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + v_- J^-(v_-) + v_+ J^+(v_+) \right),$$

where

$$J^{\pm}_a(v_{\pm}) = \frac{1}{C(R)} O(x^{\pm}, v_{\pm}) \partial_i^2 A^{\pm}_a ,$$

where $C(R)$ is the eigenvalue of Casimir operator in the representation R, $C(R) = N$ in the case of adjoint representation

The classical equations of motion for the gluon field v_μ which arise from the action are the following:

$$(D_\mu G^{\mu\nu})_a = \partial_\mu G_a^{\mu\nu} + g f_{abc} v_\mu^b G^{c\,\mu\nu} = j_a^+ \delta^{\nu +} + j_a^- \delta^{\nu -}$$

We consider the Lipatov's effective action as a gauge invariant non-linear action which describes interaction of gluons with Reggeons (new degrees of freedom).

We assume for these currents that their variation on gluon fields reproduces the Lipatov's induced currents

$$\delta\,\left(\,v_{\pm}\,J^{\pm}(v_{\pm})\,\right)\,=\,(\delta\,v_{\pm})\,j_{\mp}^{ind}(v_{\pm})\,=\,(\delta\,v_{\pm})\,j^{\pm}(v_{\pm})\,,$$

which posesses a covariant conservation property:

$$\left(\,D_{\pm}\,j_{\mp}^{ind}(v_{\pm})\,\right)^{a}\,=\,\left(D_{\pm}\,j^{\pm}(v_{\pm})\right)^{a}\,=\,0\,.$$

In fact, it is enough to determine $J_{a}^{\pm}(v_{\pm})$ currents's form.

The light-cone gauge $v^a_- = 0$. The topic of this part of report is the search of classical solutions for gluon fields in the form of the expansion in powers of the coupling constant g:

$$v^a_+ = \sum_{k=0}^{+\infty} g^k v^a_{+k}(A_+, A_-); \quad v^{ia} = \sum_{k=0}^{+\infty} g^k v^{ia}_k(A_+, A_-) .$$

We see that from the 4 equations of motion we have obtained 3 unknown fields. The last equation is transformed into the transversality condition of the effective vertex.

$$-\partial_-[\partial_i v^i_k + \partial_- v_{+k}] = \bar{j}^+_{k-1} ,$$

$$\Box v^j_k - \partial^j[\partial_i v^i_k + \partial_- v_{+k}] = \bar{j}^j_{k-1} ,$$

$$\Box v_{+k} - \partial_+[\partial_i v^i_k + \partial_- v_{+k}] = \bar{j}^-_{k-1} .$$

Here $\bar{j}^+_{k-1}$, $\bar{j}^-_{k-1}$, $\bar{j}^i_{k-1}$ are functions of $v^\nu_{k-1}(A_+, A_-)$, ..., $v^\nu_0(A_+, A_-)$, A_+ and A_-.

We have

$$-\partial_-[\partial_i v_k^i + \partial_- v_{+k}] = \bar{j}_{k-1}^+ \,,$$

$$\Box v_k^j - \partial^j[\partial_i v_k^i + \partial_- v_{+k}] = \bar{j}_{k-1}^j \,,$$

$$\Box v_{+k} - \partial_+[\partial_i v_k^i + \partial_- v_{+k}] = \bar{j}_{k-1}^- \,.$$

The condition, equivalent to the existence of the solutions:

$$\partial_\mu \bar{j}_{k-1}^\mu = \Box[\partial_i v_k^i + \partial_- v_{+k}] - \Box[\partial_i v_k^i + \partial_- v_{+k}] = 0 \,.$$

Classical solutions for v_k when requiring $\partial_\mu \bar{j}_{k-1}^\mu = 0$ are self-consistent and have the form

$$v_k^j = \Box^{-1}\left[\bar{j}_{k-1}^j - \partial^j \partial_-^{-1} \bar{j}_{k-1}^+\right] ,$$

$$v_{+k} = \Box^{-1}\left[\bar{j}_{k-1}^- - \partial_+ \partial_-^{-1} \bar{j}_{k-1}^+\right] .$$

LO equations of motion:

$$\partial_i \partial^i v_{a+} = -\partial_i^2 A_{a+}, \quad v_{a+} = A_{a+}$$

and

$$(D_+ (\partial_- v_i))_a = 0, \quad v_i^b = U^{bc}(v_+)\,\rho_{c\,i}\left(x^-, x_\perp\right), \quad D_+ U(v_+) = 0.$$

CGC approach

Self-consistency condition, induced current and definition of the $\rho_{c\,i}(x^-, x_\perp)$ function:

$$-\partial_i \partial_- v_a^i = j_a^+, \quad j_a^+ = -U^{ab}(v_+)\,\partial_i \partial_- \rho_a^i$$

similarly to CGC approach.

Current's form

From Lipatov's effective action we have:

$$\delta\,\left(v_+\,J^+\right)\,=\frac{1}{N}\delta\,tr[\,(\,v_{+x}\,O_x\,\partial_i^2\,A^+\,)\,]=-\frac{1}{N}\delta v_+^a\,tr[\,T_a\,O\,T_b\,O^T\,]\,\left(\partial_i^2 A_b^+\right)$$

that in the case of adjoint representation gives:

$$\delta\,\left(v_+\,J^+\right)=\frac{1}{N}\left(\delta v_+^a\right)\,tr\,[\,f_a\,O\,f_b\,O^T\,]\,\left(\,\partial_i^2\,A_b^+\,\right)=\frac{1}{N}\,\left(\delta v_+^a\right)\,U^{a\,b}\,\left(\,\partial_i^2\,A_b^+\,\right)$$

that provides

$$U^{a\,b}\,=\,tr\,[\,f_a\,O\,f_b\,O^T\,]$$

and

$$\partial_i\,\partial_-\,\rho_a^i\,=\,-\,\frac{1}{N}\,\partial_\perp^2\,A_a^+\,,\quad\rho_a^i\,=\,\frac{1}{N}\,\partial_-^{-1}\,\left(\partial^i\,A_-^a\right)\,,$$

(connection with CGC approach).

Inserting obtained v_{cl}, we will obtain an action which will depend only on the reggeon fields, determining the LO RFT action of the approach. Formally, due to the presence of the path-ordered exponential in the solutions, the action will include all orders of perturbative terms which can be important for large $v_+ \approx A_+$ in the processes where some large color charge is created. The expansion of these exponential must be supplemented by solution of equations of motion to corresponding orders, otherwise only part of the usual perturbative corrections will be accounted.
In general, the following expansion for the action exists:

$$S_{eff} = -\int d^4 x \left(s_1[g, A_+, A_-] + g\, s_2[g, A_+, A_-] + \cdots \right) .$$

So S_{eff} can be expanded in terms of reggeon fields A_- and A_+ as

$$\Gamma = \sum_{n,m=0} \left(A_+^{a_1} \cdots A_+^{a_n} K_{b_1 \cdots b_m}^{a_1 \cdots a_n} A_-^{b_1} \cdots A_-^{b_m} \right),$$

that determines this expression as functional of reggeon fields and provides effective vertices of the interactions of the reggeized gluons in the RFT calculus.

Substituting the LO of classical solutions into action gives us the full LO value of the kernel only for $n + m = 2$ and only a fraction of the contributions for more complex vertices.
NLO of classical solutions allows us to get a complete answer for $n + m = 3$.
With NNLO of classical solutions , we moved on to 4-reggeon interactions.

One loop expression

We define:

$$v_i^a \to v_{i\,cl}^a + \varepsilon_i^a\,, \quad v_+^a \to v_{+\,cl}^a + \varepsilon_+^a$$

and expand the light-cone Lagrangian around the non-trivial classical solutions

$$L_2 = -\frac{1}{4}F_{ij}^a F_{ij}^a + F_{i+}^a F_{i-}^a + \frac{1}{2}F_{+-}^a F_{+-}^a$$

$$-v_+^{a\,cl}\, O^{ab}(v_+^{cl})\,\left(\partial_i\,\partial_-\,\rho_b^i\right) - \frac{1}{2}\left(\frac{\delta\, U^{b\,a}(v_+)}{\delta v_+^c}\right)^{xy}_{v_+=v_+^{cl}}\left(\partial_i\,\partial_-\,\rho_a^i\right)_x \varepsilon_{+\,x}^b \varepsilon_{+\,y}^c + \ldots$$

Where

$$U_x^{a\,b}(v_+) = U_x^{a\,b}(v_{+\,0}^{cl}) + g\,\left(U_1^{a\,b}(v_{+\,0}^{cl})\right)_{xy}^{c}\,\varepsilon_{+\,y}^c + \frac{1}{2}g^2\left(U_2^{a\,b}(v_{+\,0}^{cl})\right)_{xyz}^{c\,d}\,\varepsilon_{+\,y}^c\,\varepsilon_{+\,z}^d + \ldots$$

Notations used on the previous slide

$$\left(U_1^{a\,b}\right)_{xy}^{c} = tr[\,f_a\,G^+_{xy}\,f_c\,O_y\,f_b\,O^T_x\,] + tr[\,f_c\,G^+_{yx}\,f_a\,O_x\,f_b\,O^T_y\,]\,,$$

$$\begin{aligned}\left(U_2^{a\,b}\right)_{xyz}^{c\,d} &= tr[\,f_a\,G^+_{xy}\,f_c\,G^+_{yz}\,f_d\,O_z\,f_b\,O^T_x\,] + tr[\,f_a\,G^+_{xz}\,f_d\,G^+_{zy}\,f_c\,O_y\,f_b\,O^T_x\,]\\ &+ tr[\,f_d\,G^+_{zx}\,f_a\,G^+_{xy}\,f_c\,O_y\,f_b\,O^T_z\,] + tr[\,f_c\,G^+_{yx}\,f_a\,G^+_{xz}\,f_d\,O_z\,f_b\,O^T_y\,]\\ &+ tr[\,f_d\,G^+_{zy}\,f_c\,G^+_{yx}\,f_a\,O_x\,f_b\,O^T_z\,] + tr[\,f_c\,G^+_{yz}\,f_d\,G^+_{zx}\,f_a\,O_x\,f_b\,O^T_y\,]\,,\end{aligned}$$

where

$$O_x = \delta^{a\,b} + g\int d^4y\,G^{+\,a\,a_1}_{xy}\,(v_+(y))_{a_1\,b} = 1 + g\,G^+_{xy}\,v_{+y}$$

and Green's functions of D_+ operators

$$G^+_{xy} = G^{+\,0}_{xy} + g\,G^{+\,0}_{xz}\,v_{+z}\,G^+_{zy}\,,$$

where

$$\partial_{+x}\,G^{+\,0}_{xy} = \delta_{x\,y}\,,\quad G^{+\,0}_{yx}\,\overleftarrow{\partial}_{+x} = -\delta_{x\,y}\,.$$

Final expression for the one loop effective action:

$$
\begin{aligned}
\Gamma \;&=\; \int d^4x \, \left(L_{YM}(v_i^{cl}, v_+^{cl}) - v_{+\,cl}^{a} \, J_a^{+}(v_+^{cl}) - A_+^a \left(\partial_i^2 \, A_-^a \right) \right) \\
&+\; \frac{\imath}{2} \ln \left(1 + \, G(v^{cl}) \, M(v^{cl}) \right) \, ,
\end{aligned}
$$

which is functional of the reggeized gluon fields only.

The interaction of reggeized gluons A_+ and A_- is defined as effective vertex of interactions of reggeon fields in the action:

$$
\left(K_{x\,y}^{a\,b} \right)^{+\,-} \,=\, K_{x\,y}^{a\,b} \,=\, \left(\frac{\delta^2 \, \Gamma}{\delta A_{+\,x}^{a} \, \delta A_{-\,y}^{b}} \right)_{A_+ ,\, A_- \,=\, 0} \,,
$$

we can call this vertex as interaction kernel as well.

Interaction kernels A_+, A_- in the form of an expansion $K^{bd}_{zw} = \sum_{k=0}^{\infty} K^{bd}_{zw\,k}$

The contributions to this kernel are provided by the different terms in the action which are linear with respect to A_+, A_- fields. For example, the variation of a logarithms gives:

$$-2\imath\, K^{ab}_{xy1} = \left(\frac{\delta^2 \ln\left(1 + G\,M\,\right)}{\delta A^a_{+\,x}\,\delta A^b_{-\,y}} \right)_{A_+,\,A_-\,=0} =$$

$$= \left[\left(\frac{\delta^2\, G}{\delta A^a_{+\,x} \delta A^b_{-\,y}} M + \frac{\delta\, G}{\delta A^a_{+\,x}}\,\frac{\delta\, M}{\delta A^b_{-\,y}} + \frac{\delta\, G}{\delta A^b_{-\,y}}\,\frac{\delta\, M}{\delta A^a_{+\,x}} + G\,\frac{\delta^2\, M}{\delta A^a_{+\,x} \delta A^b_{-\,y}} \right) (1 + GM)^{-1} - \right.$$
$$\left. - \left(\frac{\delta\, G}{\delta A^b_{-\,y}} M + G\,\frac{\delta\, M}{\delta A^b_{-\,y}} \right) (1 + G\,M)^{-1} \left(\frac{\delta\, G}{\delta A^a_{+\,x}} M + G\,\frac{\delta\, M}{\delta A^a_{+\,x}} \right) (1 + G\,M)^{-1} \right]_{A=0}$$

Leading order contribution:

$$K^{a\,b}_{xy\,0} = -\,\delta^{a\,b}\,\delta_{x\,y}\,\partial^2_{i\,x} = \delta^{a\,b}\,\delta_{x\,y}\,\left(\partial_i\,\partial^i\right)_x\,.$$

The propagator of the reggeon field

For the kernel

$$K^{bd}_{zw} \;=\; \sum_{k\,=\,0} K^{bd}_{zw\;k}$$

the propagator is defined as

$$\int d^4 z\;\left(\,K^{ab}_{xz}\right)^{-\,+}\;\left(\,D^{bc}_{zy}\right)_{+\,-}\;=\;\delta^{ac}\,\delta_{xy}$$

or

$$\begin{aligned}\left(D^{ac}_{xy}\right)_{+\,-} \;&=\; \left(D^{ac}_{xy}\right)_{0\,+-} - \int d^4 z \int d^4 w\;\left(D^{ab}_{xz}\right)_{0\,+-}\left(\left(\,K^{bd}_{zw}\right)^{-\,+}\right. \\ &-\; \left.\left(\,K^{bd}_{zw}\right)^{-\,+}_{0}\right)\left(D^{dc}_{wy}\right)_{+-}\;,\end{aligned}$$

where

$$\int d^4 z\;K^{ab}_{xz\,0}\,D^{bc}_{zy\,0}\;=\;\delta^{ac}\,\delta_{xy}\,.$$

The well-known NLO kernel:

$$-2\,\imath\,K_{x\,y\,1}^{a\,b}\;=\;\frac{\imath\,g^2\,N}{4\,\pi}\,\partial^2_{i\,x}\left(\int\frac{dp_-}{p_-}\int\frac{d^2p_\perp}{(2\pi)^2}\int\frac{d^2k_\perp}{(2\pi)^2}\,\frac{k_\perp^2}{p_\perp^2\;(\,p_\perp\;-\;k_\perp\,)^2}\,e^{-\imath\;k_i\,(x_i\,-\,y_i)}\right)$$

provides the reggeized gluon propagator equation after the Fourier transform:

$$\tilde{D}^{ab}(p_\perp,\,p_-)\;=\;\frac{\delta^{ab}}{p_\perp^2}\;-\;\frac{g^2\,N}{32\,\pi^3}\int\frac{dk_-^{'}}{k_-^{'}}\int d^2k_\perp\,\frac{p_\perp^2}{k_\perp^2\;(\,p_\perp\;-\;k_\perp\,)^2}\,\tilde{D}^{ab}(p_\perp,\,k_-^{'})\,,$$

here we used

$$D_0^{ab}(x,y)\;=\;\delta^{ab}\int\frac{d^4p}{(2\pi)^4}\,\frac{e^{-\imath\,p\,(x\,-\,y)}}{p_\perp^2}\,,$$

$$\epsilon(p_\perp^2)\;=\;-\,\frac{\alpha_s\,N}{4\,\pi^2}\int d^2k_\perp\,\frac{p_\perp^2}{k_\perp^2\;(\,p_\perp\;-\;k_\perp\,)^2}\,.$$

Rewriting this equation in the differential form:

$$\frac{\partial\,\tilde{D}^{ab}(p_\perp,\eta)}{\partial\,\eta} \;=\; \tilde{D}^{ab}(p_\perp,\eta)\,\epsilon(p_\perp^2)\,.$$

On the rapidity interval of interest we obtain the final expression for the propagator:

$$\tilde{D}^{ab}(p_\perp, Y) \;=\; \frac{\delta^{ab}}{p_\perp^2}\left(\frac{s}{s_0}\right)^{\epsilon(p_\perp^2)}$$

$$0 \,<\, \eta \,<\, Y \;=\; \ln(s/s_0)$$

The description of gluon and quark production amplitudes

We extend the formalism in order to incorporate the vertices of real particles production, i.e. the vertices of reggeons interaction with asymptotic gluon and quark fields.
In order to compute the production amplitudes in the quasi-multi-Regge kinematics, the classical solutions must be modified taking into account solutions of equations of motion for free gluon field v_f^μ:

$$v_{cl}^\mu \rightarrow v_{cl}^\mu + v_f^\mu$$

The v_{cl}^μ fields were calculated in our work to NNLO precision, here we consider the equations of motion for v_f^μ

$$(D_\mu G^{\mu\nu})_a = 0$$

to LO precision.

The description of gluon and quark production amplitudes

These equations provide the well-known expressions for v_+ and $v_\perp$.
The first equation provides a constraint of the theory in light-cone gauge:

$$v_{a+f} = -\partial_-^{-1}\partial^i v_{aif}\,.$$

The second equation of motion for the dynamical v_i fields:

$$\Box v_{aif} = 0\,.$$

The third equation can be considered as a check of consistency of the found solutions, it is equal to zero by construction. We see also, that the Lorentz condition for the free gluon field

$$\partial^\mu v_{a\mu f} = 0$$

is satisfied in this case as an operator equation.

S-matrix for the quark production amplitudes

Considering the fermionic part of the QCD Lagrangian, we follow the usual light-cone decomposition of the quark field. Introducing projector operators

$$\Lambda^{\pm} = \frac{\gamma^{\mp}\gamma^{\pm}}{2}\,,$$

we project out the fermion field obtaining two two-component spinors:

$$\psi_{\pm} = \Lambda^{\pm}\psi\,.$$

The free, asymptotic quark fields can be defined as solutions of equations of motion obtained from the free quarks Lagrangian

$$L_Q^f = i\sqrt{2}\psi_+^{\dagger}\partial_+\psi_+ - \frac{i}{\sqrt{2}}\psi_+^{\dagger}\Big(i\gamma^{\perp}\partial_{\perp} + m\Big)\partial_-^{-1}\Big(i\gamma^{\perp}\partial_{\perp} - m\Big)\psi_+$$

and which reads as

$$\Big(\Box + m^2\Big)\psi_{+f} = 0\,.$$

The description of gluon and quark production amplitudes

With the new classical solution the effective action becomes a functional of the Reggeon, free gluon and quark fields: $\Gamma(A_{+},\, A_{-},\, v_{f},\, \psi^{\dagger}_{+\ f},\, \psi_{+\ f})$. Now, the QCD S-matrix generating functional of the theory consists with free gluon and quark fields we can write as

$$S\left(v_{f},\, \psi^{\dagger}_{+\ f},\, \psi_{+\ f}\right) = \int dA\, e^{i\Gamma(A_{+},\, A_{-},\, v_{f},\, \psi^{\dagger}_{+\ f},\, \psi_{+\ f})}\,.$$

We can calculate any vertex of the theory which consists with the fields, vertex of interaction of $n\, A_{+} \,+\, m\, A_{-} \,+\, k\, v_{\perp\, f} \,+\, p\, \psi_{+f} \,+\, p\, \psi^{\dagger}_{+f}$ fields that could be written as:

$$\left(K_{a_{1}\cdots a_{n+m}}\right)^{c_{1}\cdots c_{k}}_{p;p} = \left(\frac{\delta^{n+m+k+2p}\Gamma(A_{+},\, A,\, v_{f},\, \psi^{f}_{+\ f},\, \psi_{+\ f})}{\delta A^{a_{1}}_{\pm}\cdots\delta A^{a_{n+m}}_{\pm}\delta v^{c_{1}}_{\perp f}\cdots\delta v^{c_{k}}_{\perp f}\delta\psi^{1}_{f}\cdots\delta\psi^{p}_{f}\delta\psi^{1\dagger}_{f}\cdots\delta\psi^{p\dagger}_{f}}\right),$$

where as usual we take all the fields equal to zero after the derivatives taken.

Extending by including the quarks

Let's add L_{quark}. We find only one correction of the order g^2 and ε^2. This addition will allow us to take into account the contribution with a quark loop to the gluon fields. Integrating with respect the fluctuation we obtain the following expression for the contribution to the action:

$$-\frac{g^2}{4} tr\Big(G_q^0(y,x)(\gamma^\nu v_{a\nu}(x)) G_q^0(x,y)(\gamma^\mu v_\mu^a(y))\Big),$$

where

$$G_q^0(x,y) = (i\gamma_\nu \partial_x^\nu + m)\Delta_{xy}, \quad \Delta_{xy} = \int \frac{d^4p}{(2\pi)^4} \frac{e^{ip(x-y)}}{p^2 - m^2 + i0}\,.$$

Then after varying by $v_{a\rho}(z)$ we obtain the following contributions to the equation of motion

$$j^\rho_{quark\ \ a}(z) = -\frac{g^2}{2} tr\Big(\gamma^\rho G_q^0(z,y)\gamma^\mu v_{a\ \ \mu}(y) G_q^0(y,z))\Big)\,.$$

We verified, that condition $\partial_\rho j^\rho_{quark\ a} = 0$ is true. Then the classical solutions have additional contributions v^i_{q2} and v_{q+2}, respectively:

$$v^i_{q2a} = \Box^{-1}\left[j^i_{quark\ 0a} - \partial^i \partial_-^{-1} j^+_{quark\ 0a}\right],$$

$$v_{q+2a} = \Box^{-1}\left[j^-_{quark\ 0a} - \partial_+ \partial_-^{-1} j^+_{quark\ 0a}\right],$$

where

$$j^\rho_{quark\ a}(z) = -\frac{g^2}{2} tr\Big(\gamma^\rho G^0_q(z,y)\gamma^\mu v_{a\ \mu}(y) G^0_q(y,z)\Big)$$

We want to estimate the contribution of quarks to the reggeon propagator

Inserting obtained classical gluon fields solutions into action, we obtain an action which depend only on the reggeon fields.
In order to calculate this action we need to know the components of the field strength tensor, with LO precision we have:

$$G^{a}_{+\,-\;0}\,=\,0\,,\;\;G^{a}_{i\,+\;0}\,=\,\partial_i\,A^{a}_{+}\,,\;\;G^{a}_{i\,-\;0}\,=\,-\,\partial_-\,v^{a}_{i0}\,,\;\;G_{i\,j\;0}\,=\,0\,,$$

and components of the field strength tensor with included quarks

$$G^{a}_{+\,-\;q2}\,=\,0\,,\;\;G^{a}_{i\,+\;q2}\,=g^2\,\partial_i\,v^{a}_{q+2}\,,\;\;G^{a}_{i\,-\;q2}\,=\,-\,g^2\partial_-\,v^{a}_{qi2}\,,\;\;G_{i\,j\;q2}\,=\,0\,,$$

that gives

$$\frac{1}{2}\,G_{\mu\,\nu\;0}\,G^{\mu\,\nu}_{q2}\,=g^2\,\left(\partial_-\,v^{a}_{i0}\right)\,\left(\partial_i\,v^{a}_{q+2}\right)\,+g^2\,\left(\partial_-\,v^{a}_{qi2}\right)\,\left(\partial_i\,A^{a}_{+}\right)\,.$$

Therefore, for the additional contributions to the effective action from the inclusion of quarks we obtain in LO:

$$S_{eff\ q2} = -\frac{g^2}{N}\int d^4x \Big[\partial_i \left(v^a_{q+2} \partial_i A^b_- \right) + N \left(\partial_- \left(v^b_{qi2} \partial_i A^a_+ \right) - v^b_{qi2} \left(\partial_i \partial_- A^a_+ \right) \right) \Big] = 0,$$

In addition, it is a direct contribution to the effective action:

$$-\frac{g^2}{4}\left(\frac{\delta^2\, tr\Big(G^0_q(y,x)(\gamma^\nu v_{a\nu}(x)) G^0_q(x,y)(\gamma^\mu v^a_\mu(y)) \Big)}{\delta A^a_{+\,x}\, \delta A^b_{-\,y}} \right)_{A_+,\, A_- = 0} .$$

After the calculations, we showed that this contribution is also zero.

Results

S. Bondarenko, L. Lipatov and A. Prygarin, Eur. Phys. J. C **77** (2017) no.8, 527.

$$v_{+0\ a} = A_{+\ a},$$

$$v_{i0\ a} = \rho_i^b(x_\perp, x^-)U_{ab},$$

$$v_{+1\ a} = -\frac{2}{g}\Box^{-1}\Big((\partial_+\partial^i U_{ab})\rho_i^b\Big),$$

$$v_{i1\ a} = -\Box^{-1}\Big[\partial^j F_{ji\ a} + \frac{1}{g}\partial_i\Big((\partial^j U_{ab})\rho_j^b\Big) - \partial_i{}^{-1} j^+_{a1}\Big].$$

$$U^{ab}(v_+) = -tr\Big[T^a\Big(Pe^{g\int_{-\infty}^{x^+} dx'^+ v_{+c}(x'^+,x^-,x_\perp)T^c}\Big)T^b\Big(Pe^{-g\int_{x^+}^{+\infty} dx'^+ v_{+d}(x'^+,x^-,x_\perp)T^d}\Big)\Big].$$

The last result is NNLO classical solution:

S. Bondarenko, S. Pozdnyakov, arXiv:1802.05508 [hep-ph].

$$v_{+2\ a} = \Box^{-1}\partial_{-}^{-1}\Big[\frac{2}{g}\partial_{+}j_{a1}^{+} + f_{abc}\Big(2\partial_{-}v_{i0}^{b}\partial_{+}v_{1}^{ic} + (\partial_{i}v_{0}^{i\ c})\partial_{-}v_{1+}^{b} + 2\partial_{i}(A_{+}^{b}\partial_{-}v_{1}^{i\ c})\Big)\Big], \quad (1)$$

$$v_{2a}^{i} = \Box^{-1}\Big[\partial_{-}^{-1}\partial^{i}\Big(L_{a2}^{+} - (\partial^{j}\rho_{j}^{b})(\frac{1}{g^{2}}\partial_{-}U_{ab})\Big) - f_{abc}\Big(\frac{1}{g}v_{j0}^{b}\Big(\partial^{j}v_{0}^{i\ c} - \partial^{i}v_{0}^{j\ c}\Big) + v_{0j}^{b}(\partial^{j}v_{1}^{i\ c} - \partial^{i}v_{1}^{j\ c}) - v_{0}^{i\ c}v_{1+}^{b} + 2A_{+}^{b}v_{1}^{i\ c} + + \partial_{j}(v_{0}^{j\ b}v_{1}^{i\ c} - v_{0}^{i\ b}v_{1}^{j\ c}) + f^{cde}v_{j0}^{b}v_{0d}^{j}v_{0e}^{i}\Big)\Big].$$

Conclusion

1. Lipatov's effective action provides systematical calculations of unitarity corrections to the scattering amplitudes at high energy in the framework of the regular QFT methods;
2. The perturbation theory is based on the knowledge of the classical solutions of equations of motion (written with precision NNLO) and loops contributions to the effective action;
3. Additional source of the corrections are the Regge Field Theory (RFT) loop's contributions. Lipatov's effective action can be considered as QCD variant of RFT and provides the base for these RFT calculations;
4. Formulated as RFT, the Lipatov's effective action provides self-consistent approach for the calculations of the complex vertices (kernels) of Reggeon-Reggeon interactions and correspondingly corrections to the reggeized gluons propagator or any other correlations functions of interests;
5. We studied the inclusion of quarks in action. The contribution to the reggeon propogator was equal to zero on g^2, as expected;
6. We can calculate any vertex vertex of the theory which consists with the fields.

1. S. Bondarenko, L. Lipatov and A. Prygarin, Eur. Phys. J. C **77** (2017) no.8, 527.
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Thank you for your attention