Exotic tetraquark mesons in large – N_c QCD

Dmitri Melikhov

D. V. Skobeltsyn Institute of Nuclear Physics, M. V. Lomonosov Moscow State University

1. **Observations of exotic tetra- and pentaquark candidates**
2. **Non-resonant explanations of the observed distributions**
3. **Phenomenologocal models for polyquark resonances**
4. **Exotic bound states in QCD (lattice QCD, Green functions at large-N_c, QCD sum rules)**
5. **Conclusions**

• **For many decades only hadron states with quantum numbers of the $\bar{q}q$ and qqq systems have been observed, although one can construct many other color singlets.**

Why not $\bar{q}qqqq$ or $\bar{q}q\bar{q}q$? **[Large-N_c QCD no such states at the leading $1/N_c$]**

In fact we know many multiquark states (deuteron, nuclei) but these are of a bit different nature.

• **First attempt — the strange narrow pentaquark $\theta^+(1540)$ predicted in 1997 and "temporarily" observed in several experiments with large significance — was finally unsuccessful.**

• **In the 21th century many exotic candidates have been reported in the experiments:**

New charmonium states which do not fall in the usual $\bar{c}c$ picture, e.g.:

$Z^+(4430)$ $J^P = 1^+$ **seen in** $B^+ \to K(\psi'\pi+)$ **with** $\Gamma \sim 40^{+18+30}_{-13-13}$ **MeV** $\bar{c}c\bar{d}u$

$Z^+(3900)$ $J^P = 1^+$ **seen in** $\Upsilon(4260) \to \pi^-(J/\psi\pi+)$ **with** $\Gamma \sim 30$ **MeV** $\bar{c}c\bar{d}u$

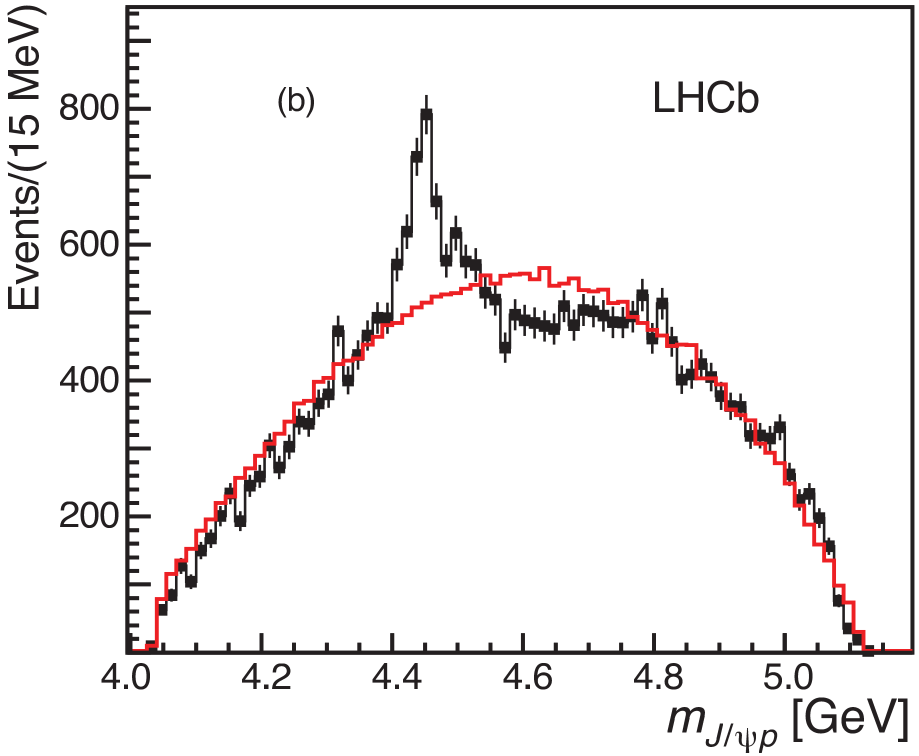
Similar charged states $\bar{b}b$ states $Z_b(10610)$ and $Z_b(10650)$.

Also many neutral states $\bar{c}c\bar{q}q$: e.g. $X(3872)$ a $J^{PC} = 1^{++}$ resonance $M = 3871.69 \pm 0.17$ MeV observed in $B^+ \to K^+(J/\psi\pi^+\pi^-)$ with the width $\Gamma < 1.2$ MeV very close to $D^0\bar{D}^{*0}$ threshold $M_{D^0} + M_{D^{*0}} - M_X = 0.11^{+0.6+0.1}_{-0.4-0.3}$ MeV.

Recently, also *pentaquark* candidates from LHCb in $\Lambda_b^0 \to K^-(J/\psi p)$

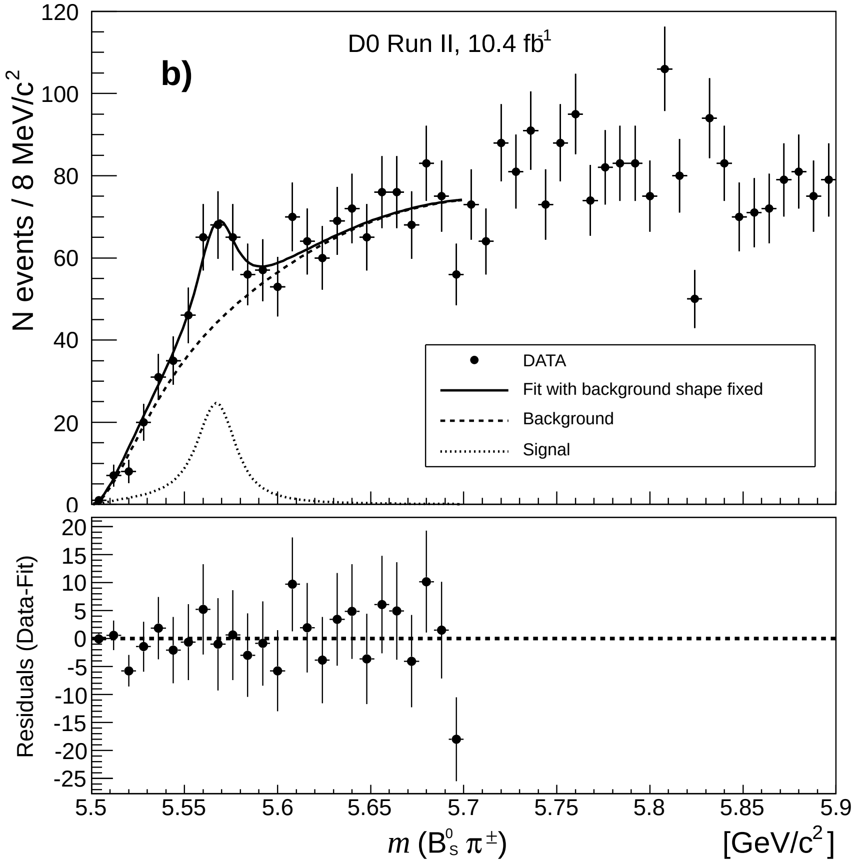
$P_c(4380)$ **($\Gamma \sim$ 200 MeV)**

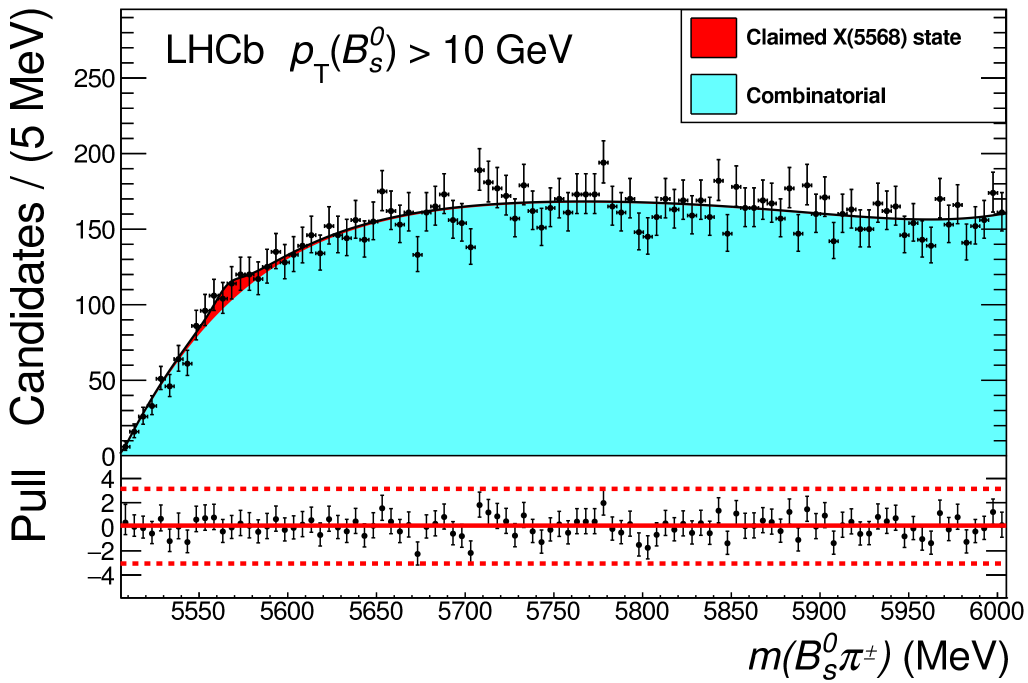
$P_c(4450)$ **($\Gamma \sim$ 30 MeV)**



LHCb, PRL(2015)

The only *flavour-exotic* candidate is $X(5568)$



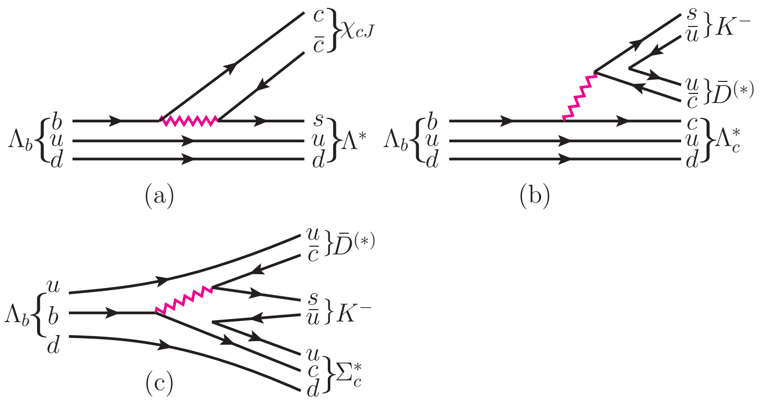


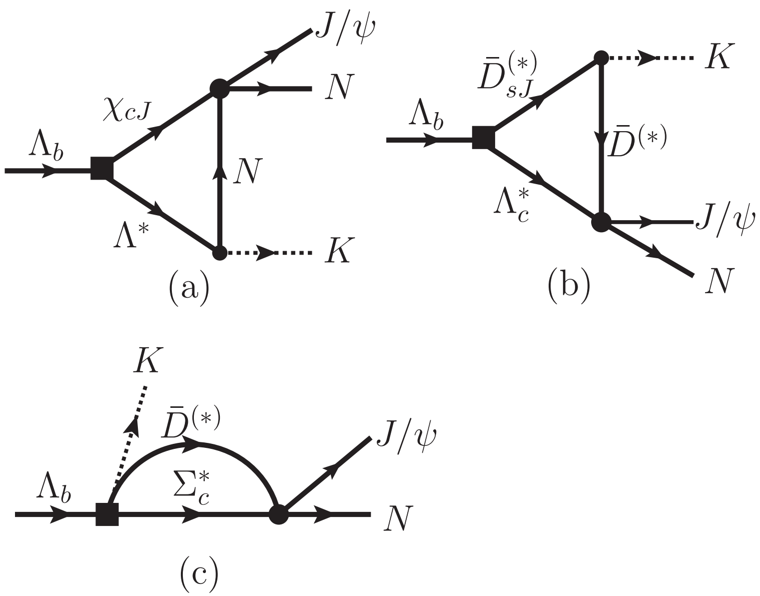
D0 vs CDF, LHCb, CMS, ATLAS (have no signals)

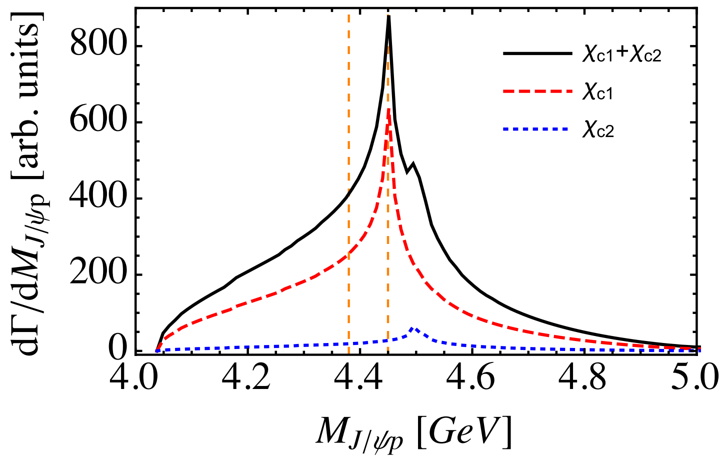
What are these structures — many scenarios.

Are they real resonances or just structures in the cross-section (anomalous thresholds, cusps etc)?

Nonresonant nature of observed distributions







Amplitudes involving hadron rescatterings (triangle diagrams + two-point functions) may lead to the resonance structures in the distributions similar to the observed ones

Phenomenological approaches to exotic states

Exotic hadrons as quark – diquark confined bound states

Naive constituent-quark model:

Constituent quarks [$m_q = 250$ MeV ($q = u, d$), $m_s = 350$ MeV]

interacting via potential (confining at large distances + OGE at short distances)

are building blocks of ordinary hadrons: *quark-antiquark mesons* **and** *three-quark baryons*.

Color-triplet–antitriplet interation via multigluon exchanges is confining at large distances

Concept of DIQUARKS: color-antitriplet $\bar{D}^a = \epsilon^{abc} q_b q_c$ made of two quarks.

Ordinary mesons: $\bar{q}^a q^a$ ($\bar{q}q$)

Ordinary baryons: $\epsilon_{abc} q^a q^b q^c = \bar{D}^a q^a$ (qqq)

Tetraquarks: $\bar{D}^a D^a$ ($\bar{q}q\bar{q}q$)

Pentaquarks: $\epsilon_{abc} \bar{D}^a \bar{D}^b \bar{q}^c$ ($\bar{q}qqqq$)

- **Why not $\epsilon_{abc} \bar{D}^a \bar{D}^b \bar{D}^c$ (confined 6-quark state)?**
- **Hierarchy of sizes: diquarks are not really compact objects, size similar to meson size**
- **Not easy to obtain narrow exotic states**

Exotic hadrons as molecular bound states

meson-meson long-distance interactions may produce a bound state (similar to the deuteron).

Mass below threshold; if relatively wide, may see the tail.

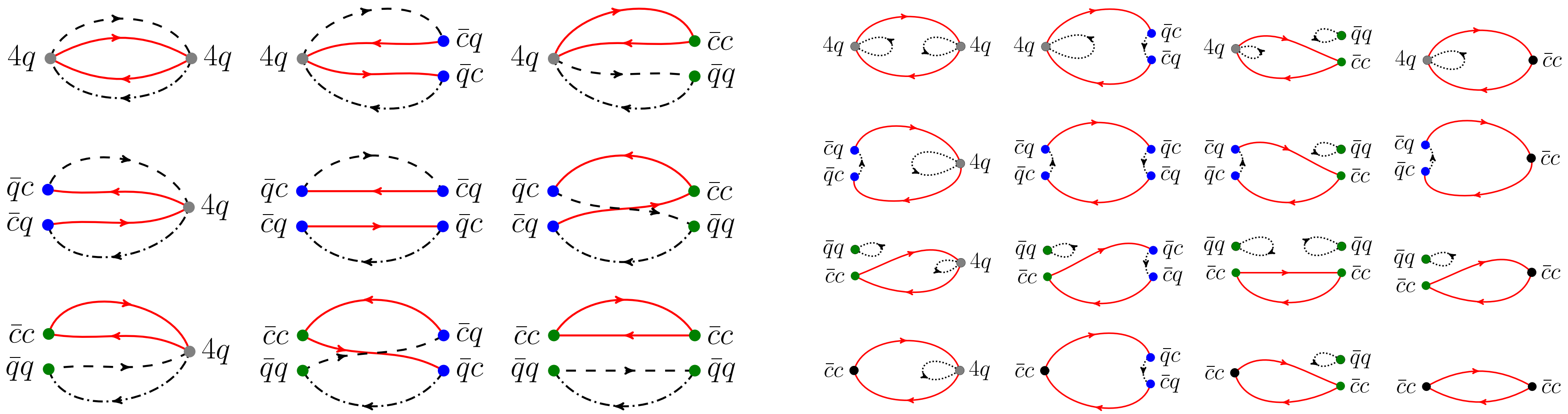
- **How to obtain from QCD the potential?**

How to systematically understand exotic states in QCD?

Lattice QCD

- **Put QCD on the lattice in a box (discrete spectrum of bound states)**
- **Consider two-point functions of a set of interpolating currents $\langle T\, J(x)\, j(0)\rangle$ in Eucledian space**
- **Identify those discrete states which yield meson-meson continuum states in the continuum limit**
- **By modifying the operator set identify the discrete continuum state and determine its properties**

Example of lattice study of $X(3872$ [Prelovsek et al, 2015: $\bar{c}c$, meson-meson, four-quark operators]



"A lattice candidate for $X(3872)$ with $I = 0$ is observed very close to the experimental state only if both $\bar{c}c$ and $\bar{D}D$ interpolators are included; the candidate is not found if diquark-antidiquark and $\bar{D}D$ are used in the absence of $\bar{c}c$. No candidate for neutral or charged $X(3872)$, or any other exotic candidates are found in the $I = 1$ channel. We also do not find signatures of exotic $\bar{c}c\bar{s}s$ candidates below 4.2 GeV". *No convincing signatures of tetraquark bound states.*

QCD at large Nc

$SU(N_c)$ **gauge theory with** $N_c \to \infty$ **and** $\alpha_s \sim 1/N_c$**. At leading order, QCD Green functions have only non-interacting mesons as intermediate states; tetraquark bound states may emerge only in** N_c**-subleading diagrams. This fact was believed to provide the theoretical explanation of the non-existence of exotic tetraquarks.**

However, even if the exotic tetraquark bound states appear only in subleading diagrams, the crucial question is their width: if narrow, they might be well observed in nature.

[W.Lucha, D.M., H.Sazdjian, "Narrow exotic tetraquark mesons in large-Nc QCD", PRD96, 014002 (2017); "Tetraquarks and two-meson states at large-Nc", EPJC77, 866 (2017).]

We discuss four-point Green functions of bilinear quark currents, depend on 6 variables p_1^2, p_2^2, $p_1'^2$, $p_2'^2$, $p = p_1 + p_2 = p_1' + p_2'$, **and the two Mandelstam variables** $s = p^2$ **and** $t = (p_1 - p_1')^2$**.**

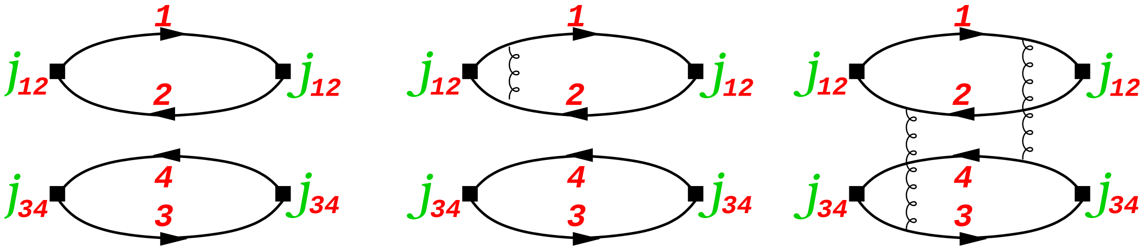
Criteria for selecting diagrams which potentially contribute to the tetraquark pole at $s = M_T^2$**:**

1. **The diagram should have a nontrivial (i.e., non-polynomial) dependence on the variable** s**.**
2. **The diagram should have a four-particle cut (i.e. threshold at** $s = (m_1 + m_2 + m_3 + m_4)^2$**), where** m_i **are the masses of the quarks forming the tetraquark bound state. The presence or absence of this cut is established by solving the Landau equations for the corresponding diagram.**

Flavour-exotic tetraquarks

Bilinear quark currents $j_{ij} = \bar{q}_i q_j$ **producing** M_{ij} **from the vacuum,** $\langle 0|j_{ij}|M_{ij}\rangle = f_{M_{ij}}$, $f_M \sim \sqrt{N_c}$.

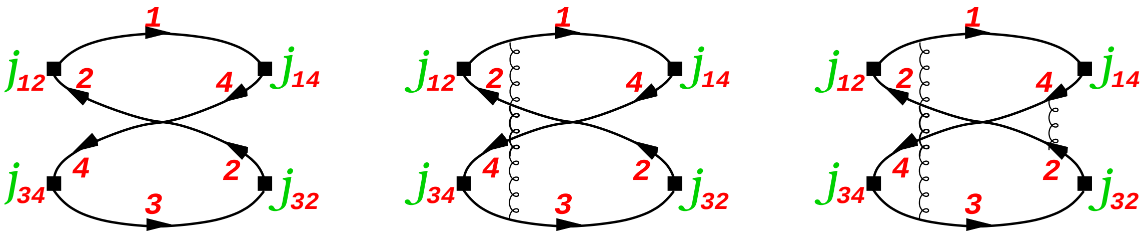
"Direct" 4-point functions $\Gamma_{\mathrm{I}}^{(\mathrm{dir})} = \langle j_{12}^\dagger j_{34}^\dagger j_{12} j_{34}\rangle$ **and** $\Gamma_{\mathrm{II}}^{(\mathrm{dir})} = \langle j_{14}^\dagger j_{32}^\dagger j_{14} j_{32}\rangle$**:**



$\sim N_c^2$ $\sim N_c^3 \alpha_s$ $\sim N_c^2 \alpha_s^2$

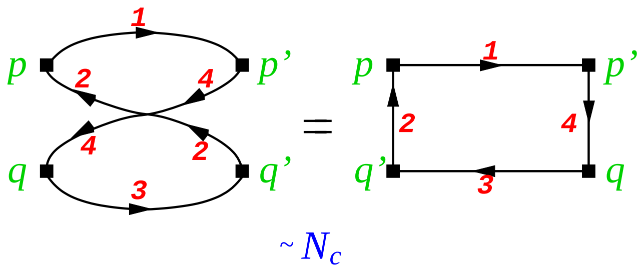
(a) (b) (c)

"Recombination" functions $\Gamma^{(\mathrm{rec})} = \langle j_{12}^\dagger j_{34}^\dagger j_{14} j_{32}\rangle$ **and** $\Gamma^{(\mathrm{rec})\dagger}$**:**

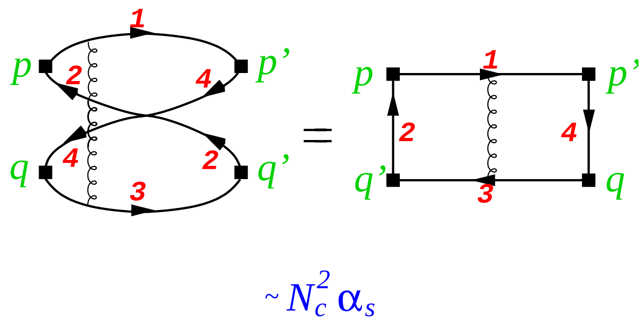


$\sim N_c$ $\sim N_c^2 \alpha_s$ $\sim N_c \alpha_s^2$

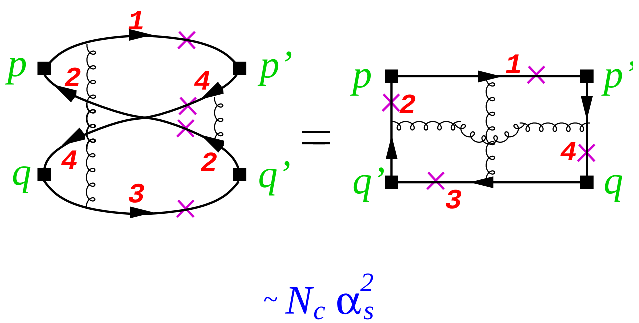
(a) (b) (c)



(a)



(b)



(c)

$$\Gamma^{(\mathrm{dir})}_{\mathrm{I},T} = \langle j^\dagger_{12} j^\dagger_{34} j_{12} j_{34}\rangle = O(N_c^0), \quad \Gamma^{(\mathrm{dir})}_{\mathrm{II},T} = \langle j^\dagger_{14} j^\dagger_{32} j_{14} j_{32}\rangle = O(N_c^0), \quad \Gamma^{(\mathrm{rec})}_{T} = \langle j^\dagger_{12} j^\dagger_{34} j_{14} j_{32}\rangle = O(N_c^{-1}).$$

The fact that “dir” and “rec” amplitudes have different behaviors in N_c requires at least two exotic poles:
T_A couples stronger to $M_{12}M_{34}$ channel, T_B couples stronger to $M_{14}M_{32}$ channel.

$$\begin{aligned}
\Gamma^{(\mathrm{dir})}_{\mathrm{I},T} &= O(N_c^0) = f_M^4\left(\frac{|A(M_{12}M_{34}\to T_A)|^2}{p^2-M^2_{T_A}} + \frac{|A(M_{12}M_{34}\to T_B)|^2}{p^2-M^2_{T_B}}\right) + \cdots, \\
\Gamma^{(\mathrm{dir})}_{\mathrm{II},T} &= O(N_c^0) = f_M^4\left(\frac{|A(M_{14}M_{32}\to T_A)|^2}{p^2-M^2_{T_A}} + \frac{|A(M_{14}M_{32}\to T_B)|^2}{p^2-M^2_{T_B}}\right) + \cdots, \\
\Gamma^{(\mathrm{rec})}_{T} &= O(N_c^{-1}) = f_M^4\left(\frac{A(M_{12}M_{34}\to T_A)A(T_A\to M_{14}M_{32})}{p^2-M^2_{T_A}} + \frac{A(M_{12}M_{34}\to T_B)A(T_B\to M_{14}M_{32})}{p^2-M^2_{T_B}}\right) + \cdots.
\end{aligned}$$

We seek tetraquarks with finite mass at large N_c:

$$\begin{aligned}
&A(T_A\to M_{12}M_{34}) = O(N_c^{-1}), \qquad A(T_A\to M_{14}M_{32}) = O(N_c^{-2}), \\
&A(T_B\to M_{12}M_{34}) = O(N_c^{-2}), \qquad A(T_B\to M_{14}M_{32}) = O(N_c^{-1}).
\end{aligned}$$

The widths $\Gamma(T_{A,B}) = O(N_c^{-2})$.

Mixing between T_A and T_B:

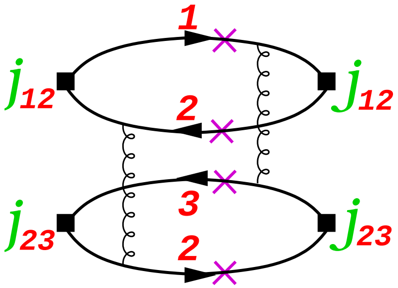
Introducing mixing parameter g_{AB}, we get additional contributions to the Green functions. Most restrictive for g_{AB} is the recombination function, for which mixing provides the additional contribution

$$\Gamma_T^{(\mathrm{rec})} = O(N_c^{-1}) = f_M^4 \left(\frac{A(M_{12}M_{34} \to T_A)}{p^2 - M_{T_A}^2} g_{AB} \frac{A(T_B \to M_{14}M_{32})}{p^2 - M_{T_B}^2} \right) + \cdots .$$

The mixing parameter $g_{AB} \le O(N_c^{-1})$: the two flavor-exotic tetraquarks of the same flavor content do not mix at large N_c.

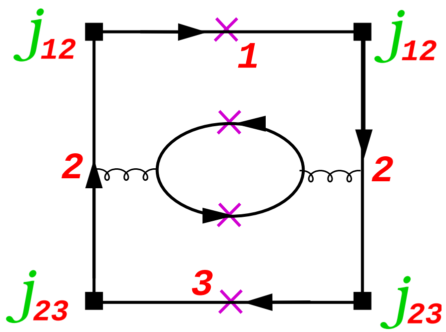
Diagrams of new topologies emerge.

For direct functions $\Gamma^{(\mathrm{dir})}_{(\mathrm{I,II}),T}$, new diagrams do not change leading large-N_c behavior:



$\sim N_c^2\,\alpha_s^2$

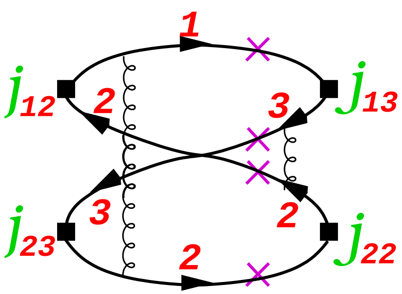
(a)



$\sim N_c^2\,\alpha_s^2$

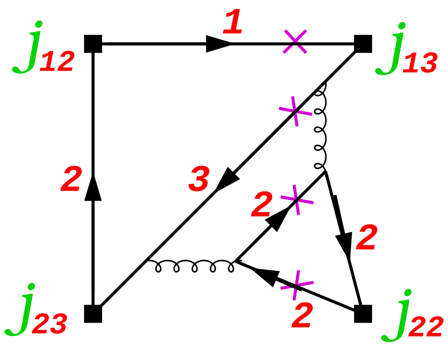
(b)

For recombination functions, the new diagram modifies leading large-N_c behavior



$\sim N_c\,\alpha_s^2$

(a)



$\sim N_c^2\,\alpha_s^2$

(b)

The new diagram modifies the leading large-N_c behavior:

$$\Gamma^{(\mathrm{dir})}_{\mathrm{I},T} = \langle j^\dagger_{12} j^\dagger_{23} j_{12} j_{23}\rangle = O(N_c^0), \quad \Gamma^{(\mathrm{dir})}_{\mathrm{II},T} = \langle j^\dagger_{13} j^\dagger_{22} j_{13} j_{22}\rangle = O(N_c^0), \quad \Gamma^{(\mathrm{rec})}_{T} = \langle j^\dagger_{12} j^\dagger_{23} j_{13} j_{22}\rangle = O(N_c^0).$$

"dir" and "rec" functions have the same behavior, and one exotic state T is enough:

$$A(T \to M_{12}M_{23}) \;=\; O(N_c^{-1}), \qquad A(T \to M_{13}M_{22}) = O(N_c^{-1}).$$

Its width is $\Gamma(T) = O(N_c^{-2})$.

T can mix with the ordinary meson M_{13}. The restriction on the mixing parameter $g_{TM_{13}}$:

$$\Gamma^{(\mathrm{dir})}_{\mathrm{I},T} = O(N_c^0) = f_M^4 \left(\frac{A(M_{12}M_{23} \to T)}{p^2 - M_T^2} g_{TM_{13}} \frac{A(M_{13} \to M_{12}M_{23})}{p^2 - M^2_{M_{13}}} \right) + \cdots .$$

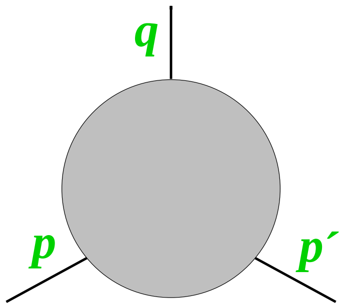
$A(M_{13} \to M_{12}M_{23}) \sim 1/\sqrt{N_c}$**, so** $g_{TM_{13}} \leq O(1/\sqrt{N_c})$**.**

The analysis of Green functions in large-N_c QCD allows one to restrict some properties of the possible exotic states.

QCD sum rules for exotic states

Strong decays from 3 – point vertex functions

• **The basic object:**



$$\Gamma(p,p',q)=\int\langle\Omega|T(J(x)j(0)j'(x')|\Omega\rangle\exp(ipx-ip'\,x')dxdx'$$

This correlator contains the triple-pole in the Minkowski region: namely

$$\Gamma(p,p',q)=\frac{ff'}{(p^2-M^2)(p'^2-M'^2)}F(q^2)+\cdots$$

where the form factor $F(q^2)$ contains pole at $q^2=M_q^2$:

$$F(q^2)=\frac{f_q g_{MM'M_q}}{(q^2-M_q^2)}+\cdots$$

$g_{MM'M_q}$ **describes the $M\to M_1M_2$ strong transition;**

f**,** f'**, and** f_{M_q} **are the decay constants of the mesons $\langle 0|j(0)|M\rangle=f_M$.**

The three-point function satisfies the double spectral representation

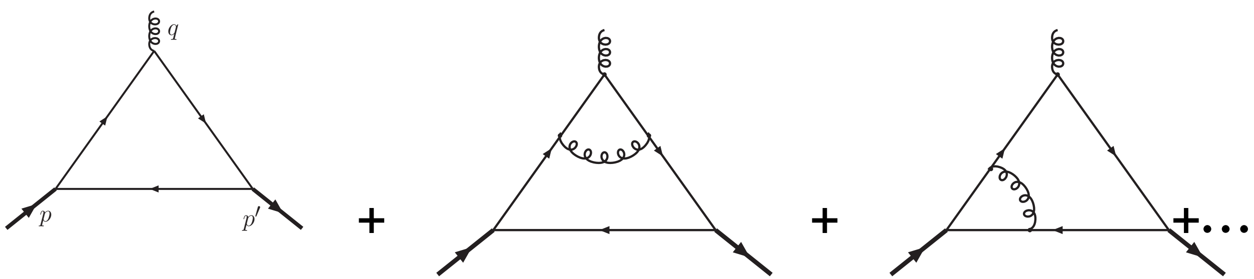
$$\Gamma(p,p',q) = \int \frac{ds}{s-p^2}\frac{ds'}{s'-p'^2}\Delta(s,s',q^2)$$

Perform double Borel transform $p^2 \to \tau$, $p'^2 \to \tau'$ and applying duality we obtain

$$\exp(-M^2\tau)\exp(-M'^2\tau')ff'F(q^2) = \int^{s_{\rm eff}} ds\,\exp(-s\tau)\int^{s'_{\rm eff}} ds'\,\exp(-s'\tau')\Delta_{\rm OPE}(s,s',q^2)$$

• **Normal hadrons:**

$$\Gamma_{\rm OPE}(p^2,p'^2,q^2) = \Gamma_0(p^2,p'^2,q^2) + \alpha_s\Gamma_1(p^2,p'^2,q^2) + \ldots$$



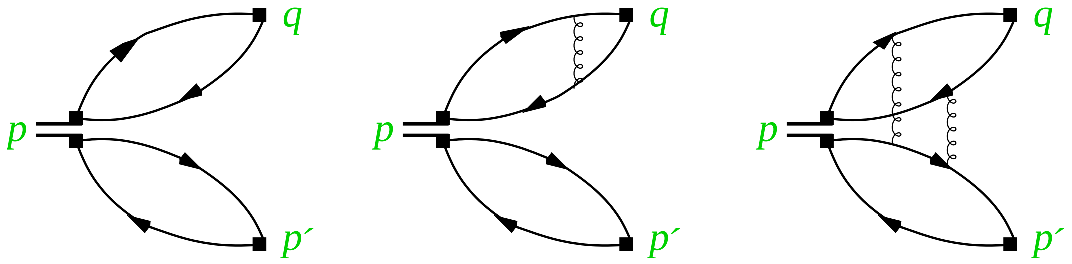
Already one-loop zero-order diagram has a nonzero double-spectral density. And therefore provides a nonzero contribution to the form factor at small and intermediate momentum transfers (and to the coupling). Radiative corrections and are crucial for large q^2 and improve the result.

• **Exotic hadrons:** $\langle T(\theta(x) j_1(0) j_2(y))\rangle$.
Many possibilities to write interpolating current for X, $\langle 0|\theta|X\rangle = f_X, \qquad f_X \neq 0.$

$$\theta = M(x)M(x), \qquad M(x) = \bar{q}(x)O\bar{q}(x)$$
$$\theta = M^A(x)M^A(x), \qquad M^A(x) = \bar{q}(x)\lambda^A O\bar{q}(x) \quad [A = 1\ldots, 8]$$
$$\theta = \bar{D}^a(x)D^a(x), \qquad D^a(x) = \epsilon^{abc}(q_c(x))^T C O q_b(x), \quad [a, b, c = 1, \ldots, 3]$$

The choice of interpolating current does not infuence the presence / absence of a bound state; dynamics decides

Color Singlet - color singlet:



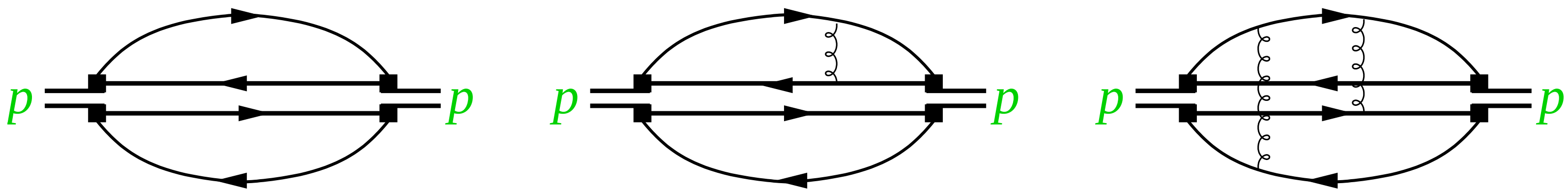
$$\Gamma_{\mathrm{OPE}}(p^2, p'^2, q^2) = \Pi(p'^2)\Pi(q^2) + (\alpha_s)^2\Gamma_{\mathrm{connected}}(p^2, p'^2, q^2)$$

After the Borel transform $p^2 \to \tau$, **the disconnected leading-order contribution vanishes.**

Thus the LO contribution is not related to the exotic-state decay.
Clear from the factorization property $\Gamma(p, p', q) = \Pi(p'^2)\Pi(q^2)$
and from the large-N_c **behaviour of the QCD diagrams.**

2 – point function of exotic currents

Singlet-singlet:



How to write down the duality relation?

$$f_X^2 \exp(-M_X^2 \tau) + \Pi^{\rm cont}_{meson-meson}(\tau) = \int ds \exp(-s\tau)\rho_{XX}(s) + \Pi_{\rm power}(\tau)$$

The right prescription:

$\boxed{\textbf{Do not include the LO (i.e. start with order } O(\alpha_s^2)\textbf{) in the singlet-singlet case: } f_X^{(1)} \sim O(\alpha_s)\textbf{.}}$

Outlook

• **Around 20 candidates for exotic resonances: $\bar{c}c\bar{q}q$ or $\bar{b}b\bar{q}q$ + $\bar{c}cqqq$.**
$X(5568)$ in $B_s\pi^\pm$ (?) is doubtful. No light-quark candidates $\bar{q}q\bar{q}q$. No convincing interpretation. Various possibilities.

• *Non-resonance explanations*:
the observed structures are due to hadron low-energy diagrams (triangles and loops). The amplitudes contains many unknown couplings. Difficult to obtain a certain conclusion.

• *Phenomenological models*:
Based on diquarks in a confining potential predict spectrum of exotic states
Based on hadron-hadron potentials predict molecular states (depending on specific potentials)

• *Lattice QCD*:
Very difficult setup; no convincing results for exotic states

• *QCD at large N_c*:
Two exotic $\bar{q}_1q_2\bar{q}_3q_4$ narrow states $\Gamma \sim O(1/N_c^2)$, each decaying into one meson-meson channel.
One cryptoexotic state $\bar{q}_1q_2\bar{q}_2q_3$ $\Gamma \sim O(1/N_c^2)$ decaying into various meson-meson channels with similar probabilities.

• *QCD sum rules*
Dozens of papers, many confirming the resonance interpretation of exotic states. However, not all conceptual issues of the method for exotic states have been fully settled.

• **Dynamics of fall-apart decays of exotic resonances has fundamental difference from dynamics of ordinary-meson decays: the appropriate contributions to Green functions describing decays of exotic states emerge only at subleading α_s orders; the leading order disconnected diagrams are** *not related to strong decays of exotic hadrons*. **This makes the calculation of α_s-corrections mandatory.**

Many efforts for otaining reliable predictions are necessary!