The post-Newtonian limit of hybrid f(R)-gravity

XXth International Seminar on High Energy Physics “Quarks-2018”
Prerequisites for the expansion of GR

• Dark matter

• Dark energy

• Incompatibility of gravity with quantum mechanics
f(R)-gravity

- Metric f(R)-gravity

\[ R = g^{\mu\nu} R_{\mu\nu} \equiv g^{\mu\nu} \left( \Gamma^\alpha_{\mu\nu,\alpha} - \Gamma^\alpha_{\mu\alpha,\nu} + \Gamma^\alpha_{\alpha\lambda} \Gamma^\lambda_{\mu\nu} - \Gamma^\alpha_{\mu\lambda} \Gamma^\lambda_{\alpha\nu} \right) \]

- Palatini f(R)-gravity

\[ \mathcal{R} \equiv g^{\mu\nu} \mathcal{R}_{\mu\nu} \equiv g^{\mu\nu} \left( \hat{\Gamma}^\alpha_{\mu\nu,\alpha} - \hat{\Gamma}^\alpha_{\mu\alpha,\nu} + \hat{\Gamma}^\alpha_{\alpha\lambda} \hat{\Gamma}^\lambda_{\mu\nu} - \hat{\Gamma}^\alpha_{\mu\lambda} \hat{\Gamma}^\lambda_{\alpha\nu} \right) \]
Hybrid f(R)-gravity

\[ S = \frac{1}{2k^2} \int d^4x \sqrt{-g} [R + f(\mathcal{R})] + S_m, \]

- \( g \) is the determinant of the metric
- \( R \) is the metric Ricci scalar
- \( \mathcal{R} \) is the Palatini curvature
- \( S_m \) is the standard matter action
- \( k^2 = 8\pi G/c^4 \)
- \( G \) is the gravitational constant
- \( c \) is the speed of light

\[ \mathcal{R}_{\mu\nu} = R_{\mu\nu} + \frac{3}{2} \frac{1}{F^2(\mathcal{R})} F(\mathcal{R})_{,\mu} F(\mathcal{R})_{,\nu} \]

\[ - \frac{1}{F(\mathcal{R})} \nabla_\mu F(\mathcal{R})_{,\nu} - \frac{1}{2} \frac{1}{F(\mathcal{R})} g_{\mu\nu} \nabla^\alpha \nabla^\alpha F(\mathcal{R}), \]

\[ F(\mathcal{R}) = \frac{df(\mathcal{R})}{d\mathcal{R}}. \]
Scalar-tensor representation

\[ S = \frac{1}{2k^2} \int d^4x \sqrt{-g} \left[ \phi R + \frac{3}{2\phi} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] + S_m \]

\( \phi \) is the scalar field,

\( V(\phi) \) is the scalar potential.
Field equations

\[ R_{\mu\nu} = \frac{1}{1 + \phi} \left( k^2 (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T) \right) \]

\[ + \frac{1}{2} g_{\mu\nu} (V(\phi) + \nabla_{\alpha} \nabla^{\alpha} \phi) + \nabla_{\mu} \nabla_{\nu} \phi - \frac{3}{2\phi} \partial_{\mu} \phi \partial_{\nu} \phi, \]

\[-\nabla_{\mu} \nabla^{\mu} \phi + \frac{1}{2\phi} \partial_{\mu} \phi \partial^{\mu} \phi + \frac{\phi [2V(\phi) - (1 + \phi)V_{\phi}]}{3} = \frac{\phi k^2}{3} T, \]

\[ V_{\phi} = \frac{dV(\phi)}{d\phi}. \]
PPN formalism

• Weak field limit,
• Asymptotically flat space-time background,
• Small velocities,
• Motion of matter would obey to the hydrodynamics equations for the perfect fluid.

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PPN and massive scalar-tensor theories

• Very massive scalar field $m_\phi r >> 1$

• Very light scalar field $m_\phi r << 1$

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$\phi = \phi_0 + \varphi$

\[
R_{\mu\nu} = \frac{1}{1 + \phi} \left( k^2 (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T) + \frac{1}{2} g_{\mu\nu} (V(\phi) + \nabla_\alpha \nabla^\alpha \phi) + \nabla_\mu \nabla_\nu \phi - \frac{3}{2\phi} \partial_\mu \phi \partial_\nu \phi \right),
\]

$-\nabla_\mu \nabla^\mu \phi + \frac{1}{2\phi} \partial_\mu \phi \partial^\mu \phi + \frac{\phi[2V(\phi) - (1 + \phi) V_\phi]}{3} = \frac{\phi k^2}{3} T.$
General form of the PPN metric

\begin{align*}
g_{00} &= -1 + 2U - 2\beta U^2 - 2\xi \Phi_w + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi) \Phi_1 + \\
&\quad + 2 \left( 3\gamma - 2\beta + 1 + \zeta_2 + \xi \right) \Phi_2 + 2(1 + \zeta_3) \Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi) \Phi_4 - \\
&\quad - (\zeta_1 - 2\xi) A - (\alpha_1 - \alpha_2 - \alpha_3) w^2 U - \alpha_2 w^i w^j U_{ij} + (2\alpha_3 - \alpha_1) w^i V_i, \\
g_{0i} &= -\frac{1}{2} (4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi) V_i - \frac{1}{2} (1 + \alpha_2 - \zeta_1 + 2\xi) W_i - \\
&\quad - \frac{1}{2} (\alpha_1 - 2\alpha_2) w^i U - \alpha_2 w^j U_{ij}, \\
g_{ij} &= (1 + 2\gamma U) \delta_{ij}.
\end{align*}

PPN parameter $\gamma$

$$\gamma = \frac{1 + \phi_0 \exp(-m_\phi r) / 3}{1 - \phi_0 \exp(-m_\phi r) / 3}$$

where

$m_\phi$ is scalar field mass,

$\phi_0$ is the asymptotical value of the scalar field far away from the local system.

*Hybrid metric-Palatini gravity, S. Capozziello etc., Universe (2015)
PPN parameter $\beta$

$$\beta = 1 - \frac{\phi_0 (1 + \phi_0)}{18} \frac{\exp(-2m_\phi r)}{(1 - \phi_0 \exp(-m_\phi r)/3)^2}$$

Other PPN parameters $\xi, \alpha_1, \alpha_2, \alpha_3, \zeta_1, \zeta_1, \zeta_1, \zeta_1$ equal to zero in hybrid gravity in considered approximation $m_\phi \ll 1/r$. 
Constraints on $\phi_0$ and $m_\phi$

$m_\phi = r \ (1\text{AU})$
Constraints on $\phi_0$ and $m_\phi$

$m_\phi = r \ (1\text{AU})$
Conclusions

• PPN formalism is applied to hybrid f(R)-gravity in the case $m_\phi r \ll 1$

• restrictions on the $m_\phi$ and $\phi_0$ were obtained in the weak-field limit

• the theory is valid in the Solar system in the case of light scalar field
Thank you for your attention!
PPN potentials

\[
U = \int \frac{\rho'}{|x - x'|} d^3x', \quad U_{ij} = \int \frac{\rho'(x - x'_i)(x - x'_j)}{|x - x'|^3} d^3x',
\]

\[
\Phi_w = \int \frac{\rho' \rho''(x - x')}{|x - x'|^3} \left( \frac{x' - x''}{|x - x''|} - \frac{x - x''}{|x' - x''|} \right) d^3x' d^3x'',
\]

\[
\Phi_1 = \int \frac{\rho' v^2}{|x - x'|} d^3x', \quad \Phi_2 = \int \frac{\rho' U'}{|x - x'|} d^3x',
\]

\[
\Phi_3 = \int \frac{\rho' \Pi'}{|x - x'|} d^3x', \quad \Phi_4 = \int \frac{p'}{|x - x'|} d^3x',
\]

\[
V_i = \int \frac{\rho' v'_i}{|x - x'|} d^3x', \quad W_i = \int \frac{\rho' v'(x - x')(x - x')_i}{|x - x'|^3} d^3x'.
\]