Strong coupling in the Galilean Genesis

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Outline

Motivation

- Inflation is now the strongest candidate of the early universe scenario that explains current cosmological observations consistently.
  A. H. Guth'1981, A. A. Starobinsky'1980
- In order to be convinced that inflation indeed occurred in the early stage of the universe, all other possibilities must be ruled out.
- It is therefore well motivated to study how good and how bad alternative possibilities are compared to inflation.

Motivation

- Non-singular stages in the early universe cannot only be something that replaces inflation, but also early-time completion of inflation just to get rid of the initial singularity.

Motivation

- We address whether healthy non-singular cosmologies can be implemented in the framework of general scalar-tensor theories.

We want to obtain...

Completion or alternative to inflation, totally healthy new early stage:

- Non-singular $\rightarrow$ VIOLATE NEC.
- Stable $\rightarrow$ AVOID ALL NO-GOs.
- No quantum gravity regime, want to use our effective theory $\rightarrow$ SURPRISE!

Null energy condition in a nutshell

- If gravity is described by general relativity and the energy-momentum tensor $T_{\mu\nu}$ of matter satisfies the NEC, that is, $T_{\mu\nu}k^{\mu}k^{\nu} \geq 0$ for every null vector k^{μ} in non-minimal coupling case,then it follows from the Einstein equations that $\dot{H} \leq 0$, where H is the Hubble parameter.
- This implies that an expanding universe yields a singularity in the past...

Let's violate it!

... while NEC violation could lead to singularity-free cosmology. However, violating the NEC in a healthy manner turns out to be challenging.

Null energy condition in a nutshell

NEC for scalar field

The NEC is satisfied for a canonical scalar field,
$T_{\mu\nu}k^\mu k^\nu = \dot{\phi}^2 \geq 0$.

- In a general non-canonical scalar-field theory whose Lagrangian is dependent on ϕ and its first derivative, the NEC can be violated, but NEC-violating cosmological solutions are unstable because the curvature perturbation has the wrong sign kinetic term.

Null energy condition in a nutshell

- Galileon theory and its generalizations
  A. Nicolis, R. Rattazzi, E. Trincherini'2009,
  C. Deffayet, G. Esposito-Farese, A. Vikman'2009,
  C. Deffayet et al'2011,
  involve the scalar field: whose Lagrangian contains second derivatives of ϕ while maintaining the second-order nature of the equation of motion and thus erasing the Ostrogradsky instability.

G. W. Horndeski 1973

International Journal of Theoretical Physics, Vol. 10, No. 6 (1974), pp. 363–384

Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space

GREGORY WALTER HORNDESKI

Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada

Received: 10 *July* 1973

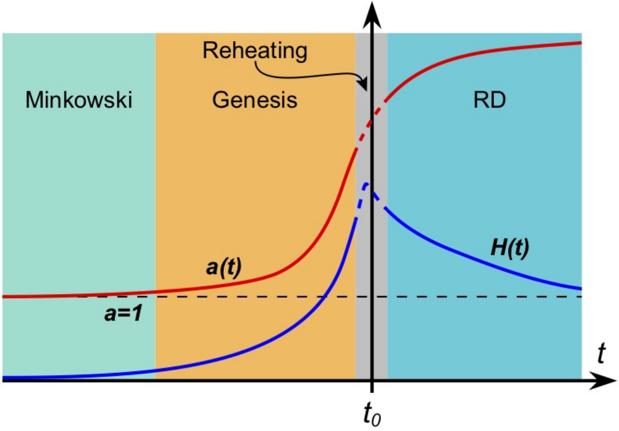
Abstract

Lagrange scalar densities which are concomitants of a pseudo-Riemannian metric-tensor, a scalar field and their derivatives of arbitrary order are considered. The most general second-order Euler–Lagrange tensors derivable from such a Lagrangian in a four-dimensional space are constructed, and it is shown that these Euler–Lagrange tensors may be obtained from a Lagrangian which is at most of second order in the derivatives

Null energy condition in a nutshell

- In contrast to the previous case of usual scalar field, it was found that the NEC and the stability of cosmological solutions are uncorrelated in Galileon-type theories.
  P. Creminelli, A. Nicolis and E. Trincherini'2010,
  P. Creminelli et al'2013,
  S. Nishi, T. Kobayashi'2015.
- This fact gives rise to healthy NEC-violating models of Galilean genesis.
  K. Hinterbichler, A. Joyce, J. Khoury, G. E. J. Miller'2012, 2013,
  D. A. Easson, I. Sawicki, A. Vikman'2013,
  S. Nishi, T. Kobayashi'2016.

Genesis



Genesis

To build the Genesis we want the following asymptotic behaviour at large times ($t \to -\infty$) for scale factor, lapse function and Hubble:

$$a \approx 1 + \frac{\chi}{\delta(-t)^{\delta}},$$

$$N \approx 1 + \frac{\eta}{(-t)^{\xi}},$$

$$H = \frac{\dot{a}}{Na} \approx \frac{\chi}{(-t)^{\delta+1}}.$$

We are also interested in asymptotic **time** behaviour at large times for all coefficients in our model, e.x. $\mathcal{F}_T$, $\mathcal{G}_T$, etc.

Instabilities. No-Go

- Although the Galileon-type theories do admit a stable early stage without an initial singularity, the genesis/bouncing universe must be interpolated to a subsequent (possibly conventional) stage and the stable early stage does not mean that the cosmological solution is stable at all times during the whole history.
- There are already several No-Go theorems for different models!
  D. Pirtskhalava, L. Santoni, E. Trincherini, P. Uttayarat'2014,
  T. Kobayashi, M. Yamaguchi, J. Yokoyama'2015,
  M. Libanov, S. Mironov, V. Rubakov'2016,
  R. Kolevatov, S. Mironov, N. Sukhov, V. Volkova'2017.
- And we actually can avoid No-Go in our case...

No-Go for full Horndeski. Tsutomu Kobayashi'17

Generic instabilities of non-singular cosmologies in Horndeski theory: a no-go theorem

Tsutomu Kobayashi[1,*]

[1] *Department of Physics, Rikkyo University, Toshima, Tokyo 171-8501, Japan*

The null energy condition can be violated stably in generalized Galileon theories, which gives rise to the possibilities of healthy non-singular cosmologies. However, it has been reported that in many cases cosmological solutions are plagued with instabilities or have some pathologies somewhere in the whole history of the universe. Recently, this was shown to be generically true in a certain subclass of the Horndeski theory. In this short paper, we extend this no-go argument to the full Horndeski theory and show that non-singular models (with flat spatial sections) in general suffer from either gradient instabilities or some kind of pathology in the tensor sector. This implies that one must go beyond the Horndeski theory to implement healthy non-singular cosmologies.

PACS numbers: 98.80.Cq, 04.50.Kd

Full Horndeski theory and No-Go

Full Horndeski theory action and lagrangian are:

$$S = \int d^4x\sqrt{-g}\mathcal{L}_H,$$

where

$$\begin{aligned}\mathcal{L}_H = {} & G_2(\phi, X) - G_3(\phi, X)\Box\phi + \\ & G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ & + G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi \\ & - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right],\end{aligned}$$

$$X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi. \tag{1}$$

Full Horndeski theory and No-Go

The perturbed metric is written in ADM 3+1 splitting

$$ds^2 = -N^2(t)dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt).$$

We also fix the following gauge:

$$-t = e^{-\phi}, N^{-1} = e^{-\phi}\sqrt{2X}.$$

The quadratic actions for h_{ij} and ζ are given, respectively, by

$$S_h^{(2)} = \frac{1}{8}\int dtd^3x\, a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\partial h_{ij})^2\right],$$

and

$$S_\zeta^{(2)} = \int dtd^3x\, a^3 \left[\mathcal{G}_S \dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2}(\partial\zeta)^2\right].$$

Full Horndeski theory and No-Go

These guys should be canonized to obtain canonical form of second-order action:

$$S^{(2)}(h) \rightarrow S^{(2)}(r),$$

and

$$S^{(2)}(\zeta) \rightarrow S^{(2)}(\psi),$$

with new canonical variables

$$r \rightarrow \sqrt{\mathcal{G}_T}\,h,$$

$$\psi \rightarrow \sqrt{\mathcal{G}_s}\,\zeta,$$

Full Horndeski theory and No-Go

We use ADM formalism (just to make calculations easier):

$$\mathcal{L} = A_2(t,N) + A_3(t,N)K + A_4(t,N)\left(K^2 - K_{ij}^2\right) + B_4(t,N)R^{(3)},$$

and introduces the following "good" function of lagrangian to obtain healthy genesis:

$$A_2 = f^{-2(\alpha+1)-\delta}a_2(N), \qquad A_3 = f^{-2\alpha-1-\delta}a_3(N),$$
$$A_4 = -B_4 = -f^{-2\alpha}.$$

Full Horndeski theory and No-Go

...we consider:

Not full Horndeski: only $G_2(\phi, X)$, $G_3(\phi, X)$, $G_4(\phi)$

$$\mathcal{L} = A_2(t,N) + A_3(t,N)K + A_4(t,N)\left(K^2 - K_{ij}^2\right) + B_4(t,N)R^{(3)},$$

$$A_2 = f^{-2(\alpha+1)-\delta} a_2(N), \qquad A_3 = f^{-2\alpha-1-\delta} a_3(N),$$
$$A_4 = -B_4 = -f^{-2\alpha}.$$

Here $f \approx c(-t)$ for $t \to -\infty$. For parameters α and δ we have $2\alpha > \delta + 1 > 1$.

Full Horndeski theory and No-Go

Kobayashi gave us a hint how to obtain stable genesis in full Horndeski:

The only possibility to avoid No-Go

$\mathcal{F}_T \to 0$ as $t \to -\infty$.

Now let us recall the expression for $\mathcal{F}_T$:

$$\mathcal{F}_T := 2G_4.$$

And also let us remind where G_4 appears in the lagrangian:

$$\mathcal{L}_H = G_2(\phi, X) - G_3(\phi, X)\Box\phi$$
$$+ \mathbf{G_4}(\phi, \mathbf{X})\mathbf{R}...$$

Horndeski genesis faces strong coupling regime...?

- Now we have arrived to the key point;

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- Now we have arrived to the key point;
- $G_4(\phi, X)$ is a coefficient multiplied by R so $\rightarrow$ the sense of M_{Pl}^2;
- And we really want $\mathcal{F}_T \rightarrow 0$, so $G_4 \rightarrow 0$ and ... $M_{Pl}^2 \rightarrow 0$ to avoid Kobayashi's No-Go!

Horndeski genesis faces strong coupling regime...?

- Now we have arrived to the key point;
- $G_4(\phi, X)$ is a coefficient multiplied by R so $\rightarrow$ the sense of M_{Pl}^2;
- And we really want $\mathcal{F}_T \rightarrow 0$, so $G_4 \rightarrow 0$ and ... $M_{Pl}^2 \rightarrow 0$ to avoid Kobayashi's No-Go!
- Usually it means strong coupling regime or quantum gravity (QG). But we still have no QG today and that is why we just can not trust our effective theory if it has strong coupling :(

Horndeski genesis faces strong coupling regime...?

$$\mathcal{L}_H = G_2(\phi, X) - G_3(\phi, X)\Box\phi + \mathbf{G_4}(\phi, \mathbf{X})\mathbf{R}...$$

Horndeski genesis faces strong coupling regime...?

- But actually we don't need to throw out our theory! We have to test whether it has strong coupling or not.
- To do so we need to consider the third-order action of perturbation (We have considered only scalar sector for now).

Strong coupling...or not?

Cubic action for the curvature perturbation:

$$S_{\psi}^{(3)} = \int Ndtd^3x \ a^3 \left(\Lambda_1 \psi'^3 + \Lambda_2 \psi \psi'^2 + ...\right) \sim \frac{\Lambda_i}{\mathcal{F}_S^{3/2}} \left(\psi^3 (')^a (\partial)^b\right).$$

Strong coupling...or not?

No strong coupling regime

- Classical motion, controlled evolution
- Want to describe our system using our effective model without unknown quantum gravity

Strong coupling...or not?

To do that we require the Universe to expand faster than processes driven by the third-order action:

$$T_{\Lambda_{\mathfrak{L}}} \ll T_{Hubble},$$

and the characteristic time of all our parameter's change must be also bigger than time of "third-order" processes's change:

$$T_{\Lambda_{\mathfrak{L}}} \ll T_{parameters}.$$

Strong coupling...or not?

What these T_{Hubble} and other times $T_{\dots}$ are?

$$T_{\Lambda_i} \sim \frac{1}{\Lambda_i},$$

$$T_{Hubble} \sim \frac{1}{H},$$

$$T_{parameters} \sim \frac{1}{\dot{H}/H} \text{ or } \frac{1}{\dot{N}/N}.$$

Strong coupling...or not?

$$T_{\Lambda_i} \sim \frac{1}{\Lambda_i} \sim (-t)^{-x},$$

$$T_{Hubble} \sim \frac{1}{H} \sim (-t)^{1+\delta},$$

$$T_{parameters} \sim \frac{1}{\dot{H}/H} \text{ or } \frac{1}{\dot{N}/N} \sim (-t).$$

Here Λ is the set of our coefficients from $S^{(3)}_{\psi}$:

$$S^{(3)}_{\psi} = \int Ndtd^3x\ a^3 \left(\Lambda_1 \psi'^3 + \Lambda_2 \psi \psi'^2 + \ldots\right) \sim \frac{\Lambda_i}{\mathcal{F}_S^{3/2}} \left(\psi^3 (')^a (\partial)^b\right).$$

Strong coupling...or not?

Finally, we have the time-dependent $\Lambda_1(t)$,$\Lambda_2(t)$, ...,$\Lambda_{18}(t)$, $H(t)$ and everything as a function of t (remember! These functions are the asymptotics at $t \to \infty$) and this dependence has a form:

$$\Lambda_i \sim (-t)^x,$$

in other words here we have only **power dependence** for all our coefficients. That is why we can rewrite our condition for **NO** strong coupling in terms of "powers":

The strongest condition is...

$$T_{\Lambda_{\mathfrak{L}}} \ll T_{parameters}.$$

$$x + 3\alpha - \frac{3}{2}\delta < a + b - 1.$$

Strong coupling...or not?

...we consider:

Not full Horndeski: only $G_2(\phi, X)$, $G_3(\phi, X)$, $G_4(\phi)$

$$\mathcal{L} = A_2(t,N) + A_3(t,N)K + A_4(t,N)\left(K^2 - K_{ij}^2\right) + B_4(t,N)R^{(3)},$$

$$A_2 = f^{-2(\alpha+1)-\delta} a_2(N), \qquad A_3 = f^{-2\alpha-1-\delta} a_3(N),$$
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Here $f \approx c(-t)$ for $t \to -\infty$. For parameters α and δ we have $2\alpha > \delta + 1 > 1$.

Strong coupling...or not?

After the analysis of all Λ_i (they just give us different values of a and b due to their structure and different x) we again can extract the strongest condition or constraint for our parameters α and β from initial lagrangian:

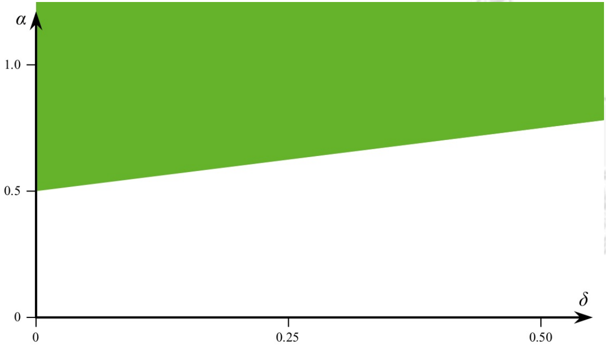
The final one...

$$\alpha < 1 - \frac{3}{2}\delta.$$

Strong coupling...or not?

Kobayashi'17

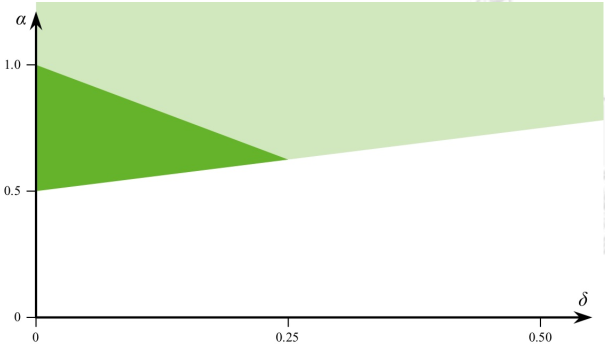
$$\alpha > \frac{1}{2} + \frac{1}{2}\delta.$$



Strong coupling...or not?

Y.A., O. Evseev, O. Melichev'18

No strong coupling if $\alpha < 1 - \frac{3}{2}\delta$.



Conclusion

- It was shown that due to NEC-violation one can build new early stage as an alternative or completion to inflation, e.g. genesis stage.
- If one violate NEC and use theories with higher order derivatives $\rightarrow$ be sure that all no-go theorems are avoided and solutions are stable for all times!
- Now one should also test his/her theory with "strong coupling" cause we still have no quantum gravity theory today.

Outlook

- Tensor sector $\rightarrow$ test!
- Try to sew our Genesis stage with next stages in a healthy way (add some matter).

THANK YOU FOR YOUR ATTENTION!