Dispersion relations and dynamic characteristics of bound states in the model of Dirac field interacting with a material plane

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The proposed by Symanzik approach for modeling of interaction of a macroscopic material body with quantum fields is considered. Its application in quantum electrodynamics enables one to establish the most general form of the action functional describing the interaction of 2-dimensional material surface with photon and fermion fields. The models making it possible to calculate the Casimir energy and Casimir-Polder potential for non-ideal conducting material are presented. Applications of the models to descriptions of of interaction of the spinor field with a material plane are considered.

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Formulation of model

The proposed by Symanzik action functional describing the interaction of the quantum field with material body has the form:

$$S(\varphi) = S_V(\varphi) + S_{def}(\varphi)$$

where

$$S_V(\varphi) = \int L(\varphi(x))d^D x, \ S_{def}(\varphi) = \int_\Gamma L_{def}(\varphi(x))d^{D'} x,$$

and Γ is a subspace of dimension $D' \leq D$ in D-dimensional space.

Formulation of model

From the basic principles of QED (gauge invariance, locality, renormalizability) it follows that for thin film without charges and currents, which shape is defined by equation $\Phi(x) = 0$, $x = (x_0, x_1, x_2, x_3)$, the action describing its interaction with photon field $A_\mu(x)$ and Dirac Fields $\bar{\psi}(x), \psi(x)$ reads

$$S_{def}(\varphi) = S_\Phi(A) + S_\Phi(\bar{\psi}, \psi).$$

The action $S_\Phi(A)$ is a surface Chern-Simon action

$$S_\Phi(A) = \frac{a}{2} \int \varepsilon^{\lambda\mu\nu\rho} \partial_\lambda \Phi(x) A_\mu(x) F_{\nu\rho}(x) \delta(\Phi(x)) dx$$

where $F_{\nu\rho}(x) = \partial_\nu A_\rho - \partial_\rho A_\nu$, $\varepsilon^{\lambda\mu\nu\rho}$ denotes totally antisymmetric tensor ($\varepsilon^{0123} = 1$), a is a constant dimensionless parameter.

Formulation of model

The fermion defect action can be written as

$$S_\Phi(\bar\psi, \psi) = \int \bar\psi(x)[\lambda + u^\mu\gamma_\mu + \gamma_5(\tau + v^\mu\gamma_\mu) + \omega^{\mu\nu}\sigma_{\mu\nu}]\psi(x)\delta(\Phi(x))dx$$

Here, γ_μ, $\mu = 0, 1, 2, 3$, are the Dirac matrices, $\gamma_5 = i\gamma_0\gamma_1\gamma_3\gamma_3$, $\sigma_{\mu\nu} = i(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$, and λ, τ, u_μ, v_μ, $\omega^{\mu\nu} = -\omega^{\nu\mu}$, $\mu, \nu = 0, 1, 2, 3$ are 16 dimensionless parameters.

It is the most general form of gauge invariant action concentrated on the defect surface being invariant in respect to reparametrization of one and not having any parameters with negative dimensions.

Formulation of model

The full action of the model, which satisfies the requirement of locality, gauge invariance and renormalizability, has the form

$$S(\bar{\psi}, \psi, A) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\hat{\partial} - m + e\hat{A})\psi$$
$$+ S_{def}(A) + S_{def}(\bar{\psi}, \psi).$$

Due to the requirements of renormalizability the fields interaction is described by standard contribution $e\bar{\psi}\hat{A}\psi$ to the QED action.

$$D_{2P}^{lm}(\vec{k}, x_3, y_3) = \frac{P^{lm}(\vec{k})}{2|\vec{k}|} \left\{ \frac{A}{C} - e^{i|\vec{k}||x_3 - y_3|} \right\} - \frac{L^{lm}(\vec{k})B}{2C|\vec{k}|^2}$$

with $D_{2P}^{l3} = D_{2P}^{3m} = 0$, $D_{2P}^{33}(\vec{k}, x_3, y_3) = -i\delta(x_3 - y_3)/|\vec{k}|^2$,

$$C = (1 + a_1 a_2(e^{2i|\vec{k}|r} - 1))^2 + (a_1 + a_2)^2,$$

$$A = (a_1 a_2 - a_1^2 a_2^2(1 - e^{2i|\vec{k}|r}))(e^{i|\vec{k}|(|x_3| + |y_3 - r|)} + e^{i|\vec{k}|(|x_3 - r| + |y_3|)})e^{i|\vec{k}|r} +$$
$$+ (a_1^2 + a_1^2 a_2^2(1 - e^{2i|\vec{k}|r}))e^{i|\vec{k}|(|x_3| + |y_3|)} +$$
$$+ (a_2^2 + a_1^2 a_2^2(1 - e^{2i|\vec{k}|r}))e^{i|\vec{k}|(|x_3 - r| + |y_3 - r|)},$$

$$B = a_1 a_2(a_1 + a_2)\left(e^{i|\vec{k}|(|x_3| + |y_3 - r|)} + e^{i|\vec{k}|(|x_3 - r| + |y_3|)}\right)e^{i|\vec{k}|r} -$$
$$- a_1(1 + a_2(a_2 + a_1 e^{2i|\vec{k}|r}))e^{i|\vec{k}|(|x_3| + |y_3|)} -$$
$$- a_2(1 + a_1(a_1 + a_2 e^{2i|\vec{k}|r})e^{i|\vec{k}|(|x_3 - r| + |y_3 - r|)}.$$

Casimir energy for two parallel planes

The energy density E_{2P} of defect is defined as

$$\ln G(0) = \frac{1}{2} \operatorname{Tr} \ln(D_{2P} D^{-1}) = -iTSE_{2P}$$

where $T = \int dx_0$ is time of existence of defect, and $S = \int dx_1 dx_2$, is area of one. It is expressed in an explicit form in terms of polylogarithm function $\mathrm{Li}_4(x)$ in the following way:

$$E_{2P} = \sum_{j=1}^{2} E_j + E_{Cas}, \ E_j = \frac{1}{2} \int \ln(1 + a_j^2) \frac{d\vec{k}}{(2\pi)^3}, \ j = 1, 2,$$

$$E_{Cas} = -\frac{1}{16\pi^2 r^3} \sum_{k=1}^{2} \mathrm{Li}_4 \left(\frac{a_1 a_2}{a_1 a_2 + i(-1)^k(a_1 + a_2) - 1} \right)$$
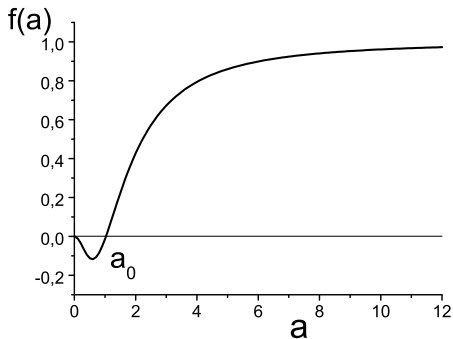
Here E_j is an infinite constant, which can be interpreted as self-energy density on the j-th planes, and E_{Cas} is an energy density of theirs interaction. Function $\text{Li}_4(x)$ is defined as

$$\text{Li}_4(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^4} = -\frac{1}{2} \int_0^{\infty} k^2 \ln(1 - xe^{-k}) dk.$$

For identical defect planes ($a_1 = a_2 = a$) the force $F_{2P}(\mathsf{r}, a)$ between them is given by

$$F_{2P}(\mathsf{r}, a) = -\frac{\partial E_{Cas}(\mathsf{r}, a)}{\partial \mathsf{r}} = -\frac{\pi^2}{240 \mathsf{r}^4} f(a).$$

Function $f(a)$ determining the Casimir forces between two parallel planes. It is even ($f(a)=f(-a)$), has the minimum $f(a_m) = -0,11723$ at $a_m = 0,5892$, and $f(a_0) = 0$ at $a_0 = 1,03246$.

Statement of problem for Dirac field

We will consider the material plane $x_3 = 0$ as a defect. In this case, in the Dirac part of the action

$$S(\overline{\psi}, \psi) = \int \overline{\psi}(x)(i\hat{\partial} - m + \Omega(x_3))\psi(x)dx,$$

the interaction of the spinor field with the plane is described with matrix $\Omega(x_3) = Q\delta(x_3)$. Since $\Omega(x_3)$ and $\delta(x_3)$ have the dimension of mass, the matrix Q is dimensionless. For homogeneous isotropic material plane in more general case, the matrix Q could be presented in the form:

$$Q = r_1 I + ir_2\gamma_5 + r_3\gamma_3 + r_4\gamma_5\gamma_3 +$$
$$+ r_5\gamma_0 + r_6\gamma_5\gamma_0 + ir_7\gamma_0\gamma_3 + ir_8\gamma_1\gamma_2$$

with I - identity 4x4 matrix, γ_3, $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ are Dirac matrices.

Modified Dirac equations

Movement of spinor particle in the field of defect $\Omega(x_3)$ is described by a modified Dirac equation

$$(i\hat{\partial} - m + \Omega(x_3))\psi(x) = 0.$$

It is one of the Euler-Lagrange equations, which is obtained by variational differentiating of the action over $\overline{\psi}(x)$. Taking the derivative over $\psi(x)$ we obtain the second equation

$$(\partial_\mu\overline{\psi}(x))\gamma^\mu + \overline{\psi}(x)(m - \Omega(x_3)) = 0.$$

The condition $\bar{\psi}(x) = \psi^*(x)\gamma_0$ fulfils if $\gamma_0\Omega^+(x) = \Omega(x)\gamma_0$. It is the case for real values of parameters r_j, $j = 1, ..., 8$.

Modified Dirac equations

Let us introduce the convenient notations. For 2×2 -matrix M with elements M_{ij}, $i, j = 1, 2$, we define the 4×4 -matrices $M^{(+)}$, $M^{(-)}$ in the following way

$$
M^{(+)} = \begin{pmatrix} M_{11} & 0 & M_{12} & 0 \\ 0 & 0 & 0 & 0 \\ M_{21} & 0 & M_{22} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad M^{(-)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & M_{11} & 0 & M_{12} \\ 0 & 0 & 0 & 0 \\ 0 & M_{21} & 0 & M_{22} \end{pmatrix}.
$$

Let $\mathcal{M}^{(\pm)}$ is the set of all the matrices $M^{(\pm)}$, then for arbitrary matrices $M_1^{(\pm)}$, $M_2^{(\mp)}$,

$$
M_1^{(\pm)} M_2^{(\mp)} = 0, \ M_1^{(\pm)} M_2^{(\pm)} \in \mathcal{M}^{(\pm)}.
$$

If we denote τ_0 the unit 2×2 - matrix, τ_j, $j = 1, 2, 3$, the Pauli matrices, and $\tau_j^{(+)}$, $\tau_j^{(-)}$ the corresponding 4×4 -matrices, then the matrix Q can be presented as

$$Q = Q^{(+)} + Q^{(-)}, \ Q^{(\pm)} = \sum_{j=0}^{3} q_j^{(\pm)} \tau_j^{(\pm)}$$

where $q_0^{(\pm)} = r_{18}^{\pm}$, $q_1^{(\pm)} = i \, r_{27}^{\pm}$, $q_2^{(\pm)} = \mp \, i \, r_{36}^{\mp}$, $q_3^{(\pm)} = \mp \, r_{45}^{\mp}$, and $r_{ij}^{\pm} = r_i \pm r_j$.

Modified Dirac equations

We denote $\psi(x)$ the solution of the modified Dirac equation, and $\psi_-(x) = \psi(x)$ for $x_3 < 0$, $\psi_+(x) = \psi(x)$ for $x_3 > 0$. The spinors $\psi(x)_\pm$ for $x_3 \neq 0$ satisfy the free Dirac equation and boundary condition

$$\lim_{x^3 \to +0} \psi_+(x) = S \lim_{x^3 \to -0} \psi_-(x),$$

One can choose the regularization procedure for $\delta(x_3)$ in such a way that the matrix S is expressed in terms of Q as

$$S = \exp\{-i\gamma^3 Q\}.$$

It follows from $Q^\dagger = \gamma_0 Q \gamma_0$ that

$$S^\dagger \gamma^0 \gamma^3 S = \gamma^0 \gamma^3$$

where $S^\dagger$ is the hermitian conjugated matrix S and

$$S = S^{(+)} + S^{(-)},$$
$$S^{(\pm)} = e^{i\chi_\pm} \left(\varsigma_{0\pm} \tau_0^{(\pm)} + i\varsigma_{1\pm} \tau_1^{(\pm)} + \varsigma_{2\pm} \tau_2^{(\pm)} + \varsigma_{3\pm} \tau_3^{(\pm)} \right),$$
$$\varsigma_{0\pm}^2 + \varsigma_{1\pm}^2 - \varsigma_{2\pm}^2 - \varsigma_{3\pm}^2 = 1$$

where $\chi_\pm$ and $\varsigma_{j\pm}$, $j = 0, 1, 2, 3$ are real parameters.

Free Dirac equations

The free Dirac equation in coordinate space reads

$$(i\hat{\partial} - m)\psi(x) = 0.$$

By substitution $\psi(x)$ in the form

$$\psi(x) = \frac{1}{(2\pi)^4} \int e^{-ipx}\psi(\bar{p})d\bar{p}, \ \bar{p} = (p^0, p^1, p^2)$$

one obtains

$$(\hat{p} - m)\psi(\bar{p}) = 0.$$

For real p_3 the considered spinor $\psi(x)$ describes the scattering state and the imaginary p_3 - the bound state.

Free Dirac equations

The general solution $\psi(\bar{p})$ of the Dirac equation can be presented as an arbitrary linear combination of linear independent spinors

$$\psi_1(\bar{p}) = \left\{ \begin{array}{c} 1 \\ 0 \\ \frac{-p_3}{m+p_0} \\ \frac{-p_1-ip_2}{m+p_0} \end{array} \right\}, \ \psi_2(\bar{p}) = \left\{ \begin{array}{c} 0 \\ 1 \\ \frac{-p_1+ip_2}{m+p_0} \\ \frac{p_3}{m+p_0} \end{array} \right\}, \ p_3 = \pm\sqrt{\bar{p}^2 - m^2}$$

for $p_0 > 0$ and

$$\psi_1'(\bar{p}) = \left\{ \begin{array}{c} \frac{p_1-ip_2}{m-p_0} \\ \frac{p_3}{p_0-m} \\ 0 \\ 1 \end{array} \right\}, \psi_2'(\bar{p}) = \left\{ \begin{array}{c} \frac{p_3}{m-p_0} \\ \frac{p_1+ip_2}{m-p_0} \\ 1 \\ 0 \end{array} \right\}.$$

for $p_0 < 0$.

Modified Dirac equations

Substituting $p_3 \to \pm i\kappa$ with $\kappa = |\kappa| = \sqrt{m^2 + p_1^2 + p_2^2 - p_0^2}$ we obtain the spinors describing the bound states

$$\psi_\pm(\bar{p}) = \psi(p)|_{p_3 \to \mp i\kappa}.$$

They can be presented as follows

$$\psi_+(\bar{p}) = a_1\psi_{1+}(\bar{p}) + a_2 e^{i\phi}\psi_{1+}(\bar{p}), \ \psi_-(\bar{p}) = d_1\psi_{1-}(\bar{p}) + d_2 e^{i\phi}\psi_{1-}(\bar{p}),$$
$$\psi'_+(\bar{p}) = a'_1\psi'_{1+}(\bar{p}) + a'_2 e^{i\phi}\psi'_{1+}(\bar{p}), \ \psi'_-(\bar{p}) = d'_1\psi'_{1-}(\bar{p}) + d'_2 e^{i\phi}\psi'_{1-}(\bar{p}).$$

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Modified Dirac equations

Here

$$\psi_{1\pm}(\bar{p}) = \left\{ \begin{array}{c} 1 \\ 0 \\ \pm ik \\ fe^{i\phi} \end{array} \right\}, \quad \psi_{\bar{2}}(\bar{p}) = \left\{ \begin{array}{c} 0 \\ 1 \\ fe^{-i\phi} \\ \mp ik \end{array} \right\},$$

$$\psi'_{1\pm}(\bar{p}) = \left\{ \begin{array}{c} -fe^{-i\phi} \\ \pm ik \\ 0 \\ 1 \end{array} \right\}, \quad \psi'_{2\pm}(\bar{p}) = \left\{ \begin{array}{c} \mp ik \\ -fe^{i\phi} \\ 1 \\ 0 \end{array} \right\},$$

$$k = \frac{\kappa}{m + |p_0|}, \; \frac{p^1 + ip^2}{m + |p_0|} = -\frac{p_1 + ip_2}{m + |p_0|} = fe^{i\phi}, \; f = |f|.$$

The spinors $\psi_\pm, \psi'_\pm$ fulfill the relations

$$\psi_+(\bar{p}) = S\psi_-(\bar{p}), \; \psi'_+(\bar{p}) = S\psi'_-(\bar{p}), \; S = e^{-i\gamma^3 Q}.$$

which can be presented as systems of linear equations for coefficients $a_1, a_2, d_1, d_2, a'_1, a'_2, d'_1, d'_2$ contained in $\psi_\pm, \psi'_\pm$.

Properties of bound states

We consider the quantities of the form

$$\bar\psi_\pm(\bar p, x_3)\Gamma\,\psi_\pm(\bar p, x_3) = \bar\psi_\pm(\bar p)\Gamma\psi_\pm(\bar p)e^{-2\kappa|x_3|}$$

with 4×4 basic Dirac matrices Γ using convenient notations

$$N_{0+} = a_1^* a_1 + a_2^* a_2,\ \ N_{1+} = a_1^* a_2 + a_2^* a_1,$$
$$N_{2+} = a_1^* a_1 - a_2^* a_2,\ \ N_{3+} = i(a_1^* a_2 - a_2^* a_1),$$
$$N_{0-} = d_1^* d_1 + d_2^* d_2,\ \ N_{1-} = d_1^* d_2 + d_2^* d_1,$$
$$N_{2-} = d_1^* d_1 - d_2^* d_2,\ \ N_{3-} = i(d_1^* d_2 - d_2^* d_1),$$
$$\vec\gamma = \{\gamma^1, \gamma^2, \gamma^3\},\ \ \vec\sigma = \{\sigma^1, \sigma^2, \sigma^3\},\ \ \sigma^j = \sum_{k,l=1}^{3}\varepsilon^{jkl}\gamma^k\gamma^l,\ j = 1, 2, 3$$

with totally antisymmetric ε^{jkl}, $\varepsilon^{123} = 1$.

For scalar and pseudoscalar invariant densities

$$\bar{\psi}_{\pm}(x)\psi_{\pm}(x) = e^{-2\kappa|x^3|}d_{\pm}(\bar{p}), \ \ \bar{\psi}_{\pm}(x)\gamma_5\psi_{\pm}(x) = ie^{-2\kappa|x^3|}d_{5\pm}(\bar{p}),$$

components of electric and axial 4-currents

$$\bar{\psi}(x)\gamma^{\mu}\psi(x) = e^{-2\kappa|x^3|}j_{\pm}^{\mu}(\bar{p}), \ \ \bar{\psi}(x)\gamma^{\mu}\gamma_5\psi(x) = e^{-2\kappa|x^3|}j_{5\pm}^{\mu}(\bar{p}),$$

for anomalous electric and magnetic dipole moments

$$\bar{\psi}_{\pm}(x)\vec{\gamma}\gamma^0\psi_{\pm}(x) = ie^{-2\kappa|x^3|}\vec{e}_{\pm}(\bar{p}), \ \ \bar{\psi}_{\pm}(x)\vec{\sigma}\psi_{\pm}(x) = ie^{-2\kappa|x^3|}\vec{m}_{\pm}(\bar{p}),$$

we obtain the following results

Properties of bound states

$$d_{\pm} = N_{0\pm}\left(1 - f^2 - k^2\right) \pm 2N_{3\pm}fk, \ d_{5\pm} = \pm 2N_{2\pm},$$

$$j_{\pm}^0 = N_{0\pm}\left(1 + f^2 + k^2\right) \mp 2N_{3\pm}fk, \ j_{5\pm}^0 = 2N_{1\pm}f,$$

$$j_{\pm}^1 = j_{\pm}^{\parallel}\cos(\phi) + j_{\pm}^{\perp}\sin(\phi), \ j_{5\pm}^1 = j_{5\pm}^{\parallel}\cos(\phi) - j_{5\pm}^{\perp}\sin(\phi),$$

$$j_{\pm}^2 = j_{\pm}^{\parallel}\sin(\phi) - j_{\pm}^{\perp}\cos(\phi), \ j_{5\pm}^2 = j_{5\pm}^{\parallel}\sin(\phi) + j_{5\pm}^{\perp}\cos(\phi),$$

$$j_{\pm}^3 = 0, \ j_{5\pm}^3 = N_{2\pm}\left(1 - f^2 + k^2\right)$$

$$j_{\pm}^{\parallel} = 2(N_{0\pm}f \mp N_{3\pm}k), \ j_{\pm}^{\perp} = \pm 2N_{1\pm}k,$$

$$j_{5\pm}^{\parallel} = N_{1\pm}\left(1 + f^2 - k^2\right), \ j_{5\pm}^{\perp} = N_{3\pm}\left(f^2 + k^2 - 1\right) \mp 2N_{0\pm}fk,$$

$$e_\pm^1 = e_\pm^\perp \sin(\phi), \ \ e_\pm^2 = -e_\pm^\perp \cos(\phi), \ \ e^3 = -2(N_{3\pm} f \mp N_{0\pm} k),$$
$$m_\pm^1 = m_\pm^\parallel \cos(\phi) - m_\pm^\perp \sin(\phi), \ \ m_\pm^2 = m_\pm^\parallel \sin(\phi) + m_\pm^\perp \cos(\phi),$$
$$m_\pm^3 = N_{2\pm}\left(k^2 - f^2 - 1\right), \ \ e^\perp = 2f N_{2\pm},$$
$$m_\pm^\parallel = iN_{1\pm}\left(f^2 - k^2 - 1\right), \ \ m_\pm^\perp = \left(N_{3\pm}\left(1 + f^2 + k^2\right) \mp 2N_{0\pm} fk\right).$$

We see also that

$$j^0_\pm p^0 - j^1_\pm p^1 - j^2_\pm p^2 = md_\pm, \ j^0_{5\pm} p^0 - j^1_{5\pm} p^1 - j^2_{5\pm} p^2 = 0,$$
$$\kappa j^3_{5\pm} = md_{5\pm} = 0, \ m \, \vec{e}_\pm \vec{m}_\pm = -\kappa j^3_{5\pm} d_\pm, \ \vec{e}_\pm \vec{m}_\pm = d_\pm d_{5\pm},$$
$$\vec{e}^2_\pm - \vec{m}^2_\pm = d^2_{5\pm} - d^2_\pm, \ d_\pm p^0 - j^0_\pm m = \kappa e^3_\pm$$

$$N_{0\pm} = \frac{(d_\pm + j^0_\pm)}{2}, \ N_{1\pm} = \frac{j^0_{5\pm}}{2f}, \ N_{2\pm} = \frac{j^3_{5\pm}(m + p^0)}{2m},$$
$$N_{3\pm} = \frac{d_\pm(1 + f^2 + k^2) + j^0_\pm(f^2 + k^2 - 1)}{4fk}.$$

Let us denote

$$a = \left\{ \begin{array}{c} a_1 \\ a_2 \end{array} \right\}, d = \left\{ \begin{array}{c} d_1 \\ d_2 \end{array} \right\},$$

and

$$K_+ = \frac{1}{f} \left(\begin{array}{cc} f & 0 \\ -ik & 1 \end{array} \right), \ K_- = \frac{1}{f} \left(\begin{array}{cc} ik & 1 \\ f & 0 \end{array} \right),$$

$$L_+ = \left(\begin{array}{cc} 1 & 0 \\ -ik & f \end{array} \right), \ L_- = \left(\begin{array}{cc} 0 & 1 \\ f & ik \end{array} \right),$$

$$s_\pm = \left(\begin{array}{cc} \varsigma_{0\pm} + \varsigma_{3\pm} & i(\varsigma_{1\pm} - \varsigma_{2\pm}) \\ i(\varsigma_{1\pm} + \varsigma_{2\pm}) & \varsigma_{0\pm} - \varsigma_{3\pm} \end{array} \right)$$

Equations for amplitudes

It follows from $\psi_+ = S\psi_-$, $S^{(\pm)} = e^{i\chi_\pm} s_\pm^{(\pm)}$ that the vectors a, d fulfill the equations

$$a = e^{i\chi_+} T_+ d, \ \ a = e^{i\chi_-} T_- d, \ \ T_\pm = K_\pm s_\pm L_\pm.$$

Hence, d satisfies the homogeneous equation

$$Td = 0,$$

where

$$T = e^{i\frac{\chi_+ + \chi_-}{2}} \left(e^{i\chi} T_+ - e^{-i\chi} T_- \right), \ \chi = \frac{\chi_+ - \chi_-}{2}$$

which has nontrivial solution, if the determinant of the matrix T is equal to zero.

Equations for amplitudes

The matrix T can be presented in the form

$$T = \frac{e^{i(\chi_+ + \chi_-)/2}}{f} \left(\begin{array}{cc} t_{11} & t_{12} \\ t_{21} & t_{22} \end{array} \right),$$

with

$$t_{11} = \left(e^{i\chi}(k(\varsigma_{1+} - \varsigma_{2+}) + \varsigma_{0+} + \varsigma_{3+}) + e^{-i\chi}(k(\varsigma_{1-} - \varsigma_{2-}) - \varsigma_{0-} + \varsigma_{3-}) \right)$$

$$t_{12} = i \left(e^{i\chi} f^2 k(\varsigma_{1+} - \varsigma_{2+}) + e^{-i\chi}(k^2(\varsigma_{1-} - \varsigma_{2-}) - 2k\varsigma_{0-} - \varsigma_{1-} - \varsigma_{2-}) \right.$$

$$t_{21} = -i \left(e^{i\chi}(k^2(\varsigma_{1+} - \varsigma_{2+}) + 2k\varsigma_{0+} - \varsigma_{1+} - \varsigma_{2+}) + e^{-i\chi} f^2 k(\varsigma_{1-} - \varsigma_{2-}) \right.$$

$$t_{22} = \left(e^{i\chi}(k(\varsigma_{1+} - \varsigma_{2+}) + \varsigma_{0+} - \varsigma_{3+}) + e^{-i\chi}(k(\varsigma_{1-} - \varsigma_{2-}) - \varsigma_{0-} - \varsigma_{3-}) \right.$$

Equations for amplitudes

Thus, $d \neq 0$ is the solution of equation $Td = 0$ if

$$t_{11}t_{22} - t_{12}t_{21} =$$
$$= 2f^2(\cos(\chi_+ - \chi_-) - \varsigma_{0-}\varsigma_{0+} + \varsigma_{1-}\varsigma_{1+} - \varsigma_{2-}\varsigma_{2+} - \varsigma_{3-}\varsigma_{3+}) -$$
$$-((k^2 - f^2)(\varsigma_{1-} - \varsigma_{2-}) - 2k\varsigma_{0-} - \varsigma_{1-} - \varsigma_{2-}) \times$$
$$\times((k^2 - f^2)(\varsigma_{1+} - \varsigma_{2+}) + 2k\varsigma_{0+} - \varsigma_{1+} - \varsigma_{2+}) = 0.$$

The general solution of equations for a, d can be written in the form

$$d = c \left\{ \begin{array}{c} t_{12} \\ -t_{11} \end{array} \right\}, \ a = c' \left\{ \begin{array}{c} t'_{12} \\ -t'_{11} \end{array} \right\}.$$

Here, c is an arbitrary constant and

$$t'_{12} = t_{12}|_{\chi_\pm \to -\chi_\pm, \ k \to -k, \ \varsigma_{0\pm} \to -\varsigma_{0\pm}},$$
$$t'_{11} = t_{11}|_{\chi_\pm \to -\chi_\pm, \ k \to -k, \ \varsigma_{0\pm} \to -\varsigma_{0\pm}},$$

$$c' = c \, f \frac{e^{2i(\chi_+ + \chi_-)}(h_{11}g_{12} - g_{11}h_{12})}{h_{12}e^{i\chi_-} - g_{12}e^{i\chi_+}},$$

$$h_{11} = k(\varsigma_{1+} - \varsigma_{2+}) + \varsigma_{0+} + \varsigma_{3+}, \ g_{11} = k(\varsigma_{1-} - \varsigma_{2-}) - \varsigma_{0-} + \varsigma_{3-},$$
$$h_{12} = f^2 k(\varsigma_{1+} - \varsigma_{2+}), \ g_{12} = k^2(\varsigma_{1-} - \varsigma_{2-}) - 2k\varsigma_{0-} - \varsigma_{1-} - \varsigma_{2-}.$$

Dispersion relation

The solvability condition (dispersion relation) can be presented as

$$(p_1^2 + p_2^2)(\cos(\chi_+ - \chi_-) - \varsigma_{0-}\varsigma_{0+} + \varsigma_{1-}\varsigma_{1+} - \varsigma_{2-}\varsigma_{2+} - \varsigma_{3-}\varsigma_{3+}) - $$
$$-2(p_0\,\varsigma_{1-} + m\,\varsigma_{2-} + \kappa\,\varsigma_{0-})(p_0\,\varsigma_{1+} + m\,\varsigma_{2+} - \kappa\,\varsigma_{0+}) = 0.$$

In virtue of $p_0^2 + \kappa^2 - p_1^2 - p_2^2 - m^2 = 0$,it follows from dispersion relation that p_0, κ, m satisfy the equation

$$(p_0^2 + \kappa^2 - m^2)(\cos(\chi_+ - \chi_-) - \varsigma_{0-}\varsigma_{0+} + \varsigma_{1-}\varsigma_{1+} - \varsigma_{2-}\varsigma_{2+} - \varsigma_{3-}\varsigma_{3+}) - $$
$$-2(p_0\,\varsigma_{1-} + m\,\varsigma_{2-} + \kappa\,\varsigma_{0-})(p_0\,\varsigma_{1+} + m\,\varsigma_{2+} - \kappa\,\varsigma_{0+}) = 0.$$

describing the relation between the dimensionless magnitudes $p_o/m, \kappa/m$ characterizing the bound state. Its solution is presented on the $(p_0 - \kappa)$-plane by hyperbola or by two straight lines.

Since for Dirac particle the physical value of p_0, κ are positive, the bound state can be realized if there are points of the $(p_0 - \kappa)$-plot presenting the dispersion relation in the region $p_0 > 0, \kappa > 0$. This part of plot can be connected or disconnected, and the possible values of p_0, κ for bound state can be both restricted and non-restricted from above.

Dispersion relation

By $\varsigma_{0\pm} = \varsigma_{2\pm} = 0$ the dispersion law has the form

$$p_0^2 - v_F^2(p_1^2 + p_2^2) = 0, \ \ v_F = \sqrt{\frac{\cos(\chi_+ - \chi_-) + \varsigma_{1-}\varsigma_{1+} - \varsigma_{3-}\varsigma_{3+}}{2(\varsigma_{1-}\varsigma_{1+})}},$$

$$\varsigma_{1\pm}^2 - \varsigma_{3\pm}^2 = 1,$$

and

$$0 \leq v_F \leq 1, \ \kappa^2 = m^2 + (p_1^2 + p_2^2)(1 - v_F^2).$$

It describe the propagation of massless particle in the defect plane with the Fermi-velocity v_F. The motion of such particles explains numerous effects in graphene.

The main results

- In the framework of the Symanzik approach, we build the model of QED field interaction with 2D material. The action of the model consist of the usual QED action and extra defect contribution. The action contains parameters, that characterize the material property.

- The characteristics of photons and Dirac particles scattering on the defect plane can be calculated in the model, also the properties of states localized near the defect plane can be investigated.

- The model and obtained on its basis results could be used for the theoretical description of the interaction of electrons, positrons and neutrons with two-dimensional materials (graphite, thin films, sputters, sharp boundaries of a solid body). Simple modifications of the model allows to take into account the effects of external electromagnetic fields.

Thank you for your attention!