Higher-derivative relativistic quantum gravity

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Creation of the fundamental quantum theory of gravitation remains one of the most important tasks, if not he most important task, of mofern theoretical physics.
In 1977 Stelle has proved renormalizability of the Lorentz invariant gravitational actions which include besides the Einstein-Hilbert term also terms with fourth derivatives of the metric, which we will call also quadratic quantum gravity.

But Stelle also made the statement that quantum gravity with fourth derivatves is unphysical because it violates either unitarity or causality. Since then this model is considered as having severe problems with physical interpretation.

Wye derive new expressions for the Lagrangian and for the graviton propagator of quadratic quantum gravity within dimensional regularization. We argue also that fourth derivative gravity is a good candidate for the fundamental quantun theory of gravitation.

Let us consider the invariant under the gauge transformations action with all possible terms quadratic in the curvature tensor

$$S_{sym} = \int d^D x \mu^{-2\epsilon} \sqrt{-g} \left(-M_{Pl}^2 R + \alpha R_{\mu\nu} R^{\mu\nu} + \beta R^2 + \delta R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \right), \tag{1}$$

where the first term is the Einstein-Hilbert action. Here $M_{Pl}^2 = 1/(16\pi G)$ is the squared Planck mass, $R_{\mu\nu\rho\sigma}$ is the Riemann tensor, $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar, α, β and δ are the dimensionless coupling constants of the Lagrangian, $D = 4 - 2\epsilon$ is the space time dimension within dimensional regularization, μ is the parameter of dimensional regularization. Usually the last term in the action (1) is missed in the literature because of the Gauss-Bonnet topological identity

$$\int d^4 x \sqrt{-g} \left(R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \right) = 0, \tag{2}$$

which is valid for space-times topologically equivalent to flat space only in four dimensions. Within dimensional regularization the term quadratic in the Riemann tensor should be added to the ation.

We work in the linearized theory around the flat space metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{3}$$

where we choose the convention $\eta_{\mu\nu} = diag(+1, -1, -1, -1)$ in four dimensions. In D dimensions $\eta_{\mu\nu}\eta^{\mu\nu} = D$. Further it is understood that indices are raised and lowered with the Minkowski metric $\eta_{\mu\nu}$.
Gauge transformation are generated by diffeomorphisms $x^\mu \to x^\mu + \zeta^\mu(x)$ and have the form

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \zeta_\nu + \partial_\nu \zeta_\mu + \left(h_{\lambda\mu}\partial_\nu + h_{\lambda\nu}\partial_\mu + (\partial_\lambda h_{\mu\nu})\right)\zeta^\lambda, \tag{4}$$

here $\zeta_\mu(x)$ are arbitrary functions.
According to Faddeev-Popov quantization one should add to the action the gauge fixing term which we choose in the form

$$S_{gf} = -\frac{1}{2\xi}\int d^D x F_\mu \partial_\nu \partial^\nu F^\mu, \tag{5}$$

where $F^\mu = \partial_\nu h^{\nu\mu}$, ξ is the gauge parameter.

One should also add the ghost term

$$S_{ghost} = \int d^D x d^D y \overline{C}_\mu(x) \frac{\delta F^\mu(x)}{\delta \zeta_\nu(y)} C_\nu(y) = \tag{6}$$

$$\int d^D x \partial^\nu \overline{C^\mu} \left[\partial_\nu C_\mu + \partial_\mu C_\nu + h_{\lambda\mu} \partial_\nu C^\lambda + h_{\lambda\nu} \partial_\mu C^\lambda + (\partial_\lambda h_{\mu\nu}) C^\lambda \right],$$

where $\overline{C}$ and C are ghost fields. Thus one gets the following generating functional for Green functions of gravitons

$$Z(J) = N \int dh_{\mu\nu} dC_\lambda d\overline{C_\rho} \exp \left[i \left(S_{sym} + S_{gf} + S_{ghost} + d^D x \mu^{-2\epsilon} J_{\mu\nu} h^{\mu\nu} \right) \right], \tag{7}$$

where as usual in the functional integral, N is the normalization factor and $J_{\mu\nu}$ is the source of the gravitational field.
We work within perturbation theory, so we make the shift of the fields

$$h_{\mu\nu} \to M_{Pl} \mu^{-\epsilon} h_{\mu\nu}. \tag{8}$$

To derive the graviton propagator we make the Fourier transfom to the momentum space and write the quadratic in $h_{\mu\nu}$ form

$$Q_{\mu\nu\rho\sigma} = \frac{1}{4}\int d^D k\ h^{\mu\nu}(-k)\Big[\left(k^2 + M_{Pl}^{-2}k^4(\alpha+4\delta)\right)P^{(2)}_{\mu\nu\rho\sigma} + k^2\left(-2+4M_{Pl}^{-2}k^2(\alpha+3\beta+\delta)\right)P^{(0-s)}_{\mu\nu\rho\sigma} + \frac{1}{\xi}M_{Pl}^{-2}k^4\left(P^{(1)}_{\mu\nu\rho\sigma} + 2P^{(0-w)}_{\mu\nu\rho\sigma}\right)\Big]h^{\rho\sigma}(k), \tag{9}$$

where $P^{(i)}_{\mu\nu\rho\sigma}$ are projectors to the spin-2, spin-1 and spin-0 components of the field correspondingly:

$$P^{(2)}_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\Theta_{\mu\rho}\Theta_{\nu\sigma} + \Theta_{\mu\sigma}\Theta_{\nu\rho}\right) - \frac{1}{3}\Theta_{\mu\nu}\Theta_{\rho\sigma}, \tag{10}$$

$$P^{(1)}_{\mu\nu\rho\sigma} = \frac{1}{2}\left(\Theta_{\mu\rho}\omega_{\nu\sigma} + \Theta_{\mu\sigma}\omega_{\nu\rho} + \Theta_{\nu\rho}\omega_{\mu\sigma} + \Theta_{\nu\sigma}\omega_{\mu\rho}\right), \tag{11}$$

$$P^{(0-s)}_{\mu\nu\rho\sigma} = \frac{1}{3}\Theta_{\mu\nu}\Theta_{\rho\sigma}, \qquad P^{(0-w)}_{\mu\nu\rho\sigma} = \omega_{\mu\nu}\omega_{\rho\sigma}. \tag{12}$$

Here $\Theta_{\mu\nu} = \eta_{\mu\nu} - k_\mu k_\nu/k^2$ and $\omega_{\mu\nu} = k_\mu k_\nu/k^2$

To obtain the graviton propagator $D_{\mu\nu\rho\sigma}$ one inverts the matrix in the square brackets of (9):

$$[Q]_{\mu\nu\kappa\lambda} D^{\kappa\lambda\rho\sigma} = \frac{1}{2}(\delta_\mu^\rho \delta_\nu^\sigma + \delta_\mu^\sigma \delta_\nu^\rho). \tag{13}$$

Thus we get for the propagator

$$D_{\mu\nu\rho\sigma} = \frac{1}{i(2\pi)^D} \left[\frac{4}{k^2} \left(\frac{1}{1 + M_{Pl}^{-2} k^2 (\alpha + 4\delta)} \right) P^{(2)}_{\mu\nu\rho\sigma} \right. \tag{14}$$

$$- \frac{2}{k^2} \left(\frac{1 + 2\epsilon \frac{1 - M_{Pl}^{-2} k^2 (\alpha + 4\beta)}{1 + M_{Pl}^{-2} k^2 (\alpha + 4\delta)}}{1 - \epsilon - M_{Pl}^{-2} k^2 \left((2\alpha + 6\beta + 2\delta) - \epsilon(\alpha + 4\beta) \right)} \right) P^{(0-s)}_{\mu\nu\rho\sigma}$$

$$\left. + 4\xi \frac{1}{M_{Pl}^{-2} k^4} \left(P^{(1)}_{\mu\nu\rho\sigma} + \frac{1}{2} P^{(0-w)}_{\mu\nu\rho\sigma} \right) \right].$$

Let us perform partial fractioning. Then the graviton propagatr takes the form

$$D_{\mu\nu\rho\sigma} = \frac{1}{i(2\pi)^D}\left[4P^{(2)}_{\mu\nu\rho\sigma}\left(\frac{1}{k^2} - \frac{1}{k^2 - M_{Pl}^2/(-\alpha - 4\delta)}\right)\right. \tag{15}$$

$$-2\frac{P^{(0-s)}_{\mu\nu\rho\sigma}}{1-\epsilon}\left(1 + 2\epsilon\frac{1 - M_{Pl}^{-2}k^2(\alpha + 4\beta)}{1 + M_{Pl}^{-2}k^2(\alpha + 4\delta)}\right)$$

$$\left(\frac{1}{k^2} - \frac{1}{k^2 - M_{Pl}^2(1-\epsilon)/(2\alpha + 6\beta + 2\delta - \epsilon(\alpha + 4\beta))}\right)$$

$$\left. + \frac{4\xi}{M_{Pl}^{-2}k^4}\left(P^{(1)}_{\mu\nu\rho\sigma} + \frac{1}{2}P^{(0-w)}_{\mu\nu\rho\sigma}\right)\right].$$

It is interesting to note that the position of one of the poles in the term with $P^{(0-s)}_{\mu\nu\rho\sigma}$ depends on the regularization parameter ϵ. The residues of both poles in this term also depend on ϵ. Thus it is clear that poles and residues of the tree level propagator do not have direct physical meaning.

In the limit of four dimensions we get for the graviton propagator

$$D_{\mu\nu\rho\sigma} = \frac{4}{i(2\pi)^D}\left[\frac{P^{(2)}_{\mu\nu\rho\sigma} - \frac{1}{2}P^{(0-s)}_{\mu\nu\rho\sigma}}{k^2} - \frac{P^{(2)}_{\mu\nu\rho\sigma}}{k^2 - M^2_{Pl}/(-\alpha - 4\delta)}\right. \tag{16}$$

$$\left. + \left(\frac{1}{2}\right)\frac{P^{(0-s)}_{\mu\nu\rho\sigma}}{k^2 - M^2_{Pl}/(2\alpha + 6\beta + 2\delta)} + \frac{\xi}{M^{-2}_{Pl}k^4}\left(P^{(1)}_{\mu\nu\rho\sigma} + \frac{1}{2}P^{(0-w)}_{\mu\nu\rho\sigma}\right)\right],$$

Within classical four-derivative gravity for a point particle with the energy-momentum tensor $T_{\mu\nu} = \delta^0_\mu\delta^0_\nu M\delta^3(x)$ the gravitational field is

$$V(r) = \frac{M}{2\pi M^2_{Pl}}\left(-\frac{1}{4r} + \frac{e^{-m_2 r}}{3r} - \frac{e^{-m_0 r}}{12r}\right), \tag{17}$$

where in our notations $m_2^2 = M^2_{Pl}/(-\alpha - 4\delta)$ and $m_0^2 = M^2_{Pl}/(2\alpha + 6\beta + 2\delta)$ are the squared masses of the massive spin-2 and spin-0 gravitons. The propagator (16) reproduces eq. (17) after the calculation of the corresponding tree level Feynman diagram describing interaction of two point-like particles.

The second term in the graviton propagator (16) has the unusial minus sign and that is why it is interpreted as the massive spin-2 ghost. To preserve renormalizability of the quantum theory one should shift all poles in propagators in Feynman integrals in the same manner $k^2 \to k^2 + i0$, hence the ghost state should be considered as the state with the negative metric. This was the reason to make the statement about violation either unitarity or causality in the model.

But this massive spin-2 state is unstable since it can decay into massless gravitons. Thus it does not appear as the asymptotic state of the S-matix. Correspondingly only particles with the positive metric participate in the scattering processes as external particles and unitarity is preserved in the theory.

It should be also mentioned that the tree level propagator will be essentially modified after the summation of one-loop corrections. Because of the mignus sign in the second term of (16) the one-loop correction due to the diagram with the massless graviton in the loop will shift the pole of the ghost from the real value $k^2 = M_{Pl}^2/(-\alpha - 4\delta)$ to the complex value $k^2 = M_{Pl}^2/(-\alpha - 4\delta) - i\Gamma$, where Γ is the decay width of the massive spin-2 graviton into the pare of massless gravitons. The complex pole is located on the unphysical Riemann sheet. This is analogous to the known virtual level in the neutron-proton system with antiparallel spins.

We conclude that the considered quadratic quantum gravity is a good candidate for fundamental quantum theory of gravitaion.

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