Baryogenesis in the $\nu$MSM: recent developments

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QUARKS-2018, Valday
June 1, 2018
Outlook

- Baryogenesis with GeV-scale right-handed neutrinos
- Freeze-out of the baryon number in low scale leptogenesis models
- A new study of the parameter space
Leptogenesis with light right-handed neutrinos

Neutrino flavour oscillations are impossible in the minimal SM.

**Extension with right-handed neutrinos**

\[
\mathcal{L} = \mathcal{L}_{SM} + \bar{N}_I \gamma^\mu \partial_\mu N_I - F_{\alpha I} \bar{L}_\alpha \tilde{\Phi} N_I - \frac{M_{IJ}}{2} \bar{N}_I^c N_J + h.c.
\]

- **Leptogenesis by** [M. Fukugita and T. Yanagida, 1986]
  Out of equilibrium decays of very heavy RH neutrino. Mass scale $\sim 10^{15}$ GeV. Might be lowered in resonant leptogenesis. The asymmetry in lepton sector is communicated to the baryon sector by sphaleron processes.

- **Baryogenesis via neutrino oscillations.** RH neutrinos with masses below the EW scale. Sphaleron freeze-out is important. [E. Akhmedov, V. Rubakov, A. Smirnov, 1998] Testable!
It was shown that two almost degenerate in mass RH neutrinos are enough for successful baryogenesis.

*T. Asaka and M. Shaposhnikov, 2005*

The third RH neutrino can be a DM candidate.

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**νMSM**

νMSM — an extension of the SM with three RH neutrinos. Active neutrino masses, DM and BAU can be addressed simultaneously.

*Canetti, Drewes, Frossard, Shaposhnikov, 2013*

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Testable!
Baryogenesis in the nuMSM

\[ \mathcal{L} = \mathcal{L}_{SM} + \bar{N}_I \gamma^\mu \partial_\mu N_I - F_{\alpha I} \bar{L}_\alpha \Phi N_I - \frac{M_{IJ}}{2} \bar{N}_I^c N_J + h.c. \]

A fermion number for HNLs coincides with helicity.

naive see-saw \( F \sim 10^{-7} - 10^{-8} \)

### Sakharov conditions

- Violation of Standard Model lepton number.
  Majorana mass violates the total lepton number, but this is suppressed as \( M_{NJ}^2 / T^2 \). scattering processes \( L_\alpha \rightarrow N_I \) through the Yukawa interactions violate SM lepton number

- \( CP \) violation. Phases in \( F_{\alpha I} \).

- Deviation from equilibrium. \( N_I \) are out of equilibrium at temperatures when sphalerons are active.
Baryogenesis in the nuMSM

Y_{\Delta L_1} = 0
Y_{\Delta L_2}, Y_{\Delta L_3} = 0
\sum_{\alpha} Y_{\Delta L_\alpha} = 0

Y_{\Delta L_1} > 0
Y_{\Delta L_2}, Y_{\Delta L_3} < 0
\sum_{\alpha} Y_{\Delta L_\alpha} = 0

Y_{\Delta L_1} > 0
Y_{\Delta L_2}, Y_{\Delta L_3} < 0
\sum_{\alpha} Y_{\Delta L_\alpha} \neq 0

The figure by Shuve and Yavin, 2014
Baryogenesis in the nuMSM

\[-F_{\alpha I} \bar{L}_\alpha \tilde{\Phi} N_I - \frac{M_{IJ}}{2} \tilde{N}_I^c N_J + h.c.\]

Experimentally observed values of active neutrino mass differences and mixings should be reproduced

\[F = \frac{i}{\nu_0} U^{PMNS} m_\nu^{1/2} \Omega m_N^{1/2},\]

\[\Omega = \begin{pmatrix} 0 & 0 \\ \cos \omega & \sin \omega \\ -\xi \sin \omega & \xi \cos \omega \end{pmatrix} \text{ for NH}\]

Important parameter

\[X_\omega = \exp(\text{Im} \omega). \quad (1)\]

Casas and Ibarra, 2001
However, several effects have to be accounted for.
In leptogenesis baryon asymmetry is reprocessed from lepton asymmetry by electroweak sphalerons

\[ B = \chi(T)(L - B) \]

The equilibrium formula for baryon asymmetry is valid as long as the sphaleron rate exceeds that of the lepton asymmetry production during all stages of BAU generation.

This is the case for high scale leptogenesis


But this is definitely not the case for the low-scale leptogenesis
The production of lepton asymmetry is described by a set of kinetic equations.

In the Higgs phase, kinetic equations generically can be written as:

\[ \dot{n}_{\nu\alpha} = f_{\alpha}(n_N, n_{\nu\alpha}), \]
\[ \dot{n}_N = g(n_N, n_{\nu\alpha}), \]

\( n_{\nu\alpha} \) (\( \alpha = e, \mu, \tau \)) are asymmetries of number densities of left-handed neutrinos, 
\( n_N \) is a matrix of number densities and correlations of HNLs and ant-HNLs.

This system is far from being realistic:
- no charged fermions;
- no sphaleron processes which are fast at temperatures above 
  \( T_{sph} \approx 131.7 \text{ GeV} \)

Structure of kinetic equations

At temperatures of low scale leptogenesis all SM species are in equilibrium:

\[ \mu_{\nu_\alpha} = \mu_{e_L,\alpha} = \mu_{e_R,\alpha} = \mu_\alpha \]

Only \( n_{\nu_\alpha} \) are changing due to interactions with HNLs:

\[ [\dot{n}_{\nu_\alpha}]_{\text{HNLs}} = [\dot{n}_\alpha]_{\text{HNLs}} \]

Also \( [\dot{n}_\alpha]_{\text{HNLs}} = [\dot{n}_{\Delta_\alpha}]_{\text{HNLs}} \), where \( n_{\Delta_\alpha} = n_\alpha - n_B/3 \)

\( (B - L \) is preserved by sphalerons)\)

\[ \dot{n}_{\Delta_\alpha} = f_\alpha(n_N, \mu_\alpha), \]
\[ \dot{n}_N = g_I(n_N, \mu_\alpha). \]

The neutrality of the electroweak plasma implies a non-trivial relation between the chemical potentials and the asymmetries

\[ \mu_\alpha = \omega_{\alpha\beta}(T)n_{\Delta_\beta} + \omega_B(T)n_B, \]

\( \omega \) – susceptibility matrices.
Susceptibilities

Thermodynamical potential

\[ \Omega(\mu, T) = \frac{1}{24} \left( 8 T^2 \mu_B^2 + 8 T^2 \mu_B \mu_Y + 6 \mu_1^2 T^2 + 6 \mu_2^2 T^2 + 6 \mu_3^2 T^2 + 22 T^2 \mu_T^2 + 22 T^2 \mu_Y^2 - 8 \mu_1 T^2 \mu_Y - 8 \mu_2 T^2 \mu_Y - 8 \mu_3 T^2 \mu_Y + 3 \langle \Phi \rangle^2 \mu_T^2 - 6 \langle \Phi \rangle^2 \mu_T \mu_Y + 3 \phi^2 \mu_Y^2 \right) \]

with

\[ \mu_Y \equiv i g_1 B_0, \quad \mu_T \equiv i g_2 A_0^3. \]


Number densities of conserved charges

\[ - \frac{\partial (\Omega/V)}{\partial \mu_\alpha} = n_\alpha, \quad - \frac{\partial (\Omega/V)}{\partial \mu_B} = n_B, \]

Neutrality conditions read

\[ \frac{\partial (\Omega/V)}{\partial \mu_Y} = 0, \quad \frac{\partial (\Omega/V)}{\partial \mu_T} = 0. \]
\[ \mu_\alpha = \omega_{\alpha\beta}(T)n_{\Delta\beta} + \omega_B(T)n_B, \]

with susceptibilities

\[ \omega(T) = \frac{1}{T^2} \begin{pmatrix} a & b & b \\ b & a & b \\ b & b & a \end{pmatrix}, \quad a = \frac{22 \left(15x^2 + 44\right)}{9 \left(17x^2 + 44\right)}, \quad b = \frac{8 \left(3x^2 + 22\right)}{9 \left(17x^2 + 44\right)} \]

and

\[ \omega_B(T) = \frac{1}{T^2} \frac{4 \left(27x^2 + 77\right)}{9 \left(17x^2 + 44\right)}, \]

where \( x = \langle \Phi \rangle / T \) — Higgs vev divided by temperature.

S. Eijima, M. Shaposhnikov and I.T., 2017

In the same way for sphalerons:

\[ n_{B^{eq}} = -\chi(T) \sum_{\Delta} n_{\Delta}, \quad \chi(T) = \frac{4 \left(27 \langle \Phi \rangle / T \right)^2 + 77}{333 \left(\langle \Phi \rangle / T \right)^2 + 869}, \]
While sphalerons are active, $T > 130$ GeV

$$\mu_\alpha(T) = \omega_{\alpha\beta}(T)n_{\Delta\beta} + \omega_B(T)n_{B^{eq}}(T),$$

Below freeze out, $T < 130$ GeV

$$\mu_\alpha(T) = \omega_{\alpha\beta}(T)n_{\Delta\beta} + \omega_B(T)n_{B^{eq}}(T_{sp}),$$

$$n_{B^{eq}} = -\chi(T)\sum_{\alpha} n_{\Delta\alpha}$$
Separate kinetic equation for $n_B$ (approach 2)


Kinetic equation for $n_B$


\[ \dot{n}_B = -\Gamma_B (n_B - n_{B^{eq}}), \]

\[ \Gamma_B = 9 \frac{869 + 333(\phi/T)^2}{792 + 306(\phi/T)^2} \cdot \frac{\Gamma_{\text{diff}}(T)}{T^3}, \]

The Chern-Simons diffusion rate in a pure gauge theory:


\[ \Gamma_{\text{diff}} \simeq \begin{cases} T^4 \cdot \exp \left( -147.7 + 0.83 T / \text{GeV} \right), & \text{broken phase,} \\ T^4 \cdot 18 \alpha_W^5, & \text{symmetric phase.} \end{cases} \]
Approach 1. A scenario of an instantaneous $B$ freeze out. Baryon number density $n_B(T) = n_{Beq}(T)$ for all temperatures above $T_{sph}$ and $n_B(T) = n_{Beq}(T_{sph})$ for all $T < T_{sph}$.

Approach 2. An approach with the separate kinetic equation for $n_B$. In this case one can follow the $n_B$ during the freeze out, but at the cost of adding a new scale into the problem.

We found that lepton asymmetry is the same in both approaches.
$n_B/n^0_B$ as function of temperature. NH.

NH, $X_\omega = 3$, $\Delta M = 10^{-11}$

T, GeV

$B/B_0$

$B/B_0$

approach 2

approach 1
$n_B/n_B^0$ as function of temperature. IH.

IH, $X_\omega = 10$, $\Delta M = 10^{-11}$

- **Approach 1**
- **Approach 2**

I. Timiryasov (EPFL)

Low-scale leptogenesis

June 1, 2018
Deviation from equilibrium

The ratio $r(T) = -B/(\sum_\alpha \Delta_\alpha)$ as function of temperature. In the equilibrium with respect to sphalerons $r(T) = \chi(T)$.
In contrast to the instantaneous freeze-out assumption, the baryon number freeze-out occurs in two steps:

1. deviation from the equilibrium value at temperatures around 140 GeV;
2. the final freeze-out at temperatures around $T_{sph} \sim 131.7$ GeV.

If a sufficient portion of lepton asymmetry was generated in the transition period, the deviation between two approaches can be significant.
Large deviation

\[
\Delta = \begin{cases} 
-1.5 \times 10^{-7} & \text{approach 1} \\
-1.0 \times 10^{-7} & \text{approach 2} \\
-5.0 \times 10^{-8} & \text{approach 1} \\
0 & \text{approach 2}\end{cases}
\]

\[
B = \begin{cases} 
2.0 \times 10^{-9} & \text{approach 1} \\
4.0 \times 10^{-9} & \text{approach 2} \\
6.0 \times 10^{-9} & \text{approach 1} \\
0 & \text{approach 2}\end{cases}
\]

\[\Delta M = 3.98 \times 10^{-11}\] for NH, \(X_\omega = 4.2\), \(\Delta M = 3.98 \times 10^{-11}\) for BH, \(X_\omega = 4.2\).
Instead of using
\[ B = \chi(T_{sph}) \sum_{\alpha} \Delta_{\alpha}(T_{sph}), \]
One can solve
\[
\frac{d(n_B(T))}{dT} \frac{dT}{dt} = -\Gamma_B(T) \left( n_B(T) + \chi(T) \sum_{\alpha} n_{\Delta_{\alpha}}(T) \right),
\]
with the source \( \sum_{\alpha} n_{\Delta_{\alpha}}(T) \) calculated within the instantaneous freeze out approach (\textbf{approach 1}).
Results of this procedure perfectly agree with those obtained within the approach 2.
Transition period between departure from equilibrium and final freeze out is important.

If one wants to ensure that the resulting BAU is correct for all parameter sets, it is necessary to solve the kinetic equation for baryon number.
Baryogenesis in the nuMSM: recent improvements

Progress in theoretical understanding

- Accurate computation of relevant rates
  - Ghiglieri and Laine, 1605.07720
  - Eijima and Shaposhnikov, 1703.06085

- Fermion number violating processes were included
  - Eijima and Shaposhnikov, 1703.06085
  - Ghiglieri and Laine, 1703.06087
  - Antusch et al., 1710.03744

- The role of sphaleron processes was clarified
  - Eijima, Shaposhnikov, I.T., 1709.07834

Studies of the parameter space

Several groups performed scans of the parameter space

- Canetti, Drewes, Frossard, Shaposhnikov, 1208.4607
- Hernández, Kekic, López-Pavón, Racker, Salvado, 1606.06719
- Drewes, Garbrecht, Gueter and Klaric, 1606.06690
- Eijima, Shaposhnikov, I.T., 180x.xxxxx
Parameter space of baryogenesis in the $\nu$MSM

<table>
<thead>
<tr>
<th>$M$, GeV</th>
<th>$\log_{10}(\Delta M/{\text{GeV}})$</th>
<th>$\text{Im}\omega$</th>
<th>$\text{Re}\omega$</th>
<th>$\delta$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.1 – 10]</td>
<td>[−11, −5]</td>
<td>[−7, 7]</td>
<td>[0, 2π]</td>
<td>[0, 2π]</td>
<td>[0, 2π]</td>
</tr>
</tbody>
</table>

Parameters of the theory: common mass; mass difference; $\text{Im}\omega$; $\text{Re}\omega$; Dirac and Majorana phases.

$$|U|^2 \equiv \sum_{\alpha I} |\Theta_{\alpha I}|^2 = \frac{1}{2M} \left[(m_2 + m_3) \left(X_\omega^2 + X_\omega^{-2}\right) + \mathcal{O}\left(\frac{\Delta M}{M}\right)\right],$$
Kinetic equations

\[
\begin{align*}
  i \frac{d \rho_{\nu \alpha}}{dt} &= -i \Gamma_{\nu \alpha} \rho_{\nu \alpha} + i \text{Tr}[\tilde{\Gamma}_{\nu \alpha} \rho_{\bar{N}}], \\
  i \frac{d \rho_{\bar{\nu} \alpha}}{dt} &= -i \Gamma^*_{\nu \alpha} \rho_{\bar{\nu} \alpha} + i \text{Tr}[\tilde{\Gamma}^*_{\nu \alpha} \rho_{N}], \\
  i \frac{d \rho_N}{dt} &= [H_N, \rho_N] - \frac{i}{2} \{\Gamma_N, \rho_N\} + i \sum_\alpha \tilde{\Gamma}_N^\alpha \rho_{\bar{\nu} \alpha}, \\
  i \frac{d \rho_{\bar{N}}}{dt} &= [H_N^*, \rho_{\bar{N}}] - \frac{i}{2} \{\Gamma_N^*, \rho_{\bar{N}}\} + i \sum_\alpha (\tilde{\Gamma}_N^\alpha)^* \rho_{\nu \alpha}.
\end{align*}
\]

Matrices of density \( \rho \) depend on momenta. A complicated system.

A simplification: integrated system \( \rho(k, t) = n(t)f(k), f(k) = 1/(e^{E(k)/T} + 1) \)

\[
\begin{align*}
  \dot{n}_{\Delta \alpha} &= -\text{Re} \tilde{\Gamma}_{\nu \alpha} \mu_{\alpha} + 2i \text{Tr}[(\text{Im} \tilde{\Gamma}_{\nu \alpha}) \mu_{+}] - \text{Tr}[(\text{Re} \tilde{\Gamma}_{\nu \alpha}) \mu_{-}], \\
  \dot{n}_+ &= -i[\text{Re} \tilde{H}_N, n_+] + \frac{1}{2} \{\text{Im} \tilde{H}_N, n_+\} - \frac{1}{2} \{\text{Re} \tilde{H}_N, n_+\} - \frac{i}{4} \{\text{Im} \tilde{H}_N, n_-\} \\
  &\quad - \frac{i}{2} \sum (\text{Im} \tilde{\Gamma}_N^\alpha) \mu_{\alpha} - S_{\text{eq}}, \\
  \dot{n}_- &= 2[\text{Im} \tilde{H}_N, n_+] - i[\text{Re} \tilde{H}_N, n_-] - i \{\text{Im} \tilde{H}_N, n_+\} - \frac{1}{2} \{\text{Re} \tilde{H}_N, n_-\} \\
  &\quad - \sum (\text{Re} \tilde{\Gamma}_N^\alpha) \mu_{\alpha}.
\end{align*}
\]

Errors of order of 50%
Numerical studies

Production of BAU in the $\nu$MSM is described by a set of 11 ordinary differential equations (which are stiff).

Previously the equations were solved using Mathematica.  

*Canetti, Drewes, Frossard, Shaposhnikov, 1208.4607*

A full scan was impossible.

We have implemented an efficient numerical procedure reducing integration time by $3-4$ orders of magnitude.
Parameter space of baryogenesis in the $\nu$MSM

Why the allowed regions have changed?

- Accurate computation of the rates (note also that after the Higgs discovery the values of the crossover temperature, sphaleron freeze-out temperature etc. were updated)
- Significant numerical improvement: system of 11 coupled ODE can now be solved more than $10^3$ times faster.
- In ref. Canetti et al., 1208.4607 Dirac and Majorana CP-phases $\delta$ and $\eta$ were fixed to some (non-optimal) values. This has reduced the allowed region in 1208.4607.
- Fast code allows to perform a thorough parameter scan.
blue line: SHiP Collaboration, Sensitivity of the SHiP experiment towards heavy neutral leptons, arXiv:180x.xxxxx, also Fig. 1 of 1805.08567
Direct detection in the SHiP experiment

SHiP – search for hidden particles. Fixed target experiment at CERN SPS. Production of HNLs in decays of $D$ and $B$ mesons.

Also NA62, MATHUSLA, (DUNE, FCC-ee - ?)
It seems that all effects important for the baryogenesis in the $\nu$MSM have been understood:

- An accurate derivation of rates
- Neutrality of plasma
- Fermion number violating processes
- Freeze-out of sphalerons

Efficient numerical methods of calculation of BAU in the $\nu$MSM were implemented.