# Proton Size Puzzle: Fat or Thin?

**“Fat and Thin” famous Anton Chekhov novel**

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**Introduction**
**Experimental data**
**Our work**
**Conclusions**

1) Phys.Part.Nucl.Lett. 14 (2017) 857
2) Phys.Lett. B776 (2018) 105; 3)arXiv:1804.09749 [hep-ph]

In 2010 the  CREMA (Charge Radius Experiments with Muonic Atoms) Collaboration measured  very precisely  the Lamb shift of muonic hydrogen. It has opened the new era of the precise investigation of the spectrum of simple atoms.

In the new experiments by this Collaboration with muonic deuterium and ions of muonic helium a charge radii of light nuclei were obtained with very high precision.

For muonic hydrogen and muonic deuterium it was shown that obtained values of the charged radii  are significantly different from those which were extracted from experiments with electronic atoms and in the scattering of the electrons with nuclei and were recommended for using by  CODATA, so-called, “PROTON CHARGE RADIUS PUZZLE”.

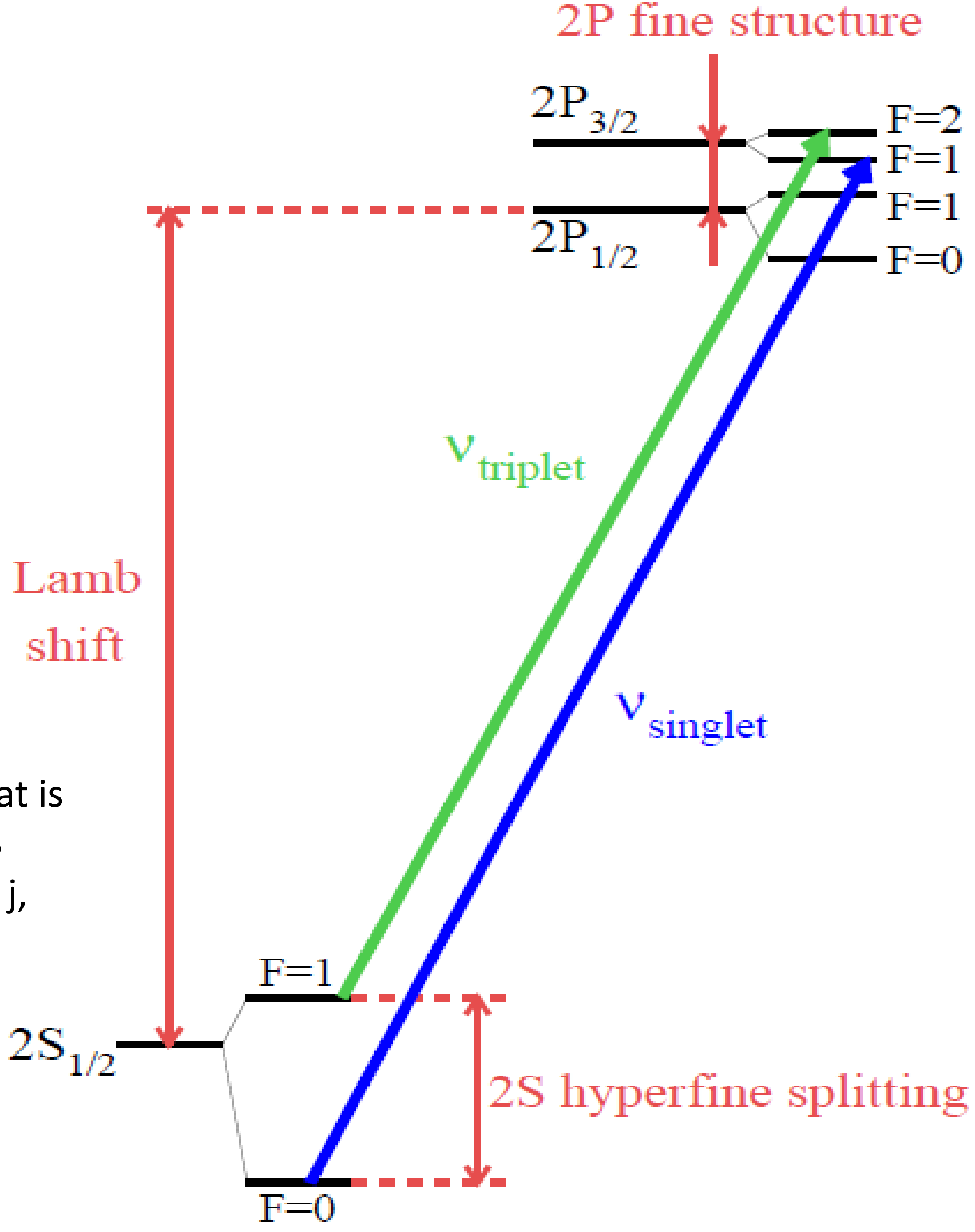
Several experimental groups plan to measure the hyperfine structure of various muonic atoms  with more high precision.

One can consider experiments with muonic atoms as a smoking gun for:

Precise measurements of the proton charge radius

Test of the Standard Model with greater accuracy

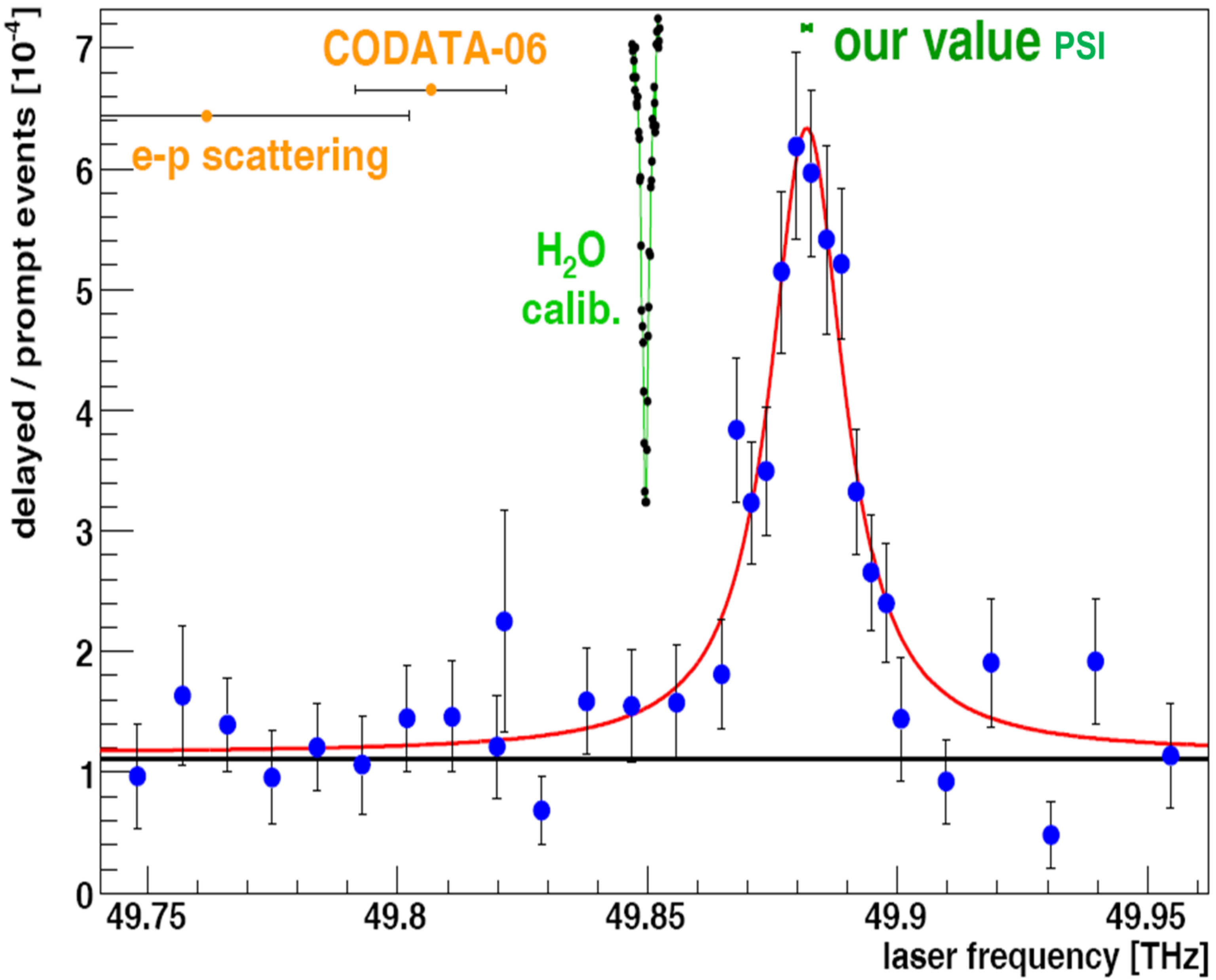
and, possibly, to reveal the source of previously unaccounted interactions between the particles forming the bound state in QED.

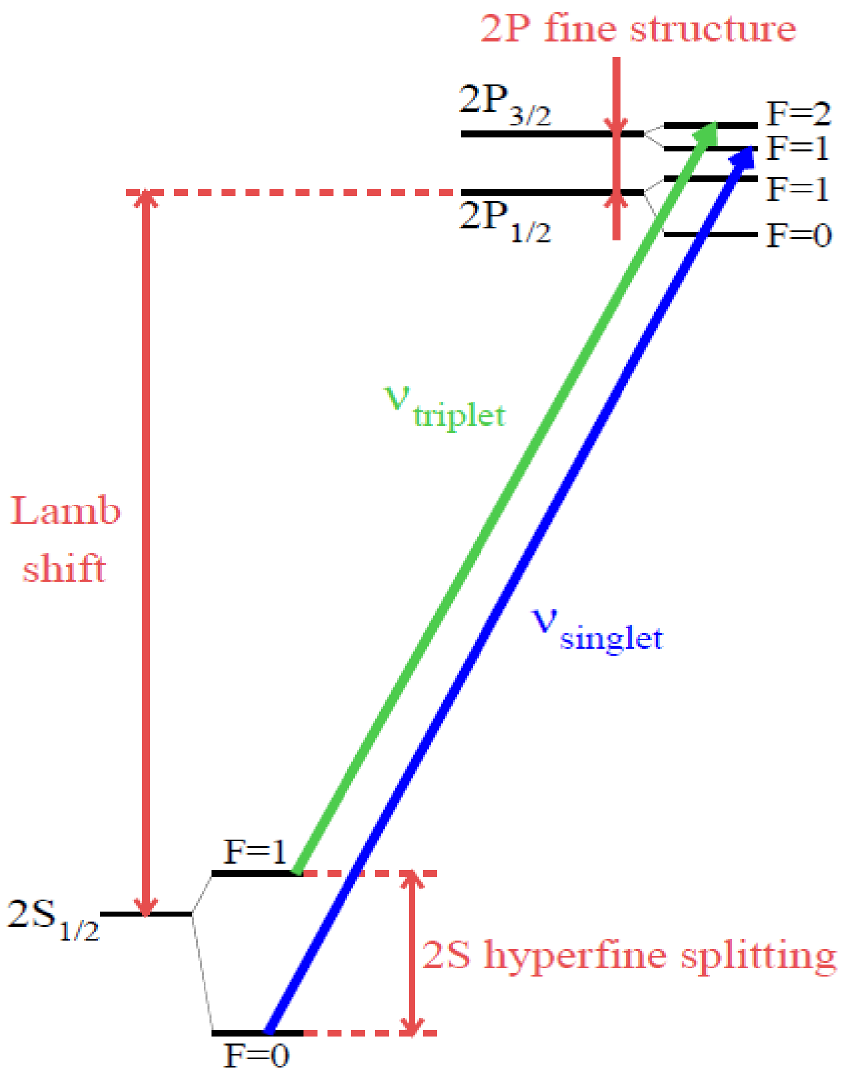



The HFS requires the spin-spin coupling that is the interaction between the nuclear spin S and the electron total angular momentum j, where f = j + S is the atom total angular momentum

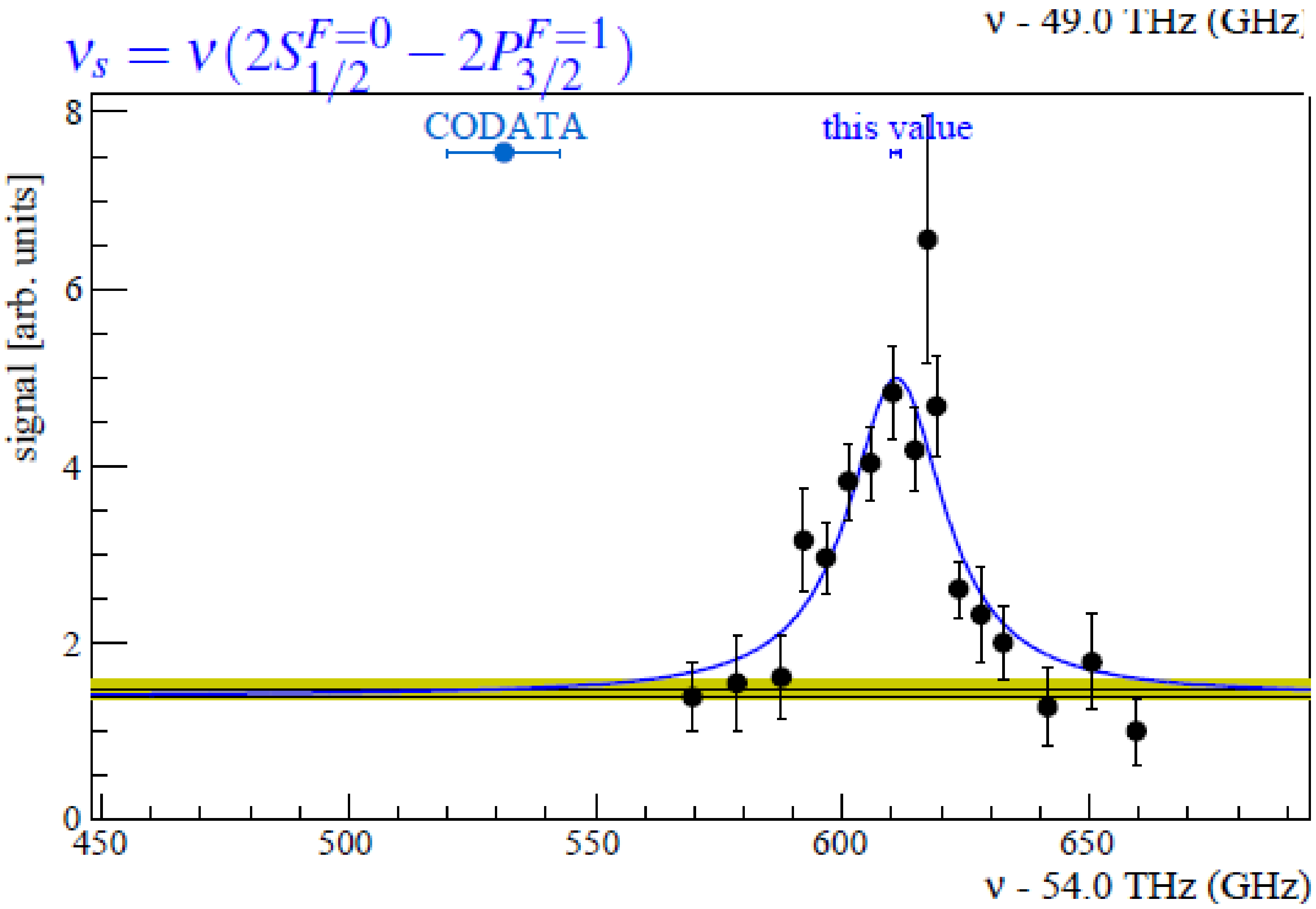
$$\nu_t = \nu(2S_{1/2}^{F=1} - 2P_{3/2}^{F=2})$$

**CREMA-10-13**

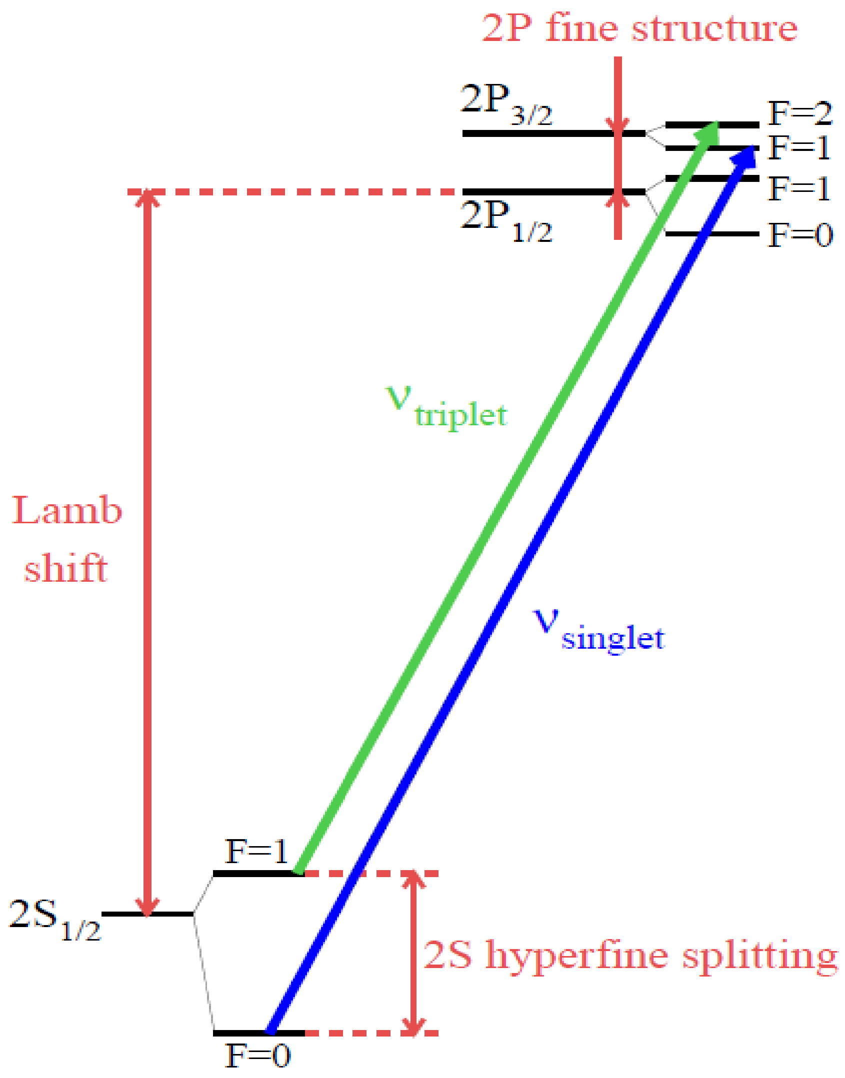


# CREMA-13



$\nu_s = \nu(2S^{F=0}_{1/2} - 2P^{F=1}_{3/2})$
ν - 49.0 THz (GHz)
CODATA
this value
signal [arb. units]
0
2
4
6
8
450
500
550
600
650
ν - 54.0 THz (GHz)

2P fine structure
$2P_{3/2}$
$2P_{1/2}$
F=2
F=1
F=1
F=0
$\nu_{triplet}$
$\nu_{singlet}$
Lamb shift
$2S_{1/2}$
F=1
F=0
2S hyperfine splitting

Two transitions measured

$$\nu_T = 49881.35(\ \ 65)\ \text{GHz}$$
$$\nu_S = 54611.16(1.05)\ \text{GHz}$$

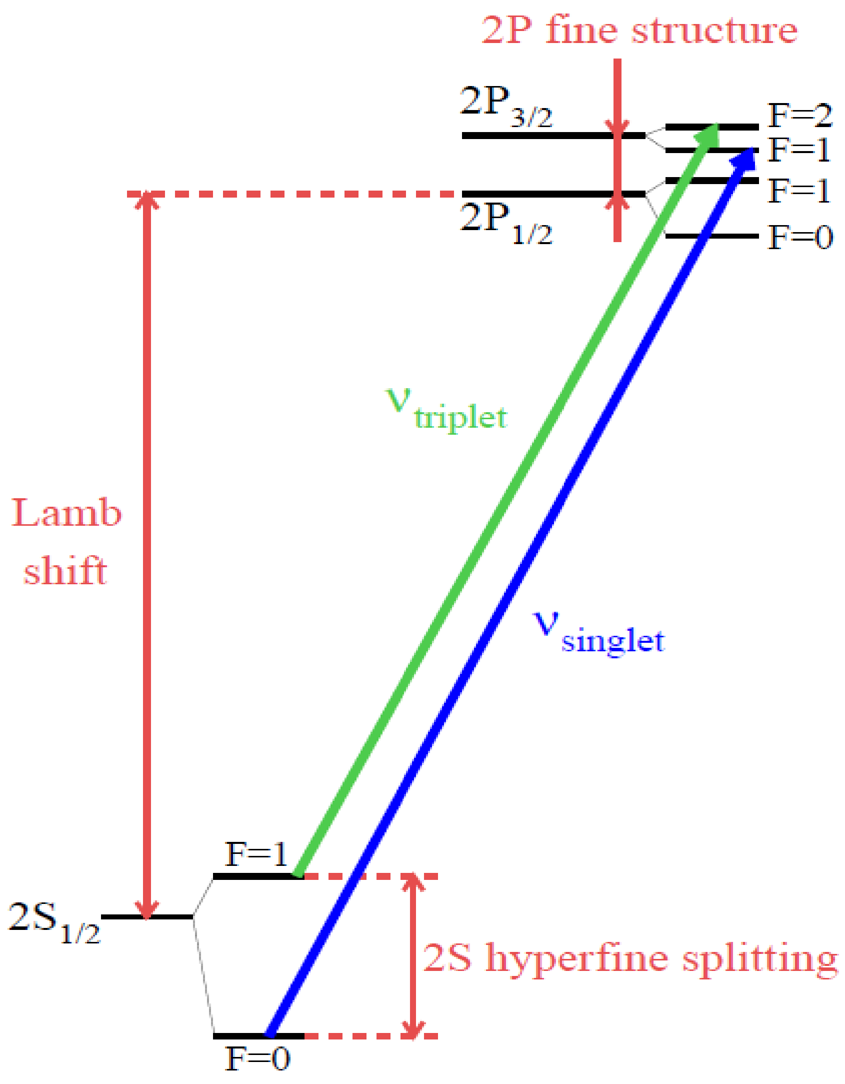
From these two transition measurements, one can independently deduce both the Lamb shift $\Delta E_L = \Delta E(2P_{1/2} - 2S_{1/2})$ and the 2S-HFS splitting ($\Delta E_{HFS}$) by the linear combinations

$$\frac{1}{4}h\nu_s + \frac{3}{4}h\nu_t = \Delta E_L + 8.8123(2)\text{meV}$$

$$h\nu_s - h\nu_t = \Delta E_{HFS} - 3.2480(2)\text{meV}$$

Then one get

$$\Delta E_{LS}^{exp} = 202.3706(23)\ \text{meV}$$
$$\Delta E_{HFS}^{exp} = 22.8089(51)\ \text{meV}$$



# Proton Size Puzzle

$r_p = 0.8751(61)\,\text{fm (CODATA-2014)}$

From spectrum of electronic atoms and the scattering of the electrons with nuclei

**Lamb shift in muonic Hydrogen (μp)** (a proton orbited by a negative muon)

muon mass $m_\mu \approx 200 \times m_e$

Bohr radius $r_\mu \approx 1/200 \times r_e$

$\mu$ inside the proton: $200^3 \approx 10^7$

***μp has much smaller Bohr radius compared to electronic hydrogen and so is much more sensitive to the finite size of the proton***

$\Delta E_{LS}^{exp} = 202.3706(23)\ \text{meV}$

CREMA experiment **Nature 2010; Science 2013**
**(0.05% precision)**

$\Delta E_{LS} = 206.0668(25) - 5.2275(10)\langle r_p^2 \rangle + O\left(\langle r_p^2 \rangle^{3/2}\right)[\text{meV}]$

**Theory summary: Antognini et al. AnnPhys 2013**
**(2% effect)**

$r_p = 0.84087(39)\,\text{fm (CREMA coll. Antognini et al., 2013)}$

**5.6 σ deviation!**

$r_E^2 = \int d^3r\, r^2\, \rho_E(r)$

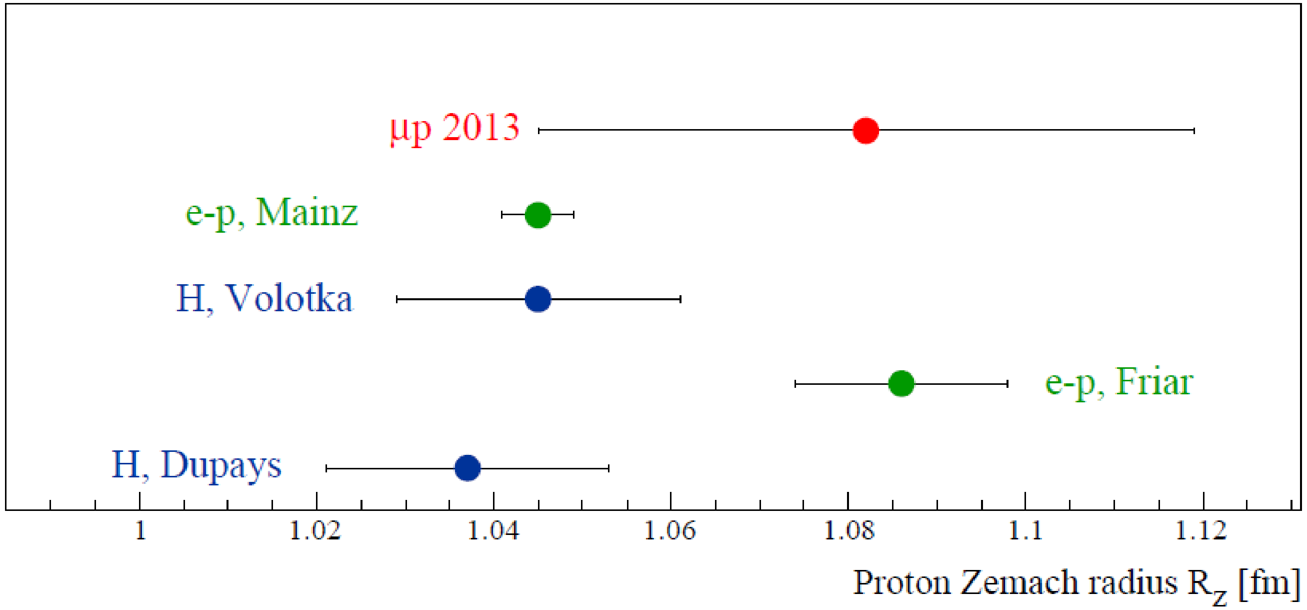
# Zemach radius

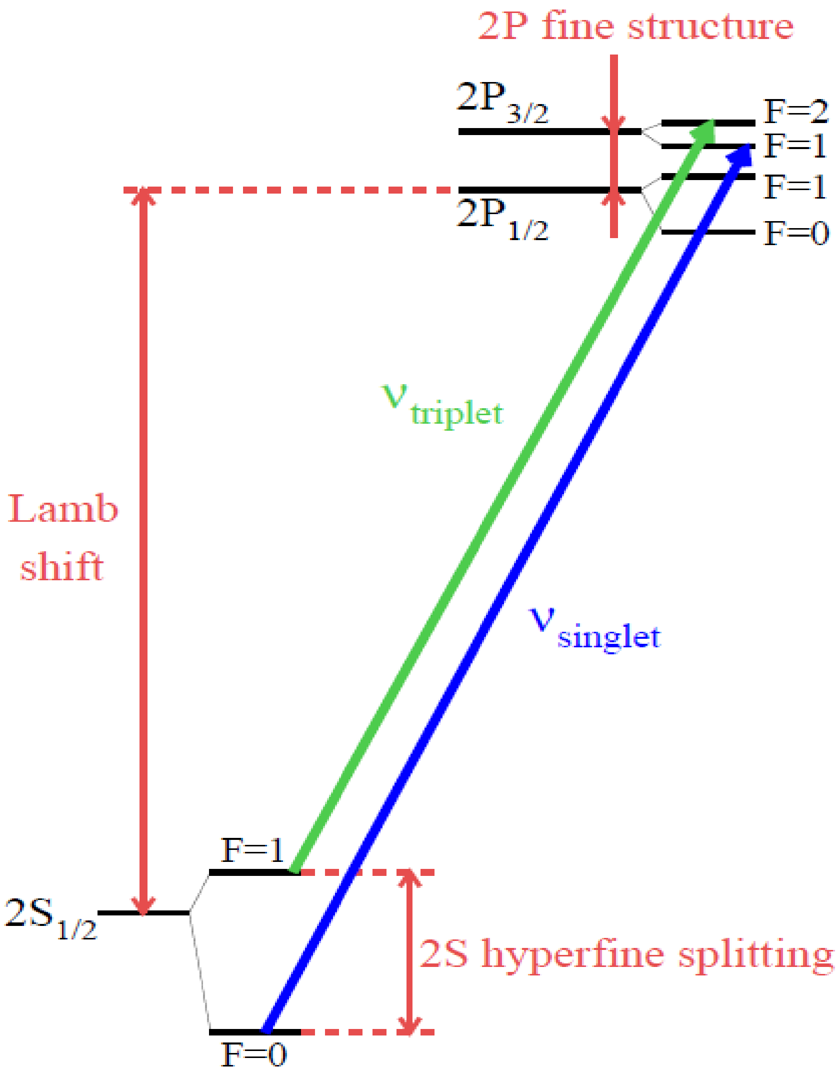
$$\Delta E^{\mathrm{exp}}_{\mathrm{2S\text{-}HFS}} = 22.8089(51)\ \ \mathrm{meV}$$

$$\Delta E^{\mathrm{th}}_{\mathrm{2S\text{-}HFS}} = 22.9843(30) - 0.1621(10) r_Z\ \ \mathrm{meV}$$

$$r_Z = \int d^3 r \int d^3 r'\ r\ \rho_E(r)\,\rho_M(r-r')$$

$$r_Z = 1.082(37)\,\mathrm{fm}\ \text{(CREMA coll. Antognini et al., 2013)}$$

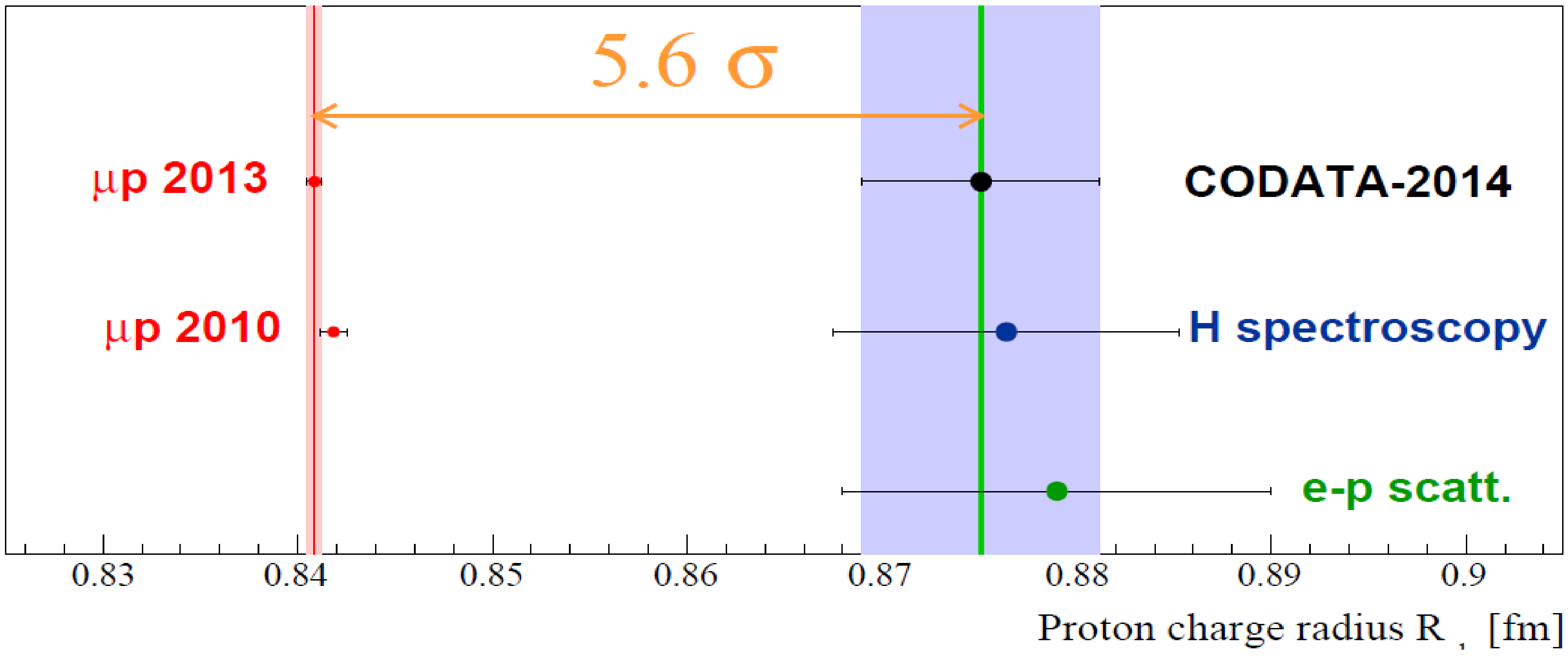


The proton rms charge radius measured with

electrons: 0.8751 ± 0.0061 fm

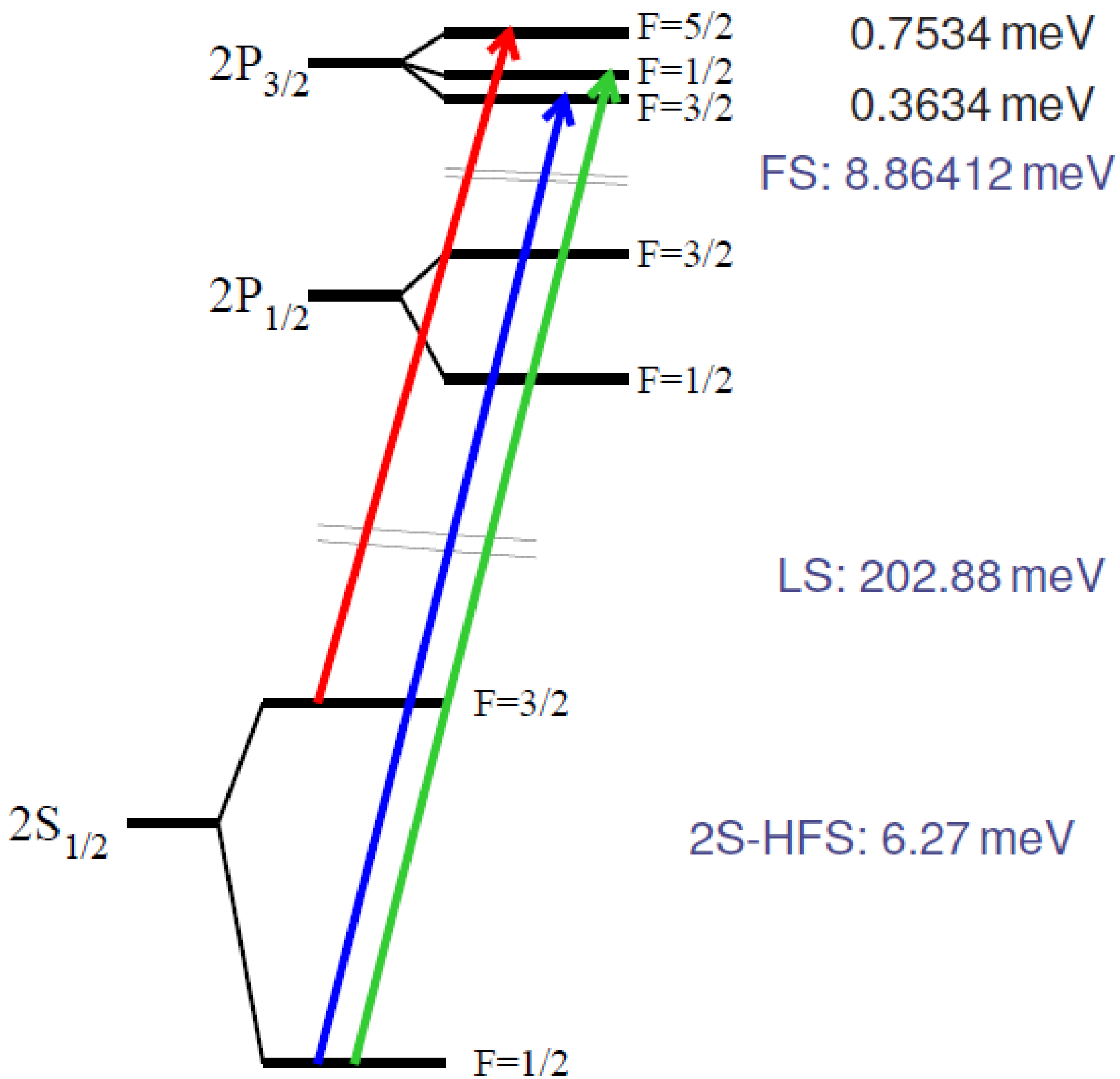
muons: 0.8409 ± 0.0004 fm




$r_p$ is 7 σ smaller than CODATA-2010

4.0 σ smaller than $r_p$ (H spectrosopy)

# Muonic deuterium CREMA-16



# Deutron charge radius

$$\Delta E_{LS}^{exp} = 202.8785(34)\ \text{meV}$$

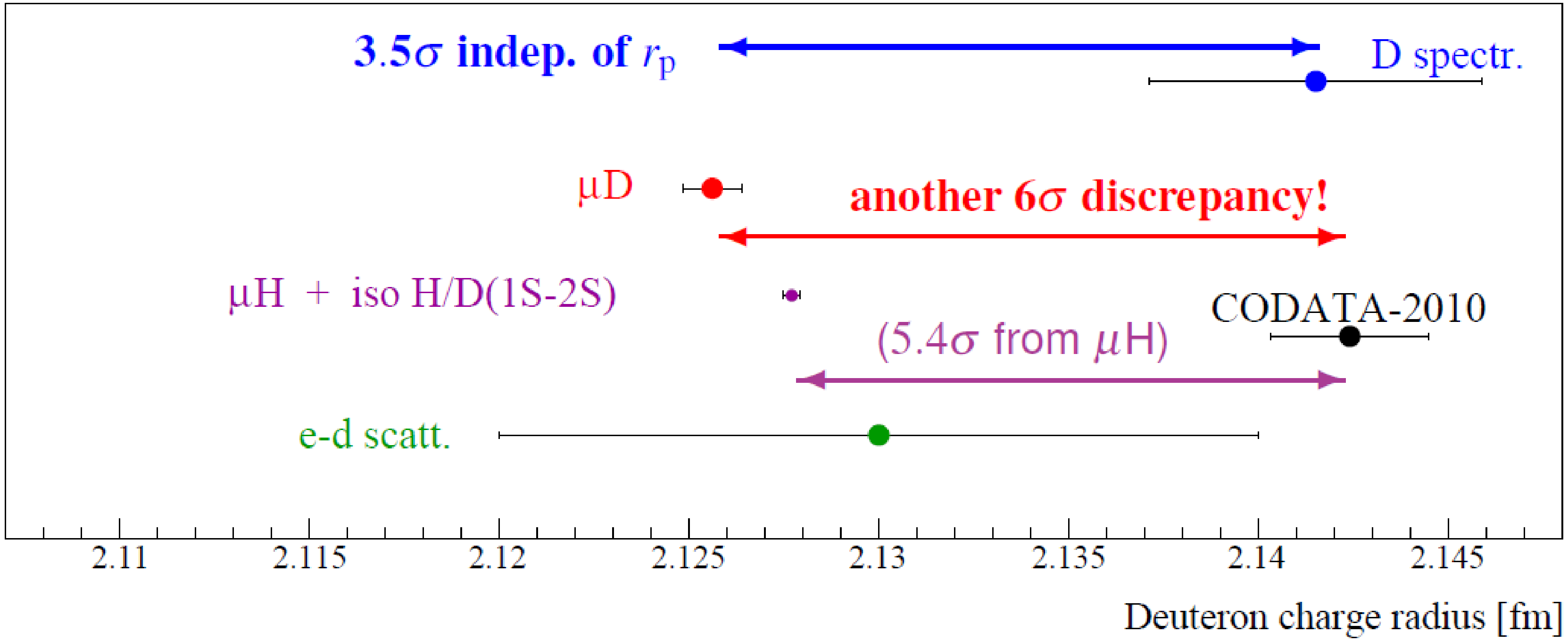
$$\Delta E_{LS}^{th} = 228.7766(10)\ \text{meV (QED)} + 1.7096(200)\ \text{meV (TPE)} - 6.1103(3) r_d^2\ \text{meV/fm}^2$$

$$\mu D \Rightarrow r_d = 2.12562(78)\ \text{meV (CREMA 2016)}$$

H/D isotope shift: $r_d^2 - r_p^2 = 3.82007(65)\ \text{fm}^2$

$r_p$ from $\mu$H gives $r_d = 2.12771(22)$ fm ← 5.4$\sigma$ from $r_p$

CODATA 2014 $r_d = 2.14130(250)$ fm




$r_d$ is 7.5 σ smaller than CODATA-2010 (99% correlated with $r_p$ !)

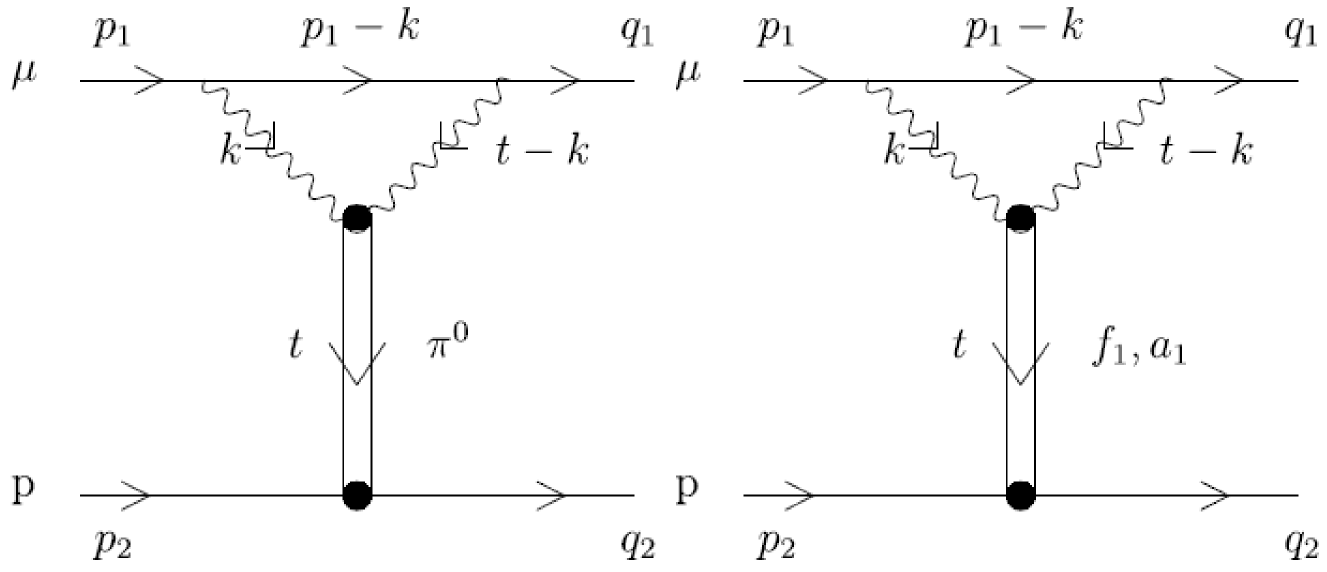
3.5 σ smaller than $r_d$ (D spectrosopy)

The leading contribution to the hyperfine splitting (HFS) in muonic hydrogen is coming from one-photon exchange [AP Martynenko, RN Faustov 2004]

$$\Delta V_B^{hfs} = \frac{8\pi\alpha\mu_p}{3m_\mu m_p}(\mathbf{S}_p\mathbf{S}_\mu)\delta(\mathbf{r}) - \frac{\alpha\mu_p(1+a_\mu)}{m_\mu m_p r^3}\left[(\mathbf{S}_p\mathbf{S}_\mu) - 3(\mathbf{S}_p\mathbf{n})(\mathbf{S}_p\mathbf{n})\right] +$$

$$\frac{\alpha\mu_p}{m_\mu m_p r^3}\left[1 + \frac{m_\mu}{m_p} - \frac{m_\mu}{2m_p\mu_p}\right](\mathbf{L}\mathbf{S}_p)$$

We calculate further the contribution to HFS coming from light pseudoscalar and axial-vector meson exchanges



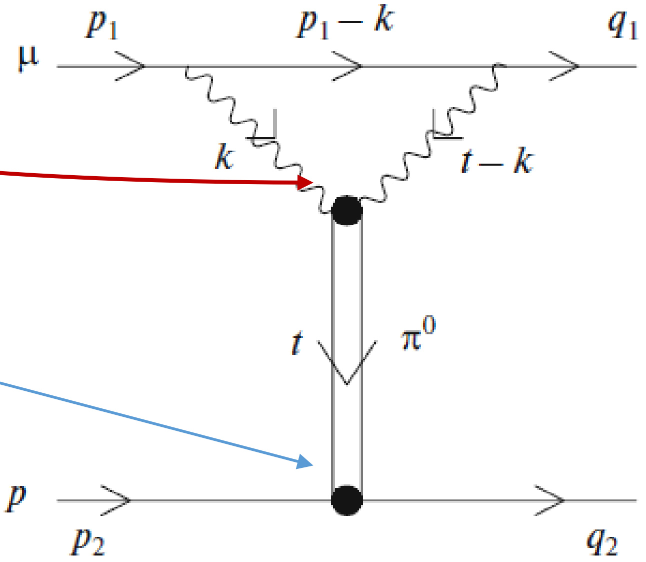
# *Pseudosclar mesons*

The effective vertex of the interaction of the PS mesons and virtual photons can be expressed in terms of the transition form factors

$$A\left(\gamma^*_{(q_1,\epsilon_1)}\gamma^*_{(q_2,\epsilon_2)}\to\pi_{(p)}\right)=ie^2\varepsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu\epsilon_2^\nu q_1^\rho q_2^\sigma\frac{1}{4\pi^2F_\pi}F_{\pi\gamma^*\gamma^*}\left(p^2;q_1^2,q_2^2\right)$$

As a result, the hyperfine part of the potential of the one-pion interaction of a muon and a proton in the S-state takes the form

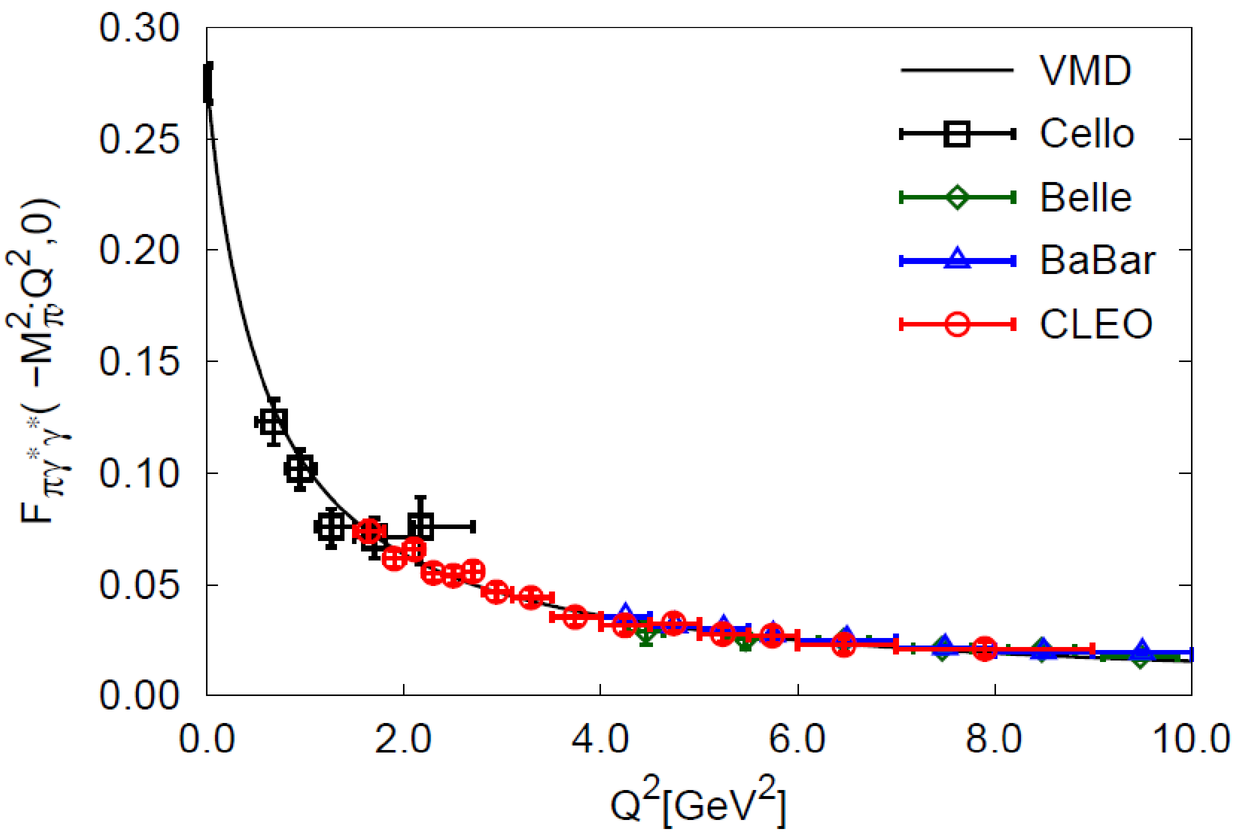
$$\Delta V^{hfs}(\mathbf{p},\mathbf{q})=\frac{\alpha^2}{6\pi^2}\frac{g_p}{m_pF_\pi}\frac{(\mathbf{p}-\mathbf{q})^2}{(\mathbf{p}-\mathbf{q})^2+m_\pi^2}\mathcal{A}(t^2),$$




where

$$\mathcal{A}(t^2)=\frac{2i}{\pi^2t^2}\int d^4k\frac{t^2k^2-(tk)^2}{k^2(k-t)^2(k^2-2kp_1)}F_{\pi\gamma^*\gamma^*}(k^2,(k-t)^2).$$

# Upper block: two-gamma transition FF into PS meson




$$F_{\pi\gamma\gamma^*}\left(Q^2,0\right)=\frac{\Lambda_\pi^2}{\Lambda_\pi^2+k^2},\quad \Lambda_\pi^{CLEO}=776\pm22\ \text{MeV},$$

$$F_{\pi\gamma^*\gamma^*}\left(Q^2,Q^2\right)=\frac{\overline{\Lambda}_\pi^2}{\overline{\Lambda}_\pi^2+k^2},\quad \overline{\Lambda}_\pi^{CLEO+QCD}=499\pm50\ \text{MeV}$$

Calculating the matrix elements with wave functions of 1S , 2S and 2P $_{1/2}$ states, we obtain the corresponding contributions to the HFS spectrum (we use data from CLEO on TFF)

$$\Delta E^{hfs}(1S)=\frac{\mu^3\alpha^5 g_A}{6F_\pi^2\pi^3}\left\{\mathcal{A}(0)\frac{4W(1+\frac{W}{m_\pi})}{m_\pi(1+\frac{2W}{m_\pi})^2}-\frac{1}{\pi}\int_0^\infty\frac{ds}{s}Im\mathcal{A}(s)\left[1+\frac{1}{4W^2(s-m_\pi^2)}\left(\frac{m_\pi^4}{(1+\frac{m_\pi}{2W})^2}-\frac{s^2}{(1+\frac{\sqrt{s}}{2W})^2}\right)\right]\right\}=-0.0017\ meV,$$

$$\Delta E^{hfs}(2S)=\frac{\mu^3\alpha^5 g_A}{48F_\pi^2\pi^3}\left\{\mathcal{A}(0)\frac{W(8+11\frac{W}{m_\pi}+8\frac{W^2}{m_\pi^2}+2\frac{W^3}{m_\pi^3})}{2m_\pi(1+\frac{W}{m_\pi})^4}-\frac{1}{\pi}\int_0^\infty\frac{ds}{s}Im\mathcal{A}(s)\right.$$
$$\left.\left[1+\frac{1}{(s-m_\pi^2)}\left(\frac{m_\pi^2(2+\frac{W^2}{m_\pi^2})}{2(1+\frac{W}{m_\pi})^4}-\frac{s(2+\frac{W^2}{s})}{2(1+\frac{W}{\sqrt{s}})^4}\right)\right]\right\}=-0.0002\ meV,$$

$$\Delta E^{hfs}_{2P_{1/2}}=\frac{\alpha^7\mu^5 g_A}{288\pi^3F_\pi^2m_\pi^2}\mathcal{A}(0)\frac{\left(9+8\frac{W}{m_\pi}+2\frac{W^2}{m_\pi^2}\right)}{(1+\frac{W}{m_\pi})^4}=0.0004\ \mu eV.$$

# Axial-Vector Mesons

The transition from initial state of two virtual photons with four-momenta k1, k2 to an axial vector meson A (JPC = 1++) with the mass $M_A$ for the small values of the relative momenta of particles in the initial and final states and small value of transfer momentum t between muon and proton is

$$T^{\mu\nu\alpha} = 8\pi i\alpha\varepsilon_{\mu\nu\alpha\tau}k^{\tau}k^{2}F^{2}_{AV\gamma^{*}\gamma^{*}}\left(M_A^2;0,0\right)$$

The final result for the HFS potential is

$$\Delta V^{hfs}_{AV}(\mathbf{p}-\mathbf{q}) = -\frac{32\alpha^2 g_{AVPP}}{3\pi^2(\mathbf{t}^2+M_A^2)}\int id^4k\frac{(2k^2+k_0^2)}{k^2(k^2-2m_\mu k_0)}F_{AV\gamma^*\gamma^*}(0,k^2,k^2)$$

By using L3 Collaboration data (production of $f_1(1285)$ and $f_1(1420)$), we can parameterize the transition form factor for the case of two photons with equal virtualities

$$F_{AV\gamma^{*}\gamma^{*}}\left(M_A^2;k^2,k^2\right) = F_{AV\gamma^{*}\gamma^{*}}\left(M_A^2;0,0\right)F^2_{AV}\left(k^2\right) \qquad F_{AV}\left(k^2\right) = \frac{\Lambda_A^2}{\Lambda_A^2+k^2}$$

with the normalization fixed from the decay width of axial-vector meson measured by L3 Collaboration. (Expect improvements From BESIII and BELLEII)

$$\Lambda^{L3}_{f_1(1285)} = 1040 \pm 78\ \text{MeV}, \quad \Lambda^{L3}_{f_1(1420)} = 926 \pm 78\ \text{MeV}$$

# Meson-Nucleon Couplings

**Within the NJL model from Triangle diagram one has**

$$F_{f_1(1285)\gamma^*\gamma^*}\left(M^2_{f_1(1285)};0,0\right)=\frac{5g_{f_1(1285)qq}}{72\pi^2M_q^2}$$

**Another couplings are related to each others by using chiral symmetry and SU(6)-model for wave function of the proton**

$$g_{f_1(1260)qq}=g_{f_1(1285)qq},$$

$$g_{f_1(1285)pp}=g_{f_1(1285)qq},\quad g_{f_1(1260)pp}=\frac{5}{3}g_{f_1(1285)qq}$$

**With Mq=300 MeV one gets**

$$g_{f_1(1285)pp}=g_{f_1(1285)qq}=g_{f_1(1260)qq}=3.40\pm1.19,$$

$$g_{f_1(1260)pp}=5.67\pm1.98,\quad g_{f_1(1420)pp}=1.51\pm0.19$$

Our result for HFS interaction can be rewritten ($a=2m_\mu/\Lambda_A$)

$$\Delta V_{AV}^{hfs}(\mathbf{p}-\mathbf{q}) = -\frac{32\alpha^2 g_{AVpp} F^{(0)}_{AV\gamma^*\gamma^*}(0,0,0)}{3\pi^2(\mathbf{t}^2+M_A^2)} I\left(\frac{m_\mu}{\Lambda_A}\right)$$

$$I\left(\frac{m_\mu}{\Lambda_A}\right) = -\frac{\pi^2\Lambda_A^2}{4(1-a_\mu^2)^{5/2}}\left[3\sqrt{1-a_\mu^2} - a_\mu^2(5-2a_\mu^2)\ln\frac{1+\sqrt{1-a_\mu^2}}{a_\mu}\right]$$

Making the Fourier transform and averaging the obtained expression with the wave functions for 1S and 2S states, we obtain the following contribution to hyperfine splitting

$$\Delta E_{AV}^{hfs}(1S) = \frac{32\alpha^5\mu^3\Lambda^2 g_{AVNN} F^{(0)}_{AV\gamma^*\gamma^*}(0,0,0)}{3M_A^2\pi^3\left(1+\frac{2W}{M_A}\right)^2} I\left(\frac{m_\mu}{\Lambda}\right),$$

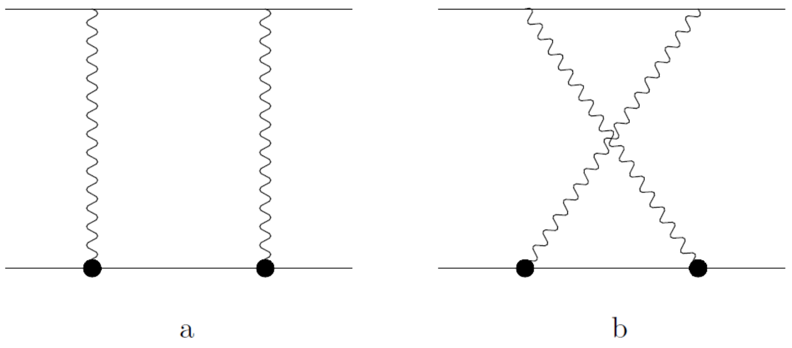
$$\Delta E_{AV}^{hfs}(2S) = \frac{2\alpha^5\mu^3\Lambda^2 g_{AVNN} F^{(0)}_{AV\gamma^*\gamma^*}(0,0,0)\left(2+\frac{W^2}{M_A^2}\right)}{3M_A^2\pi^3\left(1+\frac{W}{M_A}\right)^4} I\left(\frac{m_\mu}{\Lambda}\right)$$

where $W=\mu\alpha$ and $\mu$ is the reduced mass.

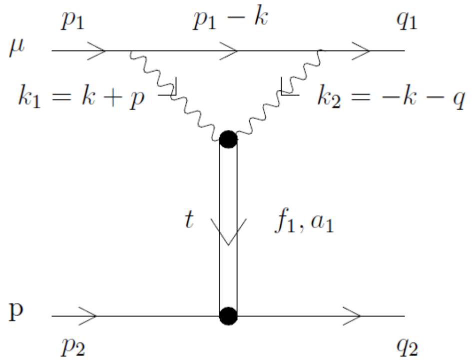
# Axial-Vector and Pseudoscalar mesons contribution to HFS of muonic hydrogen

| mesons | $I^G(J^{PC})$ | $\Lambda_A$ in MeV | $F^{(0)}_{AV\gamma^*\gamma^*}(0,0)$ in $GeV^{-2}$ | $\Delta E^{hfs}(1S)$ in meV | $\Delta E^{hfs}(2S)$ in meV |
|---|---|---|---|---|---|
| $f_1(1285)$ | $0^+(1^{++})$ | 1040 | 0.266 | $-0.0093 \pm 0.0033$ | $-0.0012 \pm 0.0004$ |
| $a_1(1260)$ | $1^-(1^{++})$ | 1040 | 0.591 | $-0.0437 \pm 0.0175$ | $-0.0055 \pm 0.0022$ |
| $f_1(1420)$ | $0^+(1^{++})$ | 926 | 0.193 | $-0.0013 \pm 0.0008$ | $-0.0002 \pm 0.0001$ |
| $\pi^0$ | $1^-(0^{-+})$ | 776 | | $-0.0017 \pm 0.0001$ | $-0.0002 \pm 0.00002$ |
| Sum | | | | $-0.0560 \pm 0.0178$ | $-0.0071 \pm 0.0024$ |

# Corrections of 2-photon interaction to the fine and hyperfine structure of P-energy levels of mu-hydrogen




$\sim \alpha^6$

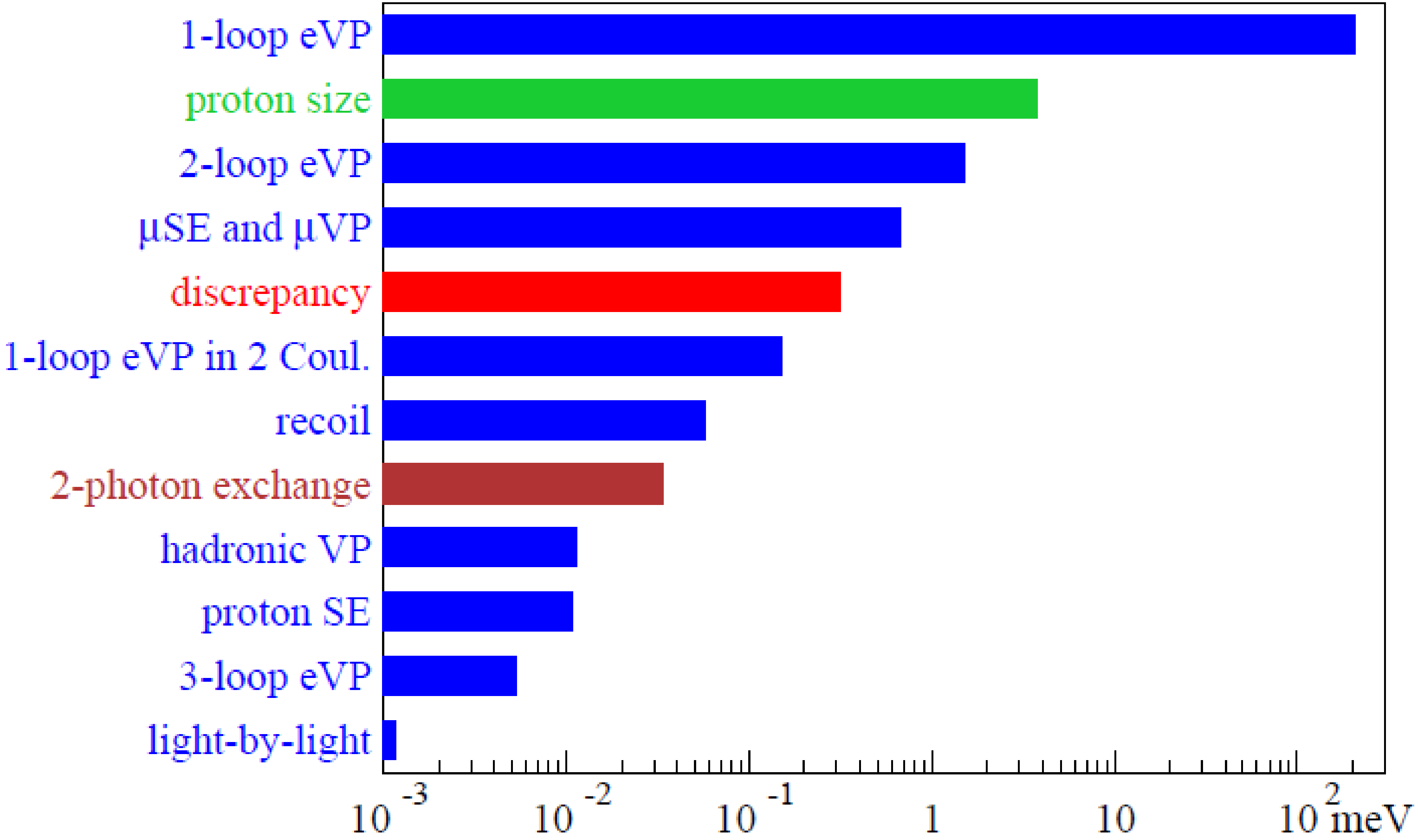



$\sim \alpha^7$

| State | Correction of $2\gamma$ exchange amplitudes, $\mu eV$ | Correction of axial meson exchange amplitudes, $\mu eV$ |
|---|---|---|
| $2^1P_{1/2}$ | -0.2027 | 0.0005 |
| $2^3P_{1/2}$ | 0.0761 | -0.0002 |
| $2^3P_{3/2}$ | 0 | -0.00005 |
| $2^5P_{3/2}$ | 0 | 0.00003 |

The corrections from two-photon exchange amplitudes are more significant for a precise comparison with the experimental data because their numerical values are of the order of 0.0001 meV.
Recall that to explain the proton radius puzzle , a contribution of about 0.3 meV is needed.

Hierarchy of contributions to Lamb shift




→ 205 meV

# Conclusions

**From CREMA experiment side**:

“Proton radius puzzle” is in fact “Z=1 radius puzzle” $r_p = 0.84087(39)\,\text{fm}$ $r_Z = 1.082(37)\,\text{fm}$

Muonic helium-3 and -4 ions show (preliminary) no big discrepancy

New projects are ongoing, one of them FAMU (Fisica Atomi MUonici) with accuracy 2 ppm

## In our theoretical work:

A new **large** contribution to the HFS of muonic hydrogen is discovered, that Induced by pseudoscalar and **axial-vector** couplings to two photon state.

While results do not influence on the proton charge radius, they provide diminishing of the Zemach radius $\boxed{r_Z = 1.040(37)\,\mathrm{fm}}$ (compare with $\boxed{r_Z = 1.082(37)\,\mathrm{fm}}$ )

It should be taking into account for the interpretation of the new data on HFS in this atom.

There are still a number of uncertainties in phenomenological input used in our calculations and some other new effects unaccounted by us. (Work is in progress)

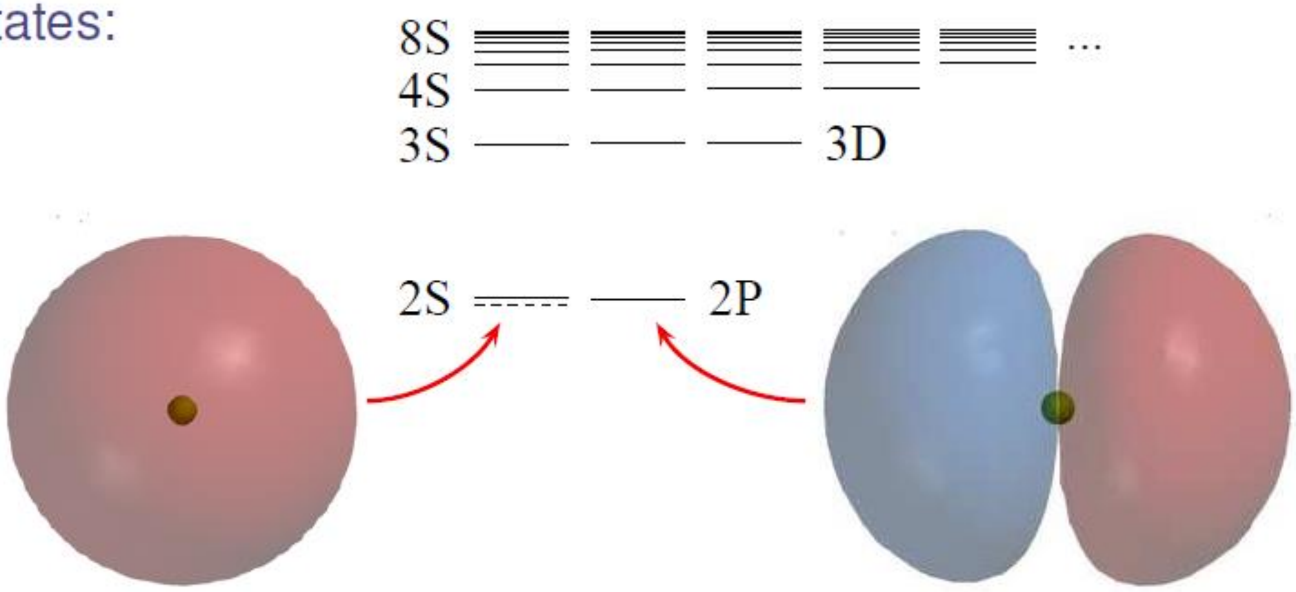
This will make it possible to better understand the existing "puzzle" of the proton charge radius, to check the Standard Model with greater accuracy and, possibly, to reveal the source of previously unaccounted interactions between the particles forming the bound state in QED.

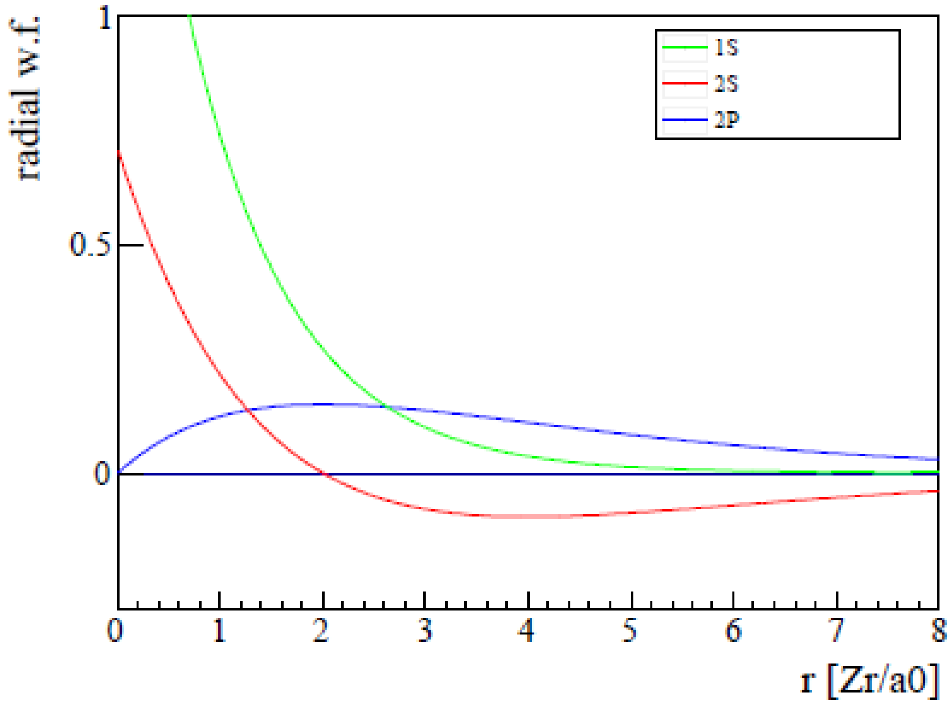
One of the ways of overcoming the crisis situation arises from a deeper theoretical analysis of the fine and hyperfine structure of muonic atom spectrum, with the verification of previously calculated contributions and the more accurate construction of the particle interaction operator in quantum field theory, the calculation of new corrections whose value for muonic atoms can increase substantially in comparison with electronic atoms. The expected results will allow to get also a new very important information about the forces which are responsible for the structure of atoms.

Below, we discuss the effects of exchanges between muon and proton which can contribute to hyperfine structure of muonic hydrogen coming from the effective light meson exchange between muon and proton induced by coupling of pion to two photons. In spite that numerically such contribution was found to be rather small, its contribution will might be important for the interpretation of new data.

- Results from muonic hydrogen and deuterium:
  - Proton charge radius: $r_p$= 0.84087 (39) fm
  - Proton Zemach radius: $R_Z$ = 1.082 (37) fm
  - Rydberg constant: $R_\infty = 3.289\,841\,960\,249\,5\ (10)^{r_p}\ (25)^{\mathrm{QED}}\ \times 10^{15}\ \mathrm{Hz/c}$
  - Deuteron charge radius: $r_d$ = 2.12771 ( 22) fm from $\mu$H + H/D(1S-2S)
  - The "Proton radius puzzle"
- muonic helium-3 and -4: charge radius 10x more precise. No big discrepancy
- H(2S-4P) gives revised Rydberg $\Rightarrow$ small $r_p$ PRELIMINARY
- New projects:
  - 1S-HFS in muonic hydrogen / $^3$He $\Leftarrow$ PSI, J-PARC, RIKEN-RAL, ...
  - LS in muonic Li, Be, B, T, ...; muonic high-Z, ...
  - 1S-2S and 2S-$n\ell$ in Hydrogen/Deuterium/Tritium, $He^+$
  - He, $H_2$, $HD^+$,...
  - Positronium $\equiv e^+e^-$, Muonium $\equiv \mu^+e^-$
  - Electron scattering: H at lower $Q^2$, D, He
  - Muon scattering: MUSE @ PSI
  - ...

tates:



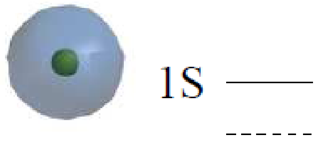

S states: max. at r=0

Electron sometimes inside the proton.

S states are shifted.

Shift ist proportional to the

size of the proton




P states: zero at r=0

Electron is not inside the proton.

Future:
HFS in muonic hydrogen and helium-3
X-ray spectroscopy of muonic radium etc.
Lamb shift in muonic Li, Be, ...
1S-2S in regular tritium (triton radius)

**CREMA 2013**

$$\Delta\tilde{E} \equiv E(2P_{3/2}^{F=2}) - E(2S_{1/2}^{F=1}) = 206.2949 \pm 0.0032\,\mathrm{meV}$$