From LO to NLO in the parton Reggeization approach.

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Outline.

Motivation

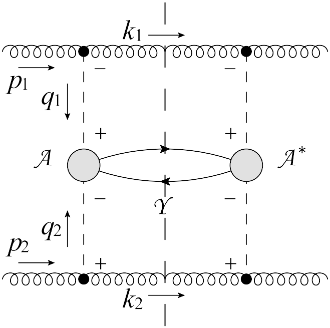
Parton Reggeization Approach (PRA) is a hybrid scheme of k_T-factorization which combines gauge-invariant matrix elements with off-shell (Reggeized) partons in the initial state with the unintegrated PDFs resumming doubly-logarithmic corrections $\sim \log^2(\mathbf{q}_T^2/Q^2)$ (Kimber-Maritn-Ryskin unPDFs).

- **The aim** of PRA is to improve the description of **multi-scale** correlational observables at the energies accessible at the LHC in comparison with the *fixed-order NLO/NNLO* calculations.
- **The wider task** is to understand the role of transverse momentum in Parton Showers (PS) and put the *Recoiling Scheme* ambiguity of PS under theoretical control.
- To provide predictions with controllable accuracy and understand our formalism better we should go to NLO.
- The LO/NLO calculation in PRA should describe **single-scale** observables with the same accuracy as LO/NLO calculation in CPM. $\Rightarrow$ We will use single-scale observables (DIS structure functions) to fix the scheme of calculations at NLO in PRA.

LO framework

Derivation of the LO factorization formula

See [A. V. Karpishkov, M. A. Nefedov, V. A. Saleev, Phys.Rev. **D96** 096019 (2017)] for details.



Inclusive process (μ^2 – hard scale of $\mathcal{Y}$):

$$p(P_1) + p(P_2) \to \mathcal{Y}(P_{\mathcal{A}}) + X,$$

auxiliary hard ($2 \to 3$) subprocess:

$$g(p_1) + g(p_2) \to g(k_1) + \mathcal{Y}(P_{\mathcal{A}}) + g(k_2),$$

Sudakov (Light-cone) decomposition:
$n_-^\mu = P_1^\mu/\sqrt{S}$, $n_+^\mu = P_2^\mu/\sqrt{S}$:

$$k^\mu = \frac{1}{2}\left(k^+ n_-^\mu + k^- n_+^\mu\right) + k_T^\mu,$$

where $k^\pm = (n_\pm k) = k^0 \pm k^3$, $n_\pm k_T = 0$ and

$$kq = \frac{1}{2}(k^+ q^- + k^- q^+) - \mathbf{k}_T \mathbf{q}_T.$$

Derivation of the LO factorization formula

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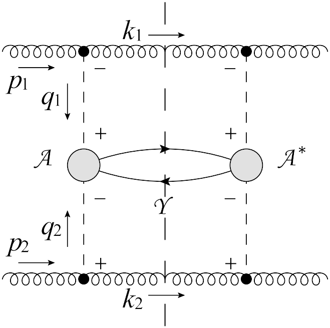


Auxiliary hard CPM subprocess:

$$g(p_1) + g(p_2) \to g(k_1) + \mathcal{Y}(P_{\mathcal{A}}) + g(k_2),$$

where $p_1^2 = 0$, $p_1^- = 0$, $p_2^2 = 0$, $p_2^+ = 0$.
Kinematic variables ($0 < z_{1,2} < 1$):

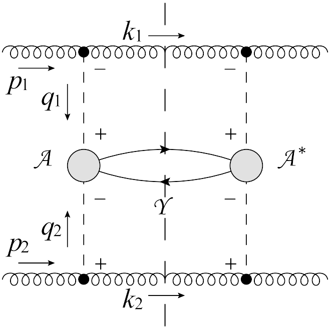
$$z_1 = \frac{p_1^+ - k_1^+}{p_1^+}, \quad z_2 = \frac{p_2^- - k_2^-}{p_2^-},$$

Two limits where $\overline{|\mathcal{M}|^2}$ factorizes:

1. **Collinear limit:** $\mathbf{k}_{T1,2}^2 \ll \mu^2$, $z_{1,2}$ – arbitrary,
2. **Multi-Regge limit:** $z_{1,2} \ll 1$, $\mathbf{k}_{T1,2}^2$ – arbitrary.

Derivation of the LO factorization formula

See [A. V. Karpishkov, M. A. Nefedov, V. A. Saleev, Phys.Rev. **D96** 096019 (2017)] for details.



Auxiliary hard CPM subprocess:

$$g(p_1) + g(p_2) \to g(k_1) + \mathcal{Y}(P_{\mathcal{A}}) + g(k_2),$$

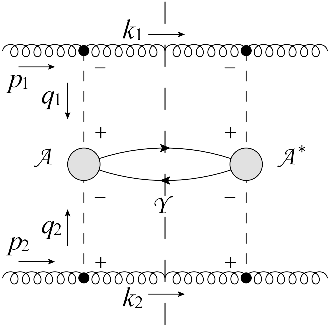
1 **Collinear limit:** $\mathbf{k}_{T1,2}^2 \ll \mu^2$, $z_{1,2}$ – arbitrary:

$$\overline{|\mathcal{M}|^2}_{\rm CL} \simeq \frac{4g_s^4}{\mathbf{k}_{T1}^2 \mathbf{k}_{T2}^2} P_{gg}(z_1) P_{gg}(z_2) \frac{\overline{|\mathcal{A}_{CPM}|^2}}{z_1 z_2},$$

where $\overline{|\mathcal{A}_{CPM}|^2}$ – amplitude $g + g \to \mathcal{Y}$ with **on-shell** initial-state partons, $P_{gg}(z)$ – DGLAP $g \to g$ splitting function.

Derivation of the LO factorization formula

See [A. V. Karpishkov, M. A. Nefedov, V. A. Saleev, Phys.Rev. **D96** 096019 (2017)] for details.



Auxiliary hard CPM subprocess:

$$g(p_1) + g(p_2) \to g(k_1) + \mathcal{Y}(P_{\mathcal{A}}) + g(k_2),$$

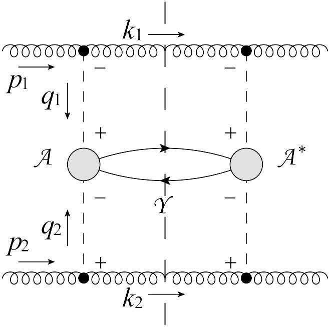
2 **Multi-Regge limit:** $z_{1,2} \ll 1$ ($\Leftrightarrow \Delta y_{1,2} \gg 1$), $\mathbf{k}_{T1,2}^2$ – arbitrary:

$$\overline{|\mathcal{M}|^2}_{\rm MRK} \simeq \frac{4g_s^4}{\mathbf{k}_{T1}^2 \mathbf{k}_{T2}^2} \tilde{P}_{gg}(z_1)\tilde{P}_{gg}(z_2) \frac{\overline{|\mathcal{A}_{PRA}|^2}}{z_1 z_2},$$

where $\tilde{P}_{gg}(z) = 2C_A/z$ and $\overline{|\mathcal{A}_{PRA}|^2}$ is the **gauge-invariant** amplitude $R_+(q_1) + R_-(q_2) \to \mathcal{Y}$ with **Reggeized** (**off-shell**) partons in the initial state.

Derivation of the LO factorization formula

See [A. V. Karpishkov, M. A. Nefedov, V. A. Saleev, Phys.Rev. **D96** 096019 (2017)] for details.



Auxiliary hard CPM subprocess:

$$g(p_1) + g(p_2) \to g(k_1) + \mathcal{Y}(P_{\mathcal{A}}) + g(k_2),$$

Modified MRK approximation: $z_{1,2}$ and $\mathbf{k}_{T1,2}^2$ – arbitrary:

$$\overline{|\mathcal{M}|^2}_{\rm mMRK} \simeq \frac{4g_s^4}{q_1^2 q_2^2} P_{gg}(z_1) P_{gg}(z_2) \frac{\overline{|\mathcal{A}_{PRA}|^2}}{z_1 z_2},$$

where $q_{1,2}^2 = \mathbf{q}_{T1,2}^2/(1-z_{1,2})$, has correct **collinear** and **Multi-Regge** limits!

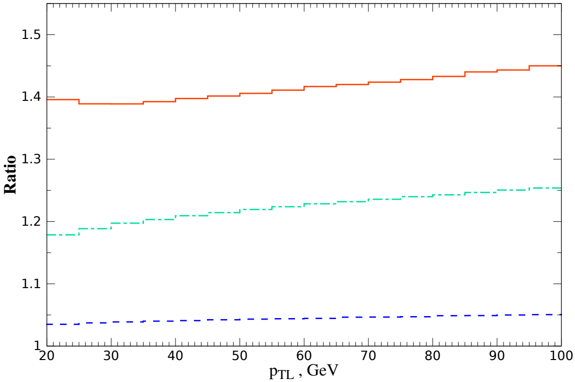
Conjecture: mMRK approximation is reasonable zero-approxiamtion for exact $\overline{|\mathcal{M}|^2}$ *away from collinear limit.*
At least, it is better than collinear limit itself.

Numerical test

Ratio $\sigma_{\rm mMRK}/\sigma_{\rm exact}$ of cross sections of the subprocess

$$g+g\to g(y_1<y_2)+g(y_2,p_{TL})+g(y_3>y_2),$$

in the mMRK approximation vs. exact result in CPM.



pp-collisions,
$\sqrt{S}=7000$ GeV.
Jet-cone radius
$R=0.5$,
$p_{T1,2,3}>10$ GeV.

Curves from top to bottom: no constraint on Δy; $\min(\Delta y_{12},\Delta y_{23})>1.5$; >3.5. **The ratio is almost flat vs. p_{TL} !**

Factorization formula

Substituting the $\overline{|\mathcal{M}|^2}_{\text{mMRK}}$ to the factorization formula of CPM and changing the variables we get:

$$d\sigma = \int_0^1 \frac{dx_1}{x_1} \int \frac{d^2\mathbf{q}_{T1}}{\pi} \tilde{\Phi}_g(x_1, t_1, \mu^2) \int_0^1 \frac{dx_2}{x_2} \int \frac{d^2\mathbf{q}_{T2}}{\pi} \tilde{\Phi}_g(x_2, t_2, \mu^2) \cdot d\hat{\sigma}_{\text{PRA}},$$

where $x_1 = q_1^+/P_1+$, $x_2 = q_2^-/P_2^-$, $\tilde{\Phi}(x, t, \mu^2)$ – "tree-level" **unintegrated PDFs**, the partonic cross-section in PRA is:

$$d\hat{\sigma}_{\text{PRA}} = \frac{\overline{|\mathcal{A}_{PRA}|^2}}{2Sx_1x_2} \cdot (2\pi)^4 \delta\left(\frac{1}{2}\left(q_1^+ n_- + q_2^- n_+\right) + q_{T1} + q_{T2} - P_{\mathcal{A}}\right) d\Phi_{\mathcal{A}}.$$

Note the usual **flux-factor** Sx_1x_2 for **off-shell** initial-state partons.

LO unintegrated PDF

The "tree-level" unPDF:

$$\tilde{\Phi}_g(x,t,\mu^2) = \frac{1}{t}\frac{\alpha_s}{2\pi}\int\limits_x^1 dz\ P_{gg}(z)\cdot\frac{x}{z}f_g\left(\frac{x}{z},\mu^2\right).$$

contains collinear divergence at $t\to 0$ and IR divergence at $z\to 1$.
In the "dressed" unPDF collinear divergence is regulated by **Sudakov formfactor** $T(t,\mu^2)$:

$$\Phi_i(x,t,\mu^2) = \frac{T_i(t,\mu^2,x)}{t}\times\frac{\alpha_s(t)}{2\pi}\int\limits_x^1 dz\ \theta_z^{\rm cut}P_{ij}(z)\frac{x}{z}f_j\left(\frac{x}{z},t\right)$$

where: $\theta_z^{\rm cut} = \theta\left((1-\Delta_{KMR}(t,\mu^2))-z\right)$, and the Kimber-Martin-Ryskin(KMR) **cut condition** [KMR, 2001]:

$$\Delta_{KMR}(t,\mu^2) = \frac{\sqrt{t}}{\sqrt{\mu^2}+\sqrt{t}},$$

follows from the **rapidity ordering** between the last emission and the hard subprocess.

LO unintegrated PDF

$$\Phi_i(x,t,\mu^2) = \frac{T_i(t,\mu^2,x)}{t} \times \frac{\alpha_s(t)}{2\pi} \int\limits_x^1 dz\ \theta_z^{\rm cut} P_{ij}(z) \frac{x}{z} f_j\left(\frac{x}{z}, t\right)$$

$$= \boxed{\frac{\partial}{\partial t}\left[T_i(t,\mu^2,x)\cdot x f_i(x,t)\right]} \leftarrow \text{derivative form of unPDF}$$

⇒ **LO normalization condition:**

$$\boxed{\int_0^{\mu^2} dt\ \Phi_i(x,t,\mu^2) = x f_i(x,\mu^2)} \leftarrow \textbf{Holds exactly!}$$

Because $T(0,\mu^2,x)=0$ and $T(\mu^2,\mu^2,x)=1$.

Sudakov formfactor

$$T_i(t,\mu^2,x) = \exp\left[-\int\limits_t^{\mu^2} \frac{dt'}{t'}\,\frac{\alpha_s(t')}{2\pi}\,(\tau_i+\Delta\tau_i)\right],$$

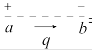
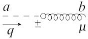
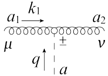
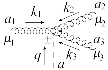
$$\tau_i = \sum_j \int\limits_0^1 dz\; \theta_z^{\rm cut}\cdot zP_{ji}(z),$$

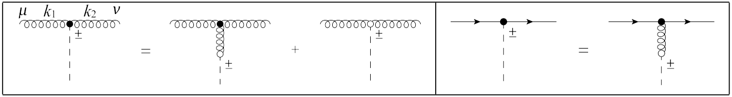
$$\Delta\tau_i = \sum_j \int\limits_0^1 dz\;\left(1-\theta_z^{\rm cut}\right)\cdot\left[zP_{ji}(z) - \underbrace{\frac{\frac{x}{z}f_j\left(\frac{x}{z},t'\right)}{xf_i(x,t')}}_{\substack{\text{similar structure in}\\ \text{the non}-\text{emission probability}\\ \text{in ISR PS}}} P_{ij}(z)\cdot\theta(z-x)\right].$$

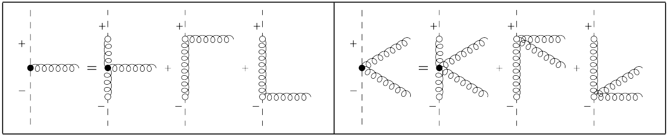
Gauge-invariant off-shell amplitudes

$\overline{|\mathcal{A}_{\rm PRA}|^2}$ is obtained from Lipatov's **gauge-invariant effective theory for MRK processes in QCD** [Lipatov 1995; Lipatov, Vyazovsky, 2001].

Some Feynman rules for **Reggeized gluons:**

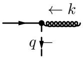
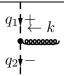
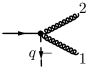
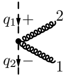
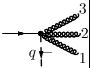
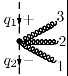
 $= \frac{-i\delta_{ab}}{2q^2}$	 $= (-iq^2) n^{\mp}_{\mu} \delta_{ab}$
	$g_s f_{aa_1a_2} \left(n^{\mp}_{\mu} n^{\mp}_{\nu}\right) \frac{q^2}{k_1^{\mp}}$
	$ig_s^2 \left(n^{\mp}_{\mu_1} n^{\mp}_{\mu_2} n^{\mp}_{\mu_3}\right) \frac{q^2}{k_3^{\mp}} \left[\frac{f_{aba_1} f_{ba_2a_3}}{k_1^{\mp}} + \frac{f_{aba_2} f_{ba_1a_3}}{k_2^{\mp}}\right]$





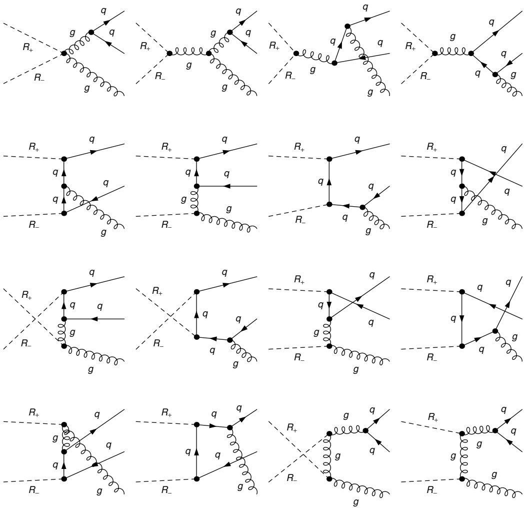
Gauge-invariant off-shell amplitudes

Some Feynman rules for Reggeized quarks:

	$ig_s T^a\left(\gamma_\mu + \hat q \frac{n^+_\mu}{k^+}\right)$		$ig_s T^a\left(\gamma_\mu + \hat q_2 \frac{n^+_\mu}{k^+} + \hat q_1 \frac{n^-_\mu}{k^-}\right)$
	$ig_s^2 (n^+_{\mu_1} n^+_{\mu_2})\hat q \left[\frac{T^{a_1}T^{a_2}}{k_1^+(k_1+k_2)^+} + \frac{T^{a_2}T^{a_1}}{k_2^+(k_1+k_2)^+}\right]$		$ig_s^2\Big[\hat q_2 (n^+_{\mu_1}n^+_{\mu_2})\left(\frac{T^{a_1}T^{a_2}}{k_1^+(k_1+k_2)^+}+\frac{T^{a_2}T^{a_1}}{k_2^+(k_1+k_2)^+}\right) - \hat q_1 (n^-_{\mu_1}n^-_{\mu_2})\left(\frac{T^{a_2}T^{a_1}}{k_1^-(k_1+k_2)^-}+\frac{T^{a_1}T^{a_2}}{k_2^-(k_1+k_2)^-}\right)\Big]$
	$ig_s^3 \hat q (n^+_{\mu_1} n^+_{\mu_2} n^+_{\mu_3})\left[\frac{T^{a_1}T^{a_2}T^{a_3}}{k_1^+(k_1+k_2)^+(k_1+k_2+k_3)^+} + (1\leftrightarrow 2 \leftrightarrow 3)\right]$		
	$ig_s^3\Big[\hat q_2 (n^+_{\mu_1}n^+_{\mu_2}n^+_{\mu_3})\left(\frac{T^{a_1}T^{a_2}T^{a_3}}{k_1^+(k_1+k_2)^+(k_1+k_2+k_3)^+} + (1\leftrightarrow 2\leftrightarrow 3)\right) + \hat q_1 (n^-_{\mu_1}n^-_{\mu_2}n^-_{\mu_3})\left(\frac{T^{a_3}T^{a_2}T^{a_1}}{k_1^-(k_1+k_2)^-(k_1+k_2+k_3)^-} + (1\leftrightarrow 2\leftrightarrow 3)\right)\Big]$		

Implementation in FeynArts.

The Feynman rules of Lipatov's EFT, up to the order $O(g_s^3,\ eg_s^2,\ e^2 g_s,\ e^3)$ are implemented in our model-file **ReggeQCD** for the package **FeynArts**.



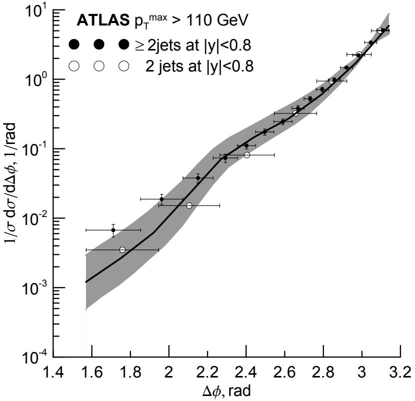
BCFW recursion and Lipatov's EFT

The approach to derive gauge-invariant scattering amplitudes with off-shell initial-state partons, using the spinor-helicity techniques and Britto-Cachazo- Feng-Witten-like recursion relations for such amplitudes, was introduced in Refs.:
1) *A. van Hameren and M. Serino*, BCFW recursion for TMD parton scattering, J. High Energy Phys. 07 (2015) 010; *K. Kutak, A. Hameren, and M. Serino*, QCD amplitudes with 2 initial spacelike legs via generalised BCFW recursion, J. High Energy Phys. 02 (2017) 009.
2) *A. van Hameren*, KaTie: For parton-level event generation with kT -dependent initial states, arXiv:1611.00680.
This formalism is equivalent to Lipatov's EFT at the tree level, but for some observables, e.g., related with heavy quarkonia, or for the generalization of the formalism to NLO, the explicit Feynman rules and the structure of EFT are more convenient.

Azimuthal decorrelations of dijets, diphotons and B-mesons

Dijet azimuthal decorrelations at the LHC

[M.A.Nefedov, V.A.Saleev, A.V.Shipilova, Phys. Rev. **D87**, 094030 (2013)].



Dijet production at the LHC ($\sqrt{S} = 7$ TeV). *Open points* – only 2 jets with $p_T > 30$ GeV at $|y| < 0.8$, *Closed points* – inclusive data.

The description in PRA is given by $2 \to 2$ subprocesses and is dominated by:

$$R_+ + R_- \to g + g.$$

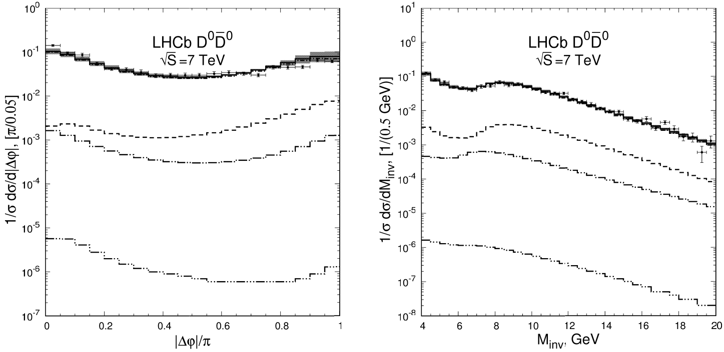
To reject the **jets from the last stage of unPDF evolution**, the cut $\max(|\mathbf{q}_{T1}|, |\mathbf{q}_{T2}|) < p_T^{\text{subleading jet}}$ is necessary.

D-meson pair production @ LHC

[A. V. Karpishkov, V. A. Saleev, A. V. Shipilova, Phys. Rev. **D 94**, 114012 (2016)]

Production of **charge-conjugated** states: mostly through

$$R_+ + R_- \to c + \bar{c}.$$

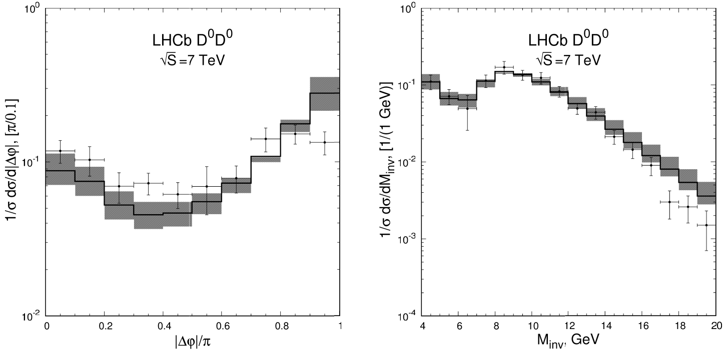


D-meson pair production @ LHC

[A. V. Karpishkov, V. A. Saleev, A. V. Shipilova, Phys. Rev. **D 94**, 114012 (2016)]

Production of **same-sign** pairs: through gluon fragmentation

$$R_+ + R_- \to g + g.$$



Diphoton production at Tevatron and the LHC

[M.A.Nefedov, V.A.Saleev, Phys. Rev. **D92**, 094033 (2015)].
The PRA calculation is at the NLO* level. $2 \to 2$ subprocesses:

$$Q_+ + \bar{Q}_- \to \gamma + \gamma, \qquad (1)$$
$$R_+ + R_- \to \gamma + \gamma, \qquad (2)$$

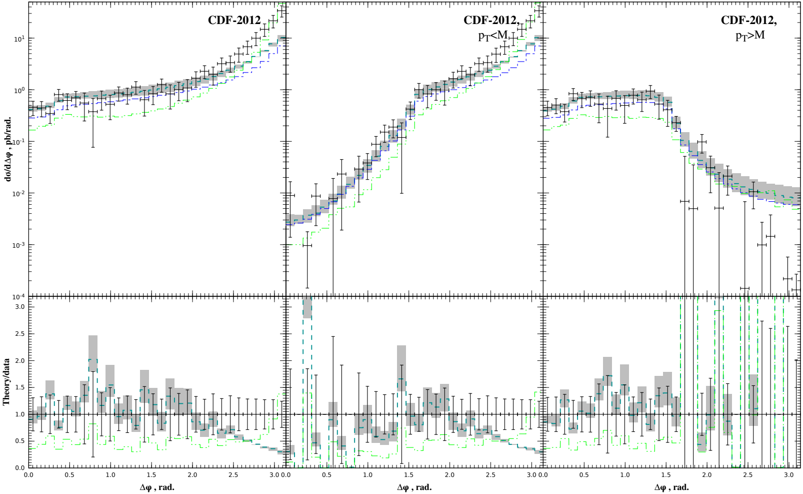
the subprocess (2) goes through **quark box**, and $t_{1,2}$-dependence of this amplitude has been calculated.
The NLO $2 \to 3$ subprocesses:

$$Q_+ + Q_- \to \gamma + \gamma + g, \qquad (3)$$
$$Q_\pm + R_\mp \to \gamma + \gamma + q, \qquad (4)$$

+**mMRK double-counting subtraction**, which practcally removes the contribution of the subprocess (3) and greatly reduces the contribution of the subprocess (4).

Diphoton production at Tevatron and the LHC

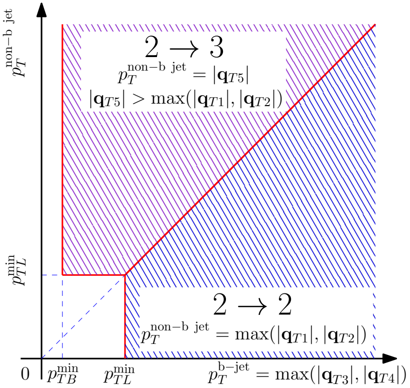
[M.A.Nefedov, V.A.Saleev, Phys. Rev. **D92**, 094033 (2015)].
Comparison with CDF data:



$B\bar{B}$-production with a jet @ LHC

[A.V.Karpishkov, M. A. Nefedov, V.A.Saleev, Phys. Rev. **D 96**, 096019 (2017)].

Complementarity of $2 \to 2$ and $2 \to 3$ contributions in the Phase-space:



Complicated kinematical situation!
The $B\bar{B}$-pair is searched in the events with a hard jet.
Our solution – "merging" of two contributions:

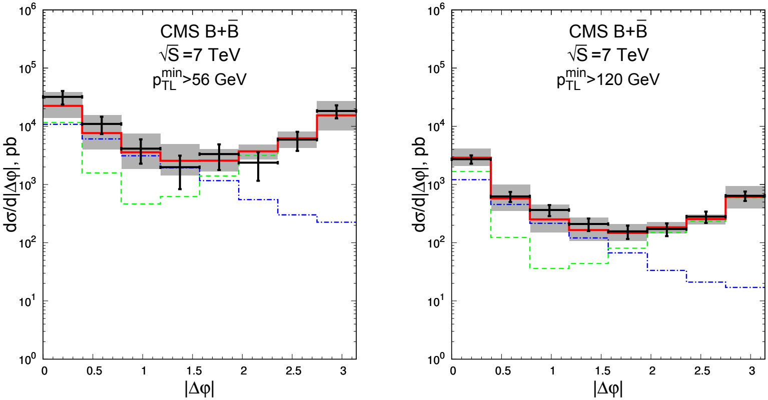
1. The hard jet $= b$-jet:

$$R_+(q_1) + R_-(q_2) \to \underset{\hookrightarrow B}{b(q_3)} + \underset{\hookrightarrow \bar{B}}{\bar{b}(q_4)},$$

2. The hard jet $\neq b$-jet:

$$R_+(q_1) + R_-(q_2) \to \underset{\hookrightarrow B}{b(q_3)} + \underset{\hookrightarrow \bar{B}}{\bar{b}(q_4)} + g(q_5),$$

Comparison with CMS data



Green line – $2 \to 2$, blue line – $2 \to 3$, orange line – sum. The dependence of **normalization and shape** of the $B\bar{B}$ azimuthal decorrelation spectrum on p_T of the **leading hard jet** is described!

Towards NLO calculations

Differences with NLO of CPM

- Real NLO corrections: **No collinear divergences** when integrating over $\mathbf{k}_T$ of additional emitted parton down to 0, unlike in CPM. Initial-state collinear divergences are regularized by transverse momentum of **initial-state** partons.
- Instead, the **double-counting** of region $\mathbf{k}_T \to 0$ or $\Delta y \to \infty$ should be subtracted from NLO contribution to the hard-scattering coefficient.
- Vertices of Lipatov's theory are **nonlocal**: contain *eikonal denominators* – $1/k^{\pm}$ $\Rightarrow$ **Rapidity divergences in real and virtual corrections!**
- Rapidity divergences are related with BFKL resummation of contributions $\sim \log 1/x$. So some elements of BFKL resummation necessarily enter at NLO.

Physical normalization condition

- **Motivation** for PRA is the **multi-scale** correlational observables. $\Rightarrow$ NLO CPM accuracy for **single-scale** observables is enough.
- We **DO NOT** want to do our own fit of unPDFs $\Rightarrow$ using ($\overline{MS}$) PDFs of CPM as collinear input. But NLO PDFs are **scheme-dependent!**
- For the single-scale observables (e.g. $F_{2/L}(x,Q^2)$) collinear factorization **is a theorem** (up to corrections $\sim (\Lambda^2_{QCD}/Q^2)^{\#}$).

$\Rightarrow$ **Physical normalization condition** at NLO:

$$f_i^{\overline{MS}}(x,\mu^2)$$
$$\swarrow \qquad\qquad \searrow$$
$$F_{2q}^{(NLO\ \mathbf{PRA})}(x,Q^2) \quad = \quad F_{2q}^{(NLO\ \mathbf{CPM})}(x,Q^2)$$
$$+O(\alpha_s^2(Q^2)) + \text{Higher twist},$$

Basic idea: Collect all terms in PRA, contributing $O(\alpha_s)$ to the normalization condition and **match** them to the NLO CPM result.

Simplest example: gluon contribution to F_2 @ NLO of PRA

[M. A. Nefedov, V. A. Saleev, hep-ph/1709.06378]

The F_2 structure function at LO ($x_B = \frac{Q^2}{2qP}$, $q_1^+ = x_1 P^+$):

$$F_2(x_B, Q^2) = \sum_j \int\limits_0^1 \frac{dx_1}{x_1} \int dt_1 \; \Phi_j(x_1, t_1, Q^2) \times C_{2j}^{(LO)} \left(z = \frac{x_B}{x_1}, \frac{t_1}{Q^2} \right)$$

where $\boxed{C_{2q}^{(LO)}\left(z, \frac{t_1}{Q^2}\right) = e_q^2 \cdot z\delta\left(\frac{(Q^2+t_1)z}{Q^2} - 1\right)}$ is calculated with the use of gauge-invariant Fadin-Sherman γQq-vertex [Fadin, Sherman, 1977]:

$$\Gamma^\mu(q_1, k) = \gamma^\mu + \hat{q}_1 \frac{n_-^\mu}{k^-}.$$

The $O(\alpha_s)$ contributions comes from:

1. t_1-dependence of **LO** contribution,
2. **NLO** subprocess
$$R_+(q_1) + \gamma^*(q) \to q + \bar{q},$$
3. Double-counting subtraction term

Role of the $\mathbf{q}_{T1}$-dependence

[M. A. Nefedov, V. A. Saleev, hep-ph/1709.06378]

$$F_2^{(LO)}(x_B,Q^2) = (e_q^2 x_B)\left[\underbrace{f_q(x_B,Q^2)}_{\text{LO CPM}} + \underbrace{\Delta F_2^{(T)}(x_B,Q^2) + \Delta F_2^{(f)}(x_B,Q^2)}_{O(\alpha_s)} + \text{pow. suppr.}\right],$$

where:

$$\Delta F_2^{(T)}(x_B,Q^2) = \int\limits_0^1 dx_1\ f_j(x_1,Q^2) \int\limits_0^{Q^2} dt_1 \underbrace{\left[1 - T_j(t_1,Q^2,x_1)\right]}_{O(\alpha_s)} \frac{\partial}{\partial t_1} C_j^{(LO)}\left(\frac{x_B}{x_1},\frac{t_1}{Q^2}\right)$$

$$\Delta F_2^{(f)}(x_B,Q^2) = \int\limits_0^1 dx_1 \int\limits_0^{Q^2} dt_1\ T_j(t_1,Q^2,x_1) \underbrace{\left[f_j(x_1,Q^2) - f_j(x_1,t_1)\right]}_{O(\alpha_s)} \frac{\partial}{\partial t_1} C_j^{(LO)}\left(\frac{x_B}{x_1},\frac{t_1}{Q^2}\right)$$

$O(\alpha_s)$-term from $\mathbf{q}_T$-dependence

[M. A. Nefedov, V. A. Saleev, hep-ph/1709.06378]
One obtains:

$$\Delta F_2^{(f_g)} = \left(\frac{\alpha_s(Q^2)}{2\pi}\right) \int\limits_0^1 \frac{dx_1}{x_1}\, f_g(x_1, Q^2) \cdot \Delta C_{qg}^{(f_g)}\left(\frac{x_B}{x_1}\right)$$

where:

$$\boxed{\Delta C_{qg}^{(f_g)}(z) = T_R\left[\xi z\left((4+\xi)z-2\right) + \left(1-2z+2z^2\right)\log\xi\right],}$$

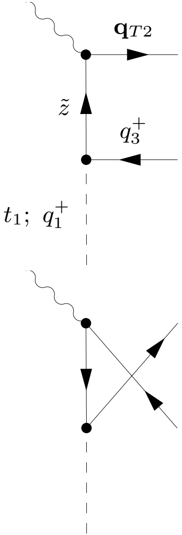
where $T_R = 1/2$ and $\xi = \min\left(1, (1-z)/z\right)$.
Other corrections:

$$\Delta F_2^{(f_q)},\ \Delta F_2^{(T)}$$

contain only $f_q(x_1, Q^2)$, and will be important for the $\gamma^\star + Q \to q + g$ subprocess and one-loop correction.

NLO subprocess $\gamma^\star + R \to q + \bar{q}$.

[M. A. Nefedov, V. A. Saleev, hep-ph/1709.06378]



Kinematics ($\tilde{z} = (q_1^+ - q_3^+)/q_1^+$,
ψ – azimuthal angle of $\mathbf{k}_T$):

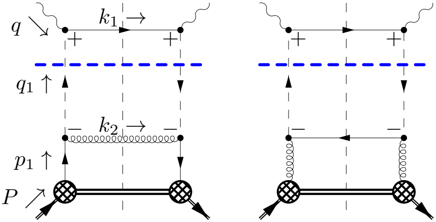
$$\mathbf{q}_{T2} = \mathbf{k}_T + \mathbf{q}_{T1}\frac{\tilde{z} - z}{1 - z},$$

$$\mathbf{k}_T^2 = \frac{(\tilde{z} - z)(1 - \tilde{z})}{1 - z}\left[\frac{Q^2}{z} - \frac{t_1}{1 - z}\right],$$

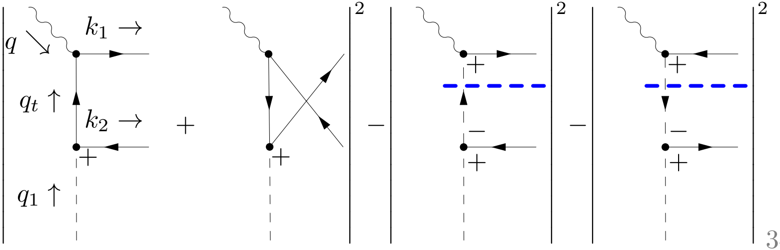
$$F_2^{(NLO,g)} = e_q^2 \cdot \frac{\alpha_s}{2\pi} \int \frac{dx_1}{x_1} \int dt_1\ \Phi_g(x_1, t_1, \mu^2) \times \underbrace{\int\limits_z^1 \frac{z \cdot d\tilde{z}}{(1 - z)} \int\limits_0^{2\pi} \frac{d\psi}{2\pi}\ \mathcal{C}_2^{(NLO,g)}}_{C_2^{(NLO,g)}\left(z = \frac{x_B}{x_1}, \frac{t_1}{Q^2}\right)}$$

Double-counting subtraction

[M. A. Nefedov, V. A. Saleev, hep-ph/1709.06378]
LO contains (horizontal blue line denotes mMRK approximation):



⇒



Physical normalization condition

[M. A. Nefedov, V. A. Saleev, hep-ph/1709.06378]
Physical normalization, $O(\alpha_s)$ gluon contribution only:

$$F_{2q}^{NLO\ \mathbf{CPM}}(x_B, Q^2) = (e_q^2 x_B)\left[f_q^{\overline{MS}}(x_B, Q^2) + \left(\frac{\alpha_s(Q^2)}{2\pi}\right) f_g^{\overline{MS}} \otimes C_{2g}^{\overline{MS}} + \text{c.c.}\right]$$

$$\|$$

$$F_{2q}^{NLO\ \mathbf{PRA}}(x_B, Q^2) = (e_q^2 x_B)\left[f_q^{PRA}(x_B, Q^2) + \left(\frac{\alpha_s(Q^2)}{2\pi}\right) f_g^{PRA} \otimes \Delta C_{qg}^{(f_g)}\right.$$

$$\left. + \left(\frac{\alpha_s(Q^2)}{2\pi}\right) \underbrace{\Phi_g^{PRA} \otimes \left(C_2^{(NLO,g)} - \Delta C_2^{(t+u,g)}\right)}_{f_g^{PRA}\otimes\left(C_{2g}^{\overline{MS}} - \Delta C_{qg}\right) + O(\alpha_s^2) + \text{Higher twist}} + \text{c.c.}\right].$$

PDFs in the **PRA scheme**:

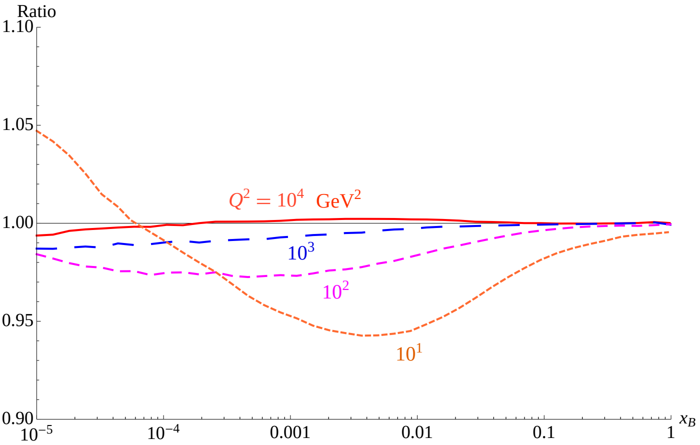
$$f_g^{PRA} = f_g^{\overline{MS}} + O(\alpha_s^2), \quad f_q^{PRA} = f_q^{\overline{MS}} + \left(\frac{\alpha_s}{2\pi}\right) f_g^{\overline{MS}} \otimes \left(\Delta C_{qg} - \Delta C_{qg}^{(f_g)}\right) + O(\alpha_s^2),$$

where **matching coefficient** ΔC_{qg} can be computed analytically $\Rightarrow$ unPDFs in PRA scheme:

$$\int_0^{\mu^2} dt\ \Phi_i^{PRA}(x, t, \mu^2) = x f_i^{PRA}(x, \mu^2),$$

Numerical test of Physical Normalization Condition

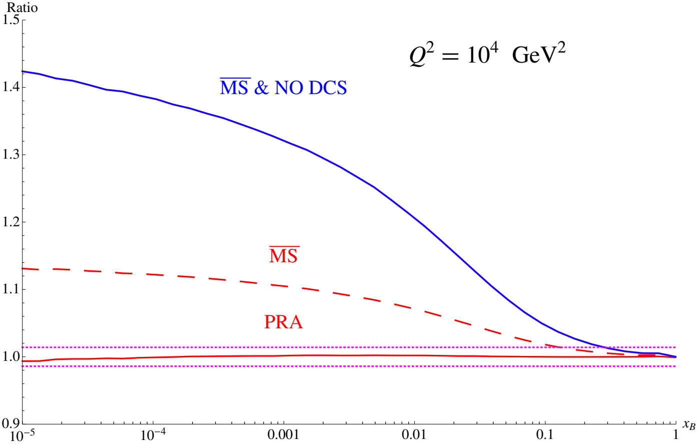
[M. A. Nefedov, V. A. Saleev, hep-ph/1709.06378]
Plot of the ratio – $F_{2q}^{NLO\ \mathbf{PRA}}(x_B, Q^2)/F_{2q}^{NLO\ \mathbf{CPM}}(x_B, Q^2)$:



Numerical test of Physical Normalization Condition

[M. A. Nefedov, V. A. Saleev, hep-ph/1709.06378]

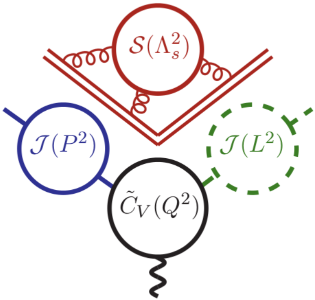
Plot of the ratio – $F_{2q}^{NLO\ \mathbf{PRA}}(x_B, Q^2)/F_{2q}^{NLO\ \mathbf{CPM}}(x_B, Q^2)$:



Magenta lines $= 1 \pm \alpha_s^2(Q^2)$

Virtual corrections in PRA

What such EFTs as SCET or Lipatov's theory are (needed for)?



Picture is taken from [hep-ph/1410.1892].

- EFT = formalism which **explicitly implements** certain factorization properties of QCD amplitudes:
  - Factorization in the **soft** and **collinear** limits – *Soft-Collinear Effective Theory (SCET).*
  - Factorization in the **Multi-Regge limit** – *Lipatov's theory.*
- As a result of kinematic approximations, **artificial logarithmic divergences** arise in different factors, but they should cancel in order-by-order in PT.
- Factorization + cancellation of artificial divergences ⇒ **Renormalization group**. The latter allows to resum **large logarithms** of scale ratios.

Rapidity divergences and regularization.

Due to the presence of the $1/q^{\pm}$-factors in the induced vertices, the loop integrals in EFT contain the light-cone (Rapidity) divergences:

$$\hat{\Sigma}_1 = q \downarrow \quad p \downarrow$$

$$= g_s^2 C_F \int \frac{d^D q}{(2\pi)^D} \left(\gamma_\mu - \hat{p} \frac{n_\mu^+}{q^+} \right) \frac{\hat{p} - \hat{q}}{q^2 (p-q)^2} \left(\gamma_\mu - \hat{p} \frac{n_\mu^-}{q^-} \right)$$

The regularization by explicit cutoff in rapidity was proposed by Lipatov [Lipatov, 1995] ($q^{\pm} = \sqrt{q^2 + \mathbf{q}_T^2} e^{\pm y}$):

$$\int \frac{dq^+ dq^-}{q^+ q^-} = \int_{y_1}^{y_2} dy \int \frac{dq^2}{q^2 + \mathbf{q}_T^2},$$

then

$$\hat{\Sigma}_1 = C_F g_s^2 \hat{p} \int \frac{\mathbf{p}_T^2 d^{D-2} \mathbf{q}_T}{\mathbf{q}_T^2 (\mathbf{p}_T - \mathbf{q}_T)^2} (y_2 - y_1) + \text{finite terms}$$

The dependence on the regulators y_i have to cancel between the contributions of the neighbouring regions order-by-order in α_s, building up the $\frac{1}{n!} (\log(s) \omega(t))^n$-terms.

Covariant regularization.

The regularization and **pole prescription** was introduced in a series of papers [Hentschinski, Vera, Chachamis *et. al.*, 2012-2013].
The regularization of the light-cone divergences is achieved by the shifting the $n^{\pm}$ vectors from the light-cone:

$$\boxed{\tilde{n}^{\pm} = n^{\pm} + e^{-\rho} n^{\mp},\ \tilde{k}^{\pm} = k^{\pm} + e^{-\rho} k^{\mp},\ \rho \to +\infty,}$$

and for the lowest-order(Rgg, Qqg) induced vertices the PV prescription is at work:

$$\frac{1}{[\tilde{k}^{\pm}]} = \frac{1}{2}\left(\frac{1}{\tilde{k}^{\pm} + i\varepsilon} + \frac{1}{\tilde{k}^{\pm} - i\varepsilon} \right).$$

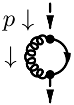
For the higher-order induced vertices, there is the nontrivial interplay between **color and kinematics** in the pole prescription.

Recently, this prescriptions has been tested at one loop for the case of **Reggeized quarks**: [M. A. Nefedov, V. A. Saleev, Mod. Phys. Lett. A, 32 1750207 (2017)].

The Reggeized quark self-energy.

[M. A. Nefedov, V. A. Saleev, Mod. Phys. Lett. A, 32 1750207 (2017)]
The self-energy graph is computed in the central kinematics ($p^+ = p^- = 0$):

$$\hat{\Sigma}_1 = q \downarrow$$



$$= g_s^2 C_F \int \frac{d^D q}{(2\pi)^D} \left(\gamma_\mu - \hat{p} \frac{n_\mu^+}{[\tilde{q}^+]} \right) \frac{\hat{p} - \hat{q}}{q^2 (p-q)^2} \left(\gamma_\mu - \hat{p} \frac{n_\mu^-}{[\tilde{q}^-]} \right),$$

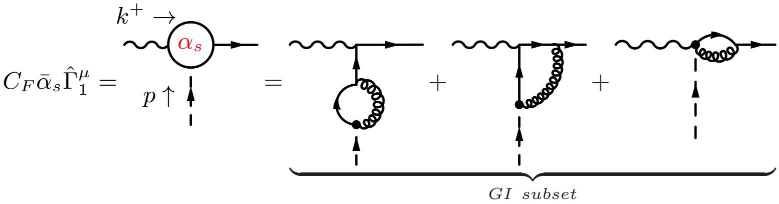
the result is:

$$\hat{\Sigma}_1 = (i\hat{p}) \frac{C_F \bar{\alpha}_s}{4\pi} \left(\frac{\mu^2}{t_1} \right)^\epsilon \left[\frac{-i\pi + 2\rho}{\epsilon} + \left(\frac{1+\epsilon}{1-2\epsilon} \right) \frac{1}{\epsilon} \right],$$

where $\bar{\alpha}_s = (4\pi)^\epsilon r_\Gamma \alpha_s$.

The unsubtracted γQq-scattering vertex.

[M. A. Nefedov, V. A. Saleev, Mod. Phys. Lett. A, 32 1750207 (2017)]



where $k^2 = 0$, $p^2 = -t_1$, $(k+q)^2 = 0$.
The e^{ρ}-divergences cancel in the sum of diagrams. The terms proportional $e^{-\epsilon\rho} \rightarrow 0$ for $\epsilon > 0 \Rightarrow$ only **logarithmic singularities** $\sim \rho$ are left.

The unsubtracted γQq-scattering vertex.

[M. A. Nefedov, V. A. Saleev, Mod. Phys. Lett. A, 32 1750207 (2017)]
The one-loop result can be expressed in terms of two gauge-invariant Lorentz structures:

$$\hat{\Gamma}_0^\mu = ee_q \bar{u}(p+k)\left(\gamma^\mu + \hat{p}\frac{n_+^\mu}{k^+}\right)\hat{n}^+,\ \hat{\Delta}_0^\mu = ee_q\left(p^\mu - \frac{t_1}{2k^+}n_+^\mu\right)\left(\bar{u}(p+k)\hat{k}\hat{n}^+\right)$$

The result:

$$\hat{\Gamma}_1^\mu = \frac{2}{t_1}\hat{\Delta}_0^\mu + \hat{\Gamma}_0^\mu\left[\underbrace{-\frac{1}{\epsilon^2} - \frac{L_1}{\epsilon}}_{IR\ part}\underbrace{+(\rho - i\pi)\left(\frac{1}{\epsilon}+L_1\right) + \frac{2L_2}{\epsilon}}_{High-Energy\ part} - \underbrace{-\left(\frac{1}{\epsilon}+L_1+3\right)}_{UV\ part} + 2L_1L_2 - \frac{L_1^2}{2} + \frac{\pi^2}{2}\right] + O(\epsilon),$$

where $L_1 = \log\left(\frac{\mu^2}{t_1}\right)$, $L_2 = \log\left(\frac{k^+}{\sqrt{t_1}}\right)$.

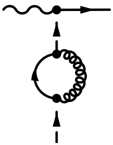
Subtracted γQq-scattering vertex.

The subtracted result:

$$\hat{\Gamma}^{\mu}_{1S} = \hat{\Gamma}^{\mu}_{1} - \delta\hat{\Gamma}^{\mu}_{1} = \frac{2}{t_1}\hat{\Delta}^{\mu}_{0} + \hat{\Gamma}^{\mu}\left[-\frac{1}{\epsilon^2} - \frac{L_1}{\epsilon} - \rho\left(\frac{1}{\epsilon} + L_1\right) + \frac{2L_2}{\epsilon} - \right.$$
$$\left. -2\left(\frac{1}{\epsilon} + L_1 + 3\right) + 2L_1L_2 - \frac{L_1^2}{2} + \frac{\pi^2}{2}\right] + O(\epsilon).$$

The subtraction term:

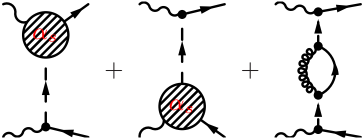
$$C_F\bar{\alpha}_s\delta\hat{\Gamma}^{\mu}_{1} =$$



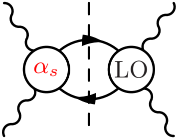
$$= \hat{\Gamma}^{\mu}_{0}\left[(2\rho - i\pi)\left(\frac{1}{\epsilon} + L_1\right) + \left(\frac{1}{\epsilon} + L_1 + 3\right)\right] + O(\epsilon).$$

The cross-check.

[M. A. Nefedov, V. A. Saleev, Mod. Phys. Lett. A, 32 1750207 (2017)]
The terms $\sim \rho$ cancel in the sum of graphs $(-\rho - \rho + 2\rho)$, as it was for gluons!



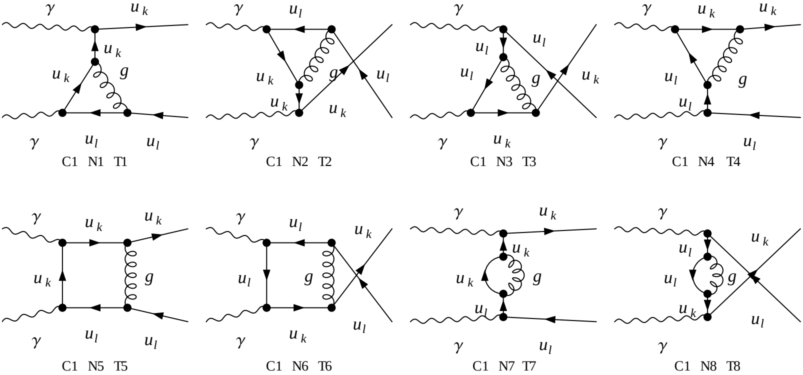
The result is **the MRK asymptotics** ($s \to \infty$, t-fixed) for the $\gamma\gamma \to q\bar{q}$-amplitude at $O(\alpha_s)$. To compare the EFT prediction with QCD result, let's compute:



$$= \left(8N_c(ee_q)^4\right) \frac{C_F \bar{\alpha}_s}{4\pi} \left[\frac{1}{\tau} C_{HE}^{(-1)} + C_{HE}^{(0)} + O(\tau)\right],$$

where $\tau = -t/s$. **The EFT predicts** $C_{HE}^{(-1)}$**.**

$\gamma \quad \gamma \quad \to \quad u_k \quad u_l$



The Regge limit of the $\gamma\gamma \to q\bar{q}$- amplitude.

[M. A. Nefedov, V. A. Saleev, Mod. Phys. Lett. A, 32 1750207 (2017)]
Both QCD(**FeynArts**+**FeynCalc**) and EFT agree on the result for $C_{HE}^{(-1)}$:

$$\begin{aligned} \mathrm{Re}\, C_{HE}^{(-1)} &= -\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\log\frac{\mu^2}{(-t)} + \left(1 + 2\log\frac{1}{\tau}\right)\left(\frac{1}{\epsilon} + \log\frac{\mu^2}{(-t)}\right) \\ &- \left[3 - \pi^2 + \log^2\frac{\mu^2}{(-t)} + 4\log\frac{1}{\tau}\right], \\ \mathrm{Im}\, C_{HE}^{(-1)} &= -\pi\left(\frac{1}{\epsilon} + \log\frac{\mu^2}{(-t)} - 2\right). \end{aligned}$$

Fermionic Glauber Operators and Quark Reggeization

Ian Moult, Mikhail P. Solon, Iain W. Stewart, and Gherardo Vita. **Fermionic Glauber Operators and Quark Reggeization**, JHEP 1802 (2018) 134.

Note added: As this paper was being finalized, Ref. [Nefedov, Saleev, 2017] appeared, which studies $\gamma\gamma \to q + \bar{q}$ amplitudes at one-loop in the Regge limit by constructing the quark Reggeization terms in the effective action formalism of Lipatov [Lipatov, 1995]. In the SCET language this corresponds to formulating an auxiliary field Lagrangian for the offshell Glauber quarks, while using the full QCD Lagrangian for other fields (without defining EFT fields for the n-collinear, soft and n^{-}-collinear sectors). Since having distinct fields for these sectors enables their factorization properties to be easily determined and studied, such as in our BFKL calculation, we believe there are certain advantages to our approach. It would be interesting to make a more explicit comparison between these formalisms.

Conclusions

- Parton Reggeization Approach has been shown to be a reliable phenomenological tool at LO
- The extension to NLO is possible and is under active development

Thank you for your attention!