No stable static spherically symmetric wormholes in Horndeski theory

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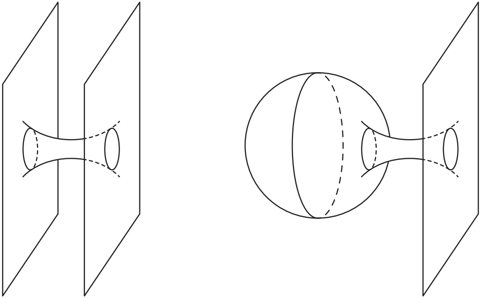
Outline

Wormholes and semiclosed worlds



Penrose theorem

- Noncompactness of the Cauchy hypersurface.
- Null Energy Condition (NEC): $G_{\mu\nu}\eta^\mu\eta^\nu \geq 0$ for any null vector η^μ, $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$; for minimal coupling to gravity: $T_{\mu\nu}\eta^\mu\eta^\nu \geq 0$.
- Trapped surface $\Rightarrow$ Singularity.

Galileons and Horndeski

«Galileon»

A. Nicolis, R. Rattazzi and E. Trincherini, Phys. Rev. D **79**, 064036 (2009) doi:10.1103/PhysRevD.79.064036, arXiv:0811.2197 [hep-th].

- Invariance under $\partial^\mu \pi \to \partial^\mu \pi + b^\mu$, where b^μ is a constant vector.
- Second order field equations despite the second order derivatives in the Lagrangian.

First article

G. W. Horndeski, Int. J. Theor. Phys. **10**, 363 (1974) doi:10.1007/BF01807638.

Horndesky theory

$$S = \int d^4x \left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right)$$

$$\begin{aligned}
\mathcal{L}_2 =& K(\phi, X),\\
\mathcal{L}_3 =& - G_3(\phi, X)\Box\phi,\\
\mathcal{L}_4 =& G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],\\
\mathcal{L}_5 =& G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi\\
&- \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right].
\end{aligned}$$

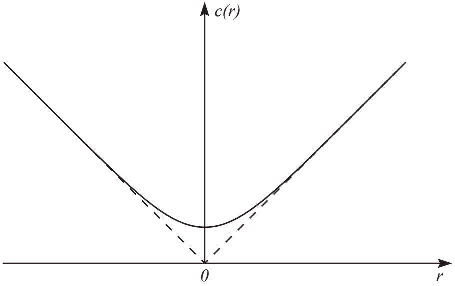
- $X = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$, $\Box\phi = \nabla_\mu\nabla^\mu\phi$;
- R – scalar curvature;
- $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ – Einstein tensor;
- metric signature $(-, +, +, +)$.

Metric

General form

$$ds^2 = -a^2(r)dt^2 + \frac{dr^2}{b^2(r)} + c^2(r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

- Gauge $a(r) = b(r)$.
- Asymptotic behaviour: $a(r) \to 1$, $c(r) \to |r|$ as $r \to \pm\infty$.
- $c(r)$ behaviour:



Spherical harmonics

$$s(t,r,\theta,\varphi)=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}s_{lm}(t,r)Y_l^m(\theta,\varphi),$$

$$V_a(t,r,\theta,\varphi)=\overset{\gamma}{\nabla}_a\Phi_1(t,r,\theta,\varphi)+E_a^b\overset{\gamma}{\nabla}_b\Phi_2(t,r,\theta,\varphi),$$

$$T_{ab}(t,r,\theta,\varphi)=\overset{\gamma}{\nabla}_a\overset{\gamma}{\nabla}_b\Psi_1+\gamma_{ab}\Psi_2(t,r,\theta,\varphi)$$
$$+\frac{1}{2}\left(E_a^c\overset{\gamma}{\nabla}_c\overset{\gamma}{\nabla}_b\Psi_3(t,r,\theta,\varphi)+E_b^c\overset{\gamma}{\nabla}_c\overset{\gamma}{\nabla}_a\Psi_3(t,r,\theta,\varphi)\right).$$

- γ_{ab} is S^2 metric;
- $\overset{\gamma}{\nabla}_a$ is covariant derivative for γ_{ab};
- $E_{ab}=\sqrt{\det\gamma}\,\varepsilon_{ab}$, ε_{ab} – Levi-Civita symbol, $\varepsilon_{\theta\varphi}=1$;
- Φ_1, Φ_2, Ψ_1, Ψ_2, Ψ_3 are scalar functions;
- $Y_l^m(\theta,\varphi)$ are spherical harmonics.

Perturbations (terms with E_{ab})

$$\delta\phi = 0, \qquad h_{tt} = 0, \qquad h_{tr} = 0, \qquad h_{rr} = 0,$$

$$h_{ta} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} h_{0,lm}(t,r) E_{ab} \partial^b Y_l^m(\theta,\varphi),$$

$$h_{ra} = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} h_{1,lm}(t,r) E_{ab} \partial^b Y_l^m(\theta,\varphi),$$

$$h_{ab} = \frac{1}{2} \sum_{l=2}^{\infty} \sum_{m=-l}^{l} h_{2,lm}(t,r) \left[E_a^c \overset{\gamma}{\nabla}_c \overset{\gamma}{\nabla}_b Y_l^m(\theta,\varphi) + E_b^c \overset{\gamma}{\nabla}_c \overset{\gamma}{\nabla}_a Y_l^m(\theta,\varphi) \right].$$

Interval covariance under $x^\mu \to x^\mu + \xi^\mu$

$$\xi_t = 0, \qquad \xi_r = 0, \qquad \xi_a = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \Lambda_{lm}(t,r) E_{ab} \partial^b Y_l^m(\theta,\varphi).$$

Interval covariance under $x^\mu \to x^\mu + \xi^\mu$

$$h_{\mu\nu} \to h_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

- $\xi_t = 0, \qquad \xi_r = 0, \qquad \xi_a = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \Lambda_{lm}(t,r) E_{ab} \partial^b Y_l^m(\theta,\varphi);$
- ∇_μ is the covariant derivative for $g_{\mu\nu}$.

$$h_{0,lm} \to h_{0,lm} + \dot{\Lambda}_{lm}(t,r),$$
$$h_{1,lm} \to h_{1,lm} + \Lambda'_{lm}(t,r) + 2\frac{c'}{c}\Lambda_{lm}(t,r),$$
$$h_{2,lm} \to h_{2,lm} + 2\Lambda_{lm}(t,r).$$

Regge-Wheeler approach with $l \geq 2$

- $h_{2,lm} = 0$;
- $m = 0$ without loss of generality.

Lagrangian for perturbations

$$\mathcal{L}^{(2)} = \frac{l(l+1)}{4(l-1)(l+2)} \left[\mathcal{A}\dot{q}^2 - \mathcal{B}q'^2 - l(l+1)\mathcal{C}q^2 - V(r)q^2 \right],$$

$$\text{where} \qquad \mathcal{A} = \frac{c^2}{a^2}\frac{\mathcal{H}^2}{\mathcal{G}}, \qquad \mathcal{B} = a^2 c^2 \frac{\mathcal{H}^2}{\mathcal{F}}, \qquad \mathcal{C} = a^2 \mathcal{H},$$

- $V(r)$ – effective potential;
- $\mathcal{C}$-part – wave propagation along the angular direction.

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Stability conditions

T. Kobayashi, H. Motohashi and T. Suyama,
Phys. Rev. D **85**, 084025 (2012); **96**, 109903(E) (2017),
doi:10.1103/PhysRevD.96.109903, doi:10.1103/PhysRevD.85.084025,
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Stability conditions

- $\mathcal{F} = 2\left(G_4 + \frac{a^2}{2}\phi' X' G_{5X} - XG_{5\phi}\right) > 0$ – radial direction,

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- $\mathcal{H} = 2\left[G_4 - 2XG_{4X} + X\left(a^2\frac{c'}{c}\phi' G_{5X} + G_{5\phi}\right)\right] > 0$ – angular dir.

Lagrangian for perturbations

$$\mathcal{L}^{(2)} = \frac{l(l+1)}{4(l-1)(l+2)}\left[\mathcal{A}\dot{q}^2 - \mathcal{B}q'^2 - l(l+1)\mathcal{C}q^2 - V(r)q^2\right],$$

$$\text{where} \qquad \mathcal{A} = \frac{c^2}{a^2}\frac{\mathcal{H}^2}{\mathcal{G}}, \qquad \mathcal{B} = a^2c^2\frac{\mathcal{H}^2}{\mathcal{F}}, \qquad \mathcal{C} = a^2\mathcal{H},$$

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$$\delta\phi = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \delta\phi_{lm}(t,r) Y_l^m(\theta,\varphi), \;\; h_{tt} = a^2 \sum_{l=0}^{\infty}\sum_{m=-l}^{l} H_{0,lm}(t,r) Y_l^m(\theta,\varphi),$$

$$h_{tr} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} H_{1,lm}(t,r) Y_l^m(\theta,\varphi), h_{rr} = \frac{1}{a^2} \sum_{l=0}^{\infty}\sum_{m=-l}^{l} H_{2,lm}(t,r) Y_l^m(\theta,\varphi),$$

$$h_{ta} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \beta_{lm}\partial_a Y_l^m(\theta,\varphi), \qquad h_{ra} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \alpha_{lm}\partial_a Y_l^m(\theta,\varphi),$$

$$h_{ab} = c^2 \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \Big[K_{lm}(t,r)\gamma_{ab} Y_l^m(\theta,\varphi) + G_{lm}(t,r)\overset{\gamma}{\nabla}_a \overset{\gamma}{\nabla}_b Y_l^m(\theta,\varphi)\Big].$$

Interval covariance under $x^\mu \to x^\mu + \xi^\mu$

$$h_{\mu\nu} \to h_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

- ∇_μ is the covariant derivative for $g_{\mu\nu}$.

Gauge functions

$$\xi_0 = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} T_{lm}(t,r) Y_l^m(\theta,\varphi),$$

$$\xi_r = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} R_{lm}(t,r) Y_l^m(\theta,\varphi),$$

$$\xi_a = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \Theta_{lm}(t,r) \partial_a Y_l^m(\theta,\varphi).$$

Gauge transformations of perturbations

$$H_{0,lm}(t,r) \to H_{0,lm}(t,r) + \frac{2}{a^2}\dot{T}_{lm}(t,r) - 2\frac{a'}{a}b^2 R_{lm}(t,r),$$

$$H_{1,lm}(t,r) \to H_{1,lm}(t,r) + \dot{R}(t,r) + T'(t,r) - 2\frac{a'}{a}T(t,r),$$

$$H_{2,lm}(t,r) \to H_{2,lm}(t,r) + 2b^2 R'_{lm}(t,r) - 2bb' R_{lm}(t,r),$$

$$\beta_{lm}(t,r) \to \beta_{lm}(t,r) + T_{lm}(t,r) + \dot{\Theta}_{lm}(t,r),$$

$$\alpha_{lm}(t,r) \to \alpha_{lm}(t,r) + R_{lm}(t,r) + \Theta'_{lm}(t,r) - 2\frac{c'}{c}\Theta_{lm}(t,r),$$

$$K_{lm}(t,r) \to K_{lm}(t,r) + 2b^2\frac{c'}{c}R_{lm}(t,r),$$

$$G_{lm}(t,r) \to G_{lm}(t,r) + \frac{2}{c^2}\Theta_{lm}(t,r).$$

- Gauge fixing: $\beta(t,r)=0$, $K(t,r)=0$ and $G(t,r)=0$.

Lagrangian for perturbations

$$\mathcal{L} = \frac{1}{2}\mathcal{K}_{ij}\dot{v}^i\dot{v}^j - \frac{1}{2}\mathcal{G}_{ij}v^{i\prime}v^{j\prime} - \frac{1}{2}\mathcal{Q}_{ij}v^i v^{j\prime} - \frac{1}{2}\mathcal{M}_{ij}v^i v^j,$$

- $i = \overline{1,2},\ j = \overline{1,2};$
- $v^1 \equiv \psi,\ v^2 \equiv \delta\phi;$
- $H_0 = -\dfrac{2}{2cc'\mathcal{H} + \Xi\phi'}\left(\dfrac{1}{a^2}\psi - c'\Xi\,\delta\phi' - l(l+1)c'\mathcal{H}\,\alpha\right);$
- $$\Xi = 2c^2\Big[-XG_{3X} + 2a^2\frac{c'}{c}\phi'\left\{G_{4X} + 2XG_{4XX} - (XG_{5\phi})_X\right\} + G_{4\phi} + 2XG_{4\phi X} - \frac{1}{c^2}XG_{5X} + a^2\frac{c'^2}{c^2}(3XG_{5X} + 2X^2G_{5XX})\Big].$$

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Stability conditions

T. Kobayashi, H. Motohashi and T. Suyama, Phys. Rev. D **89**, 084042 (2014) doi:10.1103/PhysRevD.89.084042 arXiv:1402.6740 [gr-qc]

- $\mathcal{K}_{11} > 0$, $\det\mathcal{K} > 0$ – no ghosts.

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Stability condition $\det \mathcal{K} > 0$

$$\det \mathcal{K} = \frac{4(l-1)(l+2)\left(2cc'\mathcal{H} + \Xi\phi'\right)^2 \mathcal{F}\left(2\mathcal{P}_1 - \mathcal{F}\right)}{l(l+1)a^4\mathcal{H}^2\phi'^2\left(2cc'\mathcal{H} + \Xi\phi'\right)^2} > 0,$$

$$\text{where } \mathcal{P}_1 = \frac{(2cc'\mathcal{H} + \Xi\phi')}{2c^2\mathcal{H}^2} \cdot \frac{d}{dr}\left[\frac{c^4\mathcal{H}^4}{\left(2cc'\mathcal{H} + \Xi\phi'\right)^2}\right].$$

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$$\boxed{2\mathcal{P}_1 - \mathcal{F} > 0}$$

No-go theorem

Stability conditions

- odd sector: $\mathcal{F} > 0$, $\mathcal{H} > 0$;
- even sector: $2\mathcal{P}_1 - \mathcal{F} > 0$.

$$Q = \frac{2cc'\mathcal{H} + \Xi\phi'}{c^2\mathcal{H}^2} \;\Rightarrow\; 2\mathcal{P}_1 - \mathcal{F} = -2\frac{Q'}{Q^2} - \mathcal{F} > 0,$$

$$\frac{Q'}{Q^2} < -\frac{1}{2}\mathcal{F} \xrightarrow{\int \text{ from } r \text{ to } r' > r} Q^{-1}(r) - Q^{-1}(r') < -\frac{1}{2}\int_r^{r'} \mathcal{F}\, dr.$$

No-go theorem

- $Q^{-1}(r) < 0$ at some r:

$$Q^{-1}(r') > Q^{-1}(r) + \frac{1}{2}\int_r^{r'} \mathcal{F}\, dr.$$

- $Q^{-1}(r') > 0$ at some r':

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If $\int_r^{r'} \mathcal{F}\, dr$ diverges at $r \to \pm\infty$, Q has to be singular.

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If $\int_r^{r'} \mathcal{F}\, dr$ diverges at $r \to \pm\infty$, Q has to be singular.

general relativity restores away from the wormhole

$$\begin{cases} G_4 \to M_{Pl}^2/2 \\ G_5 \to 0 \end{cases} \text{at } r \to \pm\infty \Rightarrow \mathcal{F} \to M_{Pl}^2.$$

Singular Q

$$Q = \frac{\overbrace{2cc'\mathcal{H}}^{\mathbf{1}} + \overbrace{\Xi\phi'}^{\mathbf{2}}}{c^2\mathcal{H}^2} \quad \text{– singular.}$$

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$$\left.\begin{array}{l} \bullet\ c' \text{ is not singular} \\ \bullet\ \text{if } \mathcal{H} \text{ is singular } Q \to 0 \text{ not to } \infty \end{array}\right\} \Rightarrow \mathbf{1} \text{ is not singular;}$$

Singular Q

$$Q = \frac{\overbrace{2cc'\mathcal{H}}^{\mathbf{1}} + \overbrace{\Xi\phi'}^{\mathbf{2}}}{c^2\mathcal{H}^2} \quad \text{– singular.}$$

- c is bounded from below
- $\mathcal{H} > 0$

$\Big\} \Rightarrow$ denominator is not equal to zero;

- c' is not singular
- if $\mathcal{H}$ is singular $Q \to 0$ not to ∞

$\Big\} \Rightarrow$ **1** is not singular;

- Ξ is singular
- ϕ' is singular

$\Big\}$ the Lagrangian is singular.

Conclusion

- The analisys of both even and odd sector was made for $l \geq 2$ within the Regge-Wheler approach.
- Stability conditions were obtained. They coincide with arXiv:1202.4893 [gr-qc] and arXiv:1402.6740 [gr-qc].
- No-go theorem proved: there are no static sperically symetric Lorentzian wormholes in Horndeski theory.

Thank you for your attention!