Applications of the Worldline instanton method: photon decay and Breit-Wheeler pair production in external electromagnetic field at zero and nonzero temperature

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Introduction

Breit-Wheeler pair production

Breit-Wheeler process, $\gamma\gamma \to e^+e^-$, has a threshold $s = 4m^2$ in vacuum. However, in external electric field it may occur below threshold. How to describe this process by semiclassical means?

- May be potentially tested in the electric field of intensive lasers

Photon decay in external e/m field in thermal media

How to describe the process? Complicated calculations in traditional approach.

- Applications in laser physics and astrophysics?

Semiclassical method of "Worldline instantons" may be applied

- Geometrically clear tunneling picture
- Valid only in the semiclassical limit of exponential suppression

Idea of semiclassical "Worldline Instanton" approach

- Particle production in external field ϕ_{ext}

$$\Gamma \propto \mathrm{Im} \int_{p.b.c} Dx_\mu e^{-S[x_\mu, \phi_{ext}]}.$$

- Path integral in saddle point approximation.
  E.o.m.: $\left.\frac{\delta S}{\delta x_\mu}\right|_{x_\mu^{cl}} = 0$ + periodic b.c.
- Classical solution x_μ^{cl} — closed trajectory.
  $\Gamma \propto e^{-S[x_\mu^{cl}]}$ if semiclassical condition $S[x_\mu^{cl}] \gg 1$ is satisfied.
- Fluctuations near classical solution $\delta x_\mu = x_\mu - x_\mu^{cl}$.
  Integral over fluctuations → pre-exponential factor.
- Negative mode in 2nd variation $\delta^2 S[x_\mu]$ → imaginary prefactor → particle production

Affleck, Alvarez Manton '82, Dunne Schubert '05'06, Monin '05, Monin, Voloshin '10 etc.

The Schwinger effect via "worldline instantons"

Scalar QED: $S_E = \int d^4x \left(-\frac{1}{4}F_{\mu\nu}^2 + |D_\mu\phi|^2 + m^2|\phi|^2\right).$

A_μ — classical external field. $Z[A_\mu] = \int D\phi^* D\phi e^{-S_E[A_\mu]} = e^{-W[A_\mu]}.$

Effective action in Schwinger *proper time* representation: *Schwinger, 1951*

$$W[A_\mu] = \int d^4x \left(-\frac{1}{4}F_{\mu\nu}^2 + \int_0^\infty \frac{ds}{s} e^{-m^2 s} \mathrm{Tr}\left(e^{D_\mu^2 s}\right)\right).$$

Operator $\left(-D_\mu^2\right)$ can be interpreted as QM Hamiltonian.

Affleck, Alvarez Manton 1982

$$\mathrm{Tr}\left(e^{sD_\mu^2}\right) = \int d^4x \langle x_\mu | e^{-s\left(-D_\mu^2\right)} | x_\mu \rangle = \int_{p.b.c.} Dx_\mu e^{-\int_0^s d\tau \left(\frac{\dot{x}_\mu^2}{4} + ieA_\mu \dot{x}_\mu\right)}.$$

The Schwinger effect (vacuum pair production): $\Gamma = \mathrm{Im}\, W[A_\mu]$

The Schwinger effect via "worldline instantons"

$$\Gamma \propto \mathrm{Im} \int_0^\infty \frac{ds}{s} e^{-sm^2} \int_{p.b.c.} Dx_\mu e^{-\int_0^1 d\tau \left(\frac{\dot{x}_\mu^2}{4s} + ieA_\mu \dot{x}_\mu \right)}.$$

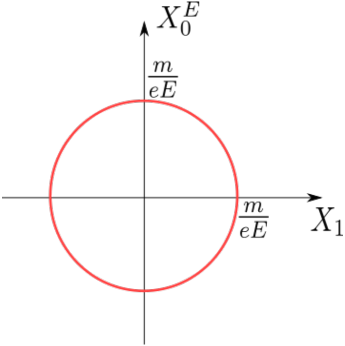
Solve integrals over x_μ and s in the saddle point approximation

Uniform constant electric field E.

The leading solution is a circle:

$$x_0 = \frac{m}{eE}\sin(2\pi\tau), \qquad x_1 = \frac{m}{eE}\cos(2\pi\tau),$$

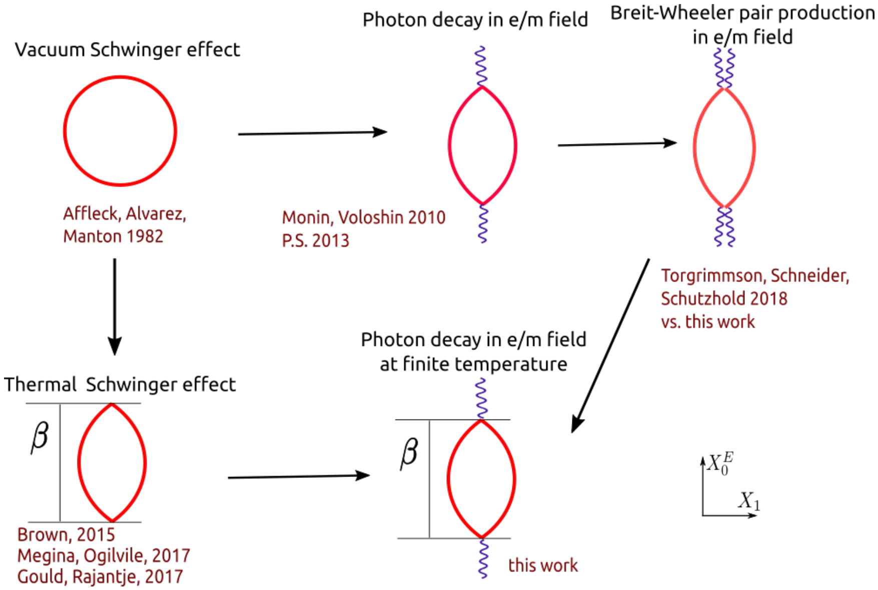
$$x_2 = x_3 = 0, \qquad s = \frac{2\pi}{eE}.$$



The action on the solution x_μ is $S = \frac{\pi m^2}{eE}$.

$$\Gamma = \frac{(eE)^2}{(2\pi)^3} e^{-\frac{\pi m^2}{eE}}.$$

Generalizations to external photons and finite temperature



How to deal with external photons in Worldline formalism

Each photon vertex — functional dervative over A_μ. In scalar QED

$$V[k_\mu, \varepsilon_\mu(k)] = \int_0^1 d\tau \, \varepsilon_\mu(k)\dot{x}_\mu(\tau)e^{ik^\mu x^\mu(\tau)}.$$

N-point photon amplitude *Schubert, 2001*

$$M[k_1, \varepsilon(k_1); \ldots ; k_N, \varepsilon(k_N)] = (-ie)^N \int_0^\infty \frac{ds}{s} e^{-m^2 s}(4\pi s)^{-2} \langle V[k_1, \varepsilon(k_1)] \ldots V[k_N, \varepsilon(k_N)] \rangle$$

Here $\langle V_1 \ldots V_N \rangle \equiv \int_{p.b.c} Dx_\mu \, [V_1 \ldots V_N] \, \exp\left(-\int_0^s d\tau \left(\frac{\dot{x}_\mu^2}{4} + ieA_\mu \dot{x}_\mu\right)\right).$

New extra terms $ik_j^\mu x^\mu(\tau_j)$ in the exponent from V_j, $j = 1..N$.

The Optical theorem

Cross-sections of pair production are proportional to the imaginary part of the corresponding amplitude (depending on the number of initial photons):

- The photon decay: $\Gamma_{\gamma\to e^+e^-} = \frac{1}{2\omega} \, \mathrm{Im}\, \mathcal{M}\left[k, \varepsilon(k); k, \varepsilon^*(k)\right].$
- Breit-Wheeler process:

$$\sigma_{\gamma\gamma\to e^+e^-} = \frac{1}{2E_{c.m.}p_{c.m.}} \, \mathrm{Im}\, \mathcal{M}\left[k_1, \varepsilon(k_1); k_2, \varepsilon(k_2); k_1, \varepsilon^*(k_1); k_2, \varepsilon^*(k_2)\right].$$

Monin, Voloshin 2010 (arXiv:1001.3354)

P.S. 2013 (arXiv:1301.5707)

Ext. photon $k_\mu = (\omega, 0, \omega, 0)$

Optical theorem:
$\Gamma \propto \mathrm{Im}\left(\Pi_{\mu\nu}(k)\varepsilon^*_\mu(k)\varepsilon_\nu(k)\right)$.

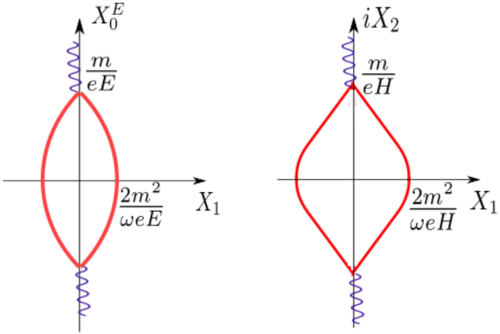
Photon: insertion
$\oint d\tau \dot{x}_\mu(\tau)\, e^{-ik_\mu x_\mu(\tau)}$
into the path integral for Γ.

Classical solution —
two arcs of circle (electric)
two hyperbolas (magnetic)

$$\Gamma \propto e^{-S}.$$

Electric field

Magnetic field



$$S = \frac{8m^3}{3\omega eE} \qquad S = \frac{8m^3}{3\omega eH}$$

in the limit $\omega \gg 2m$

Breit-Wheeler process $\gamma\gamma \to e^+e^-$ in external electric field

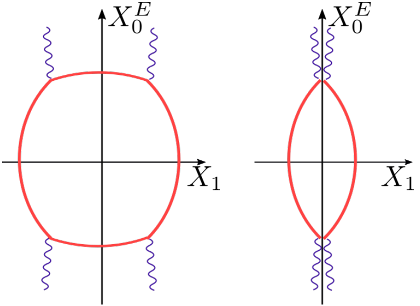
Breit-Wheeler cross-section is proportional to the imaginary part of 4-point photon amplitude:

$$\sigma_{\gamma\gamma\to e^+e^-}(k_1,k_2) \propto \operatorname{Im}\int_0^\infty \frac{ds}{s} e^{-m^2 s}\int_{p.b.c} Dx_\mu \prod_{j=1}^{4}\left(\oint d\tau_j \dot{x}_\mu \varepsilon_\mu(k_j)\right)\cdot$$

$$\cdot\exp\left(-\int_0^s d\tau\left(\frac{\dot{x}_\mu^2}{4}+ieA_\mu\dot{x}_\mu\right)-ik_1^\mu x_\mu(\tau_1)-ik_2^\mu x_\mu(\tau_2)+ik_1^\mu x_\mu(\tau_3)+ik_2^\mu x_\mu(\tau_4)\right).$$

General solution of E.O.M.s – 4 arcs of a circle, connected at 4 points $\tau_1 ... \tau_4$, see Fig, left panel.

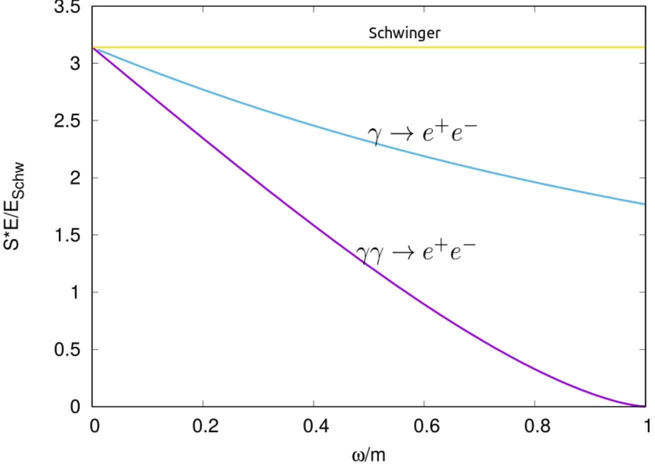
On E.O.M.s up & down arcs shrink into points, see Fig, right panel.
(the same for N photons in the initial state)



Breit-Wheeler pair production. Head-on collision.

Configuration: $k_1^\mu = (\omega, 0, \omega, 0)$, $k_2^\mu = (\omega, 0, -\omega, 0)$, $\vec{E} = (E, 0, 0)$.

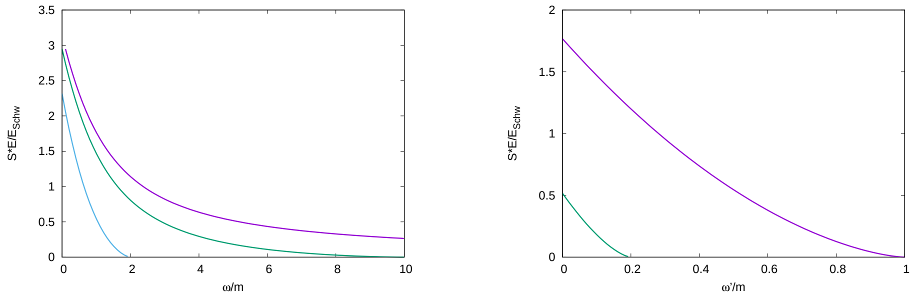
Suppression exponent: $$S[x_\mu^{cl}] = \frac{m^2}{eE}\left[\pi - 2\arcsin\frac{\omega}{m} - 2\frac{\omega}{m}\sqrt{1-\left(\frac{\omega}{m}\right)^2}\right]$$



Breit-Wheeler pair production. Head-on collision.

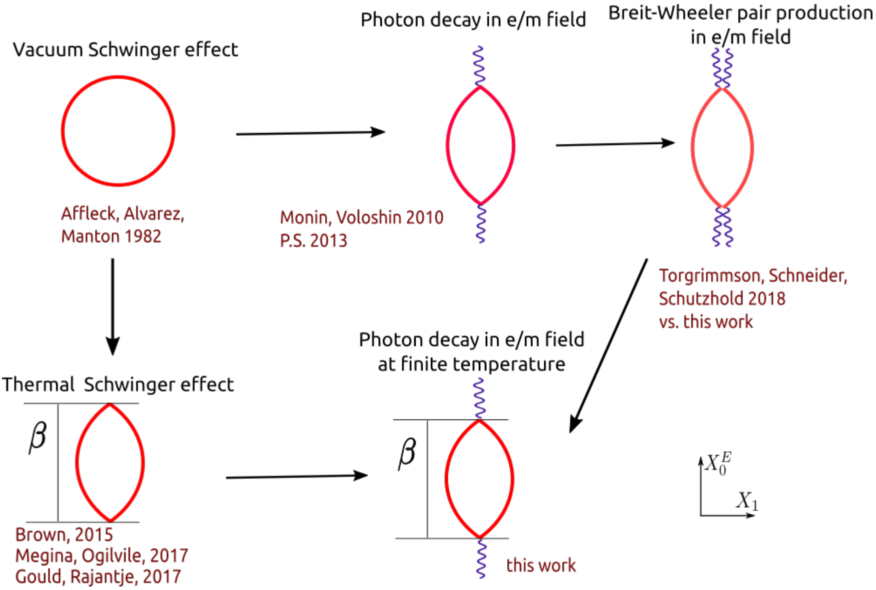
Configuration: $k_1^\mu = (\omega, 0, \omega, 0)$, $k_2^\mu = (\omega', 0, -\omega', 0)$, $\vec{E} = (E, 0, 0)$.

$$S[x_\mu^{cl}] = \frac{m^2}{eE}\left[\frac{\eta}{2} - \frac{(\omega+\omega')^2}{2m^2}\tan\frac{\eta}{4} + \frac{\eta}{8}\frac{(\omega-\omega')^2}{m^2}\right], \qquad \text{where} \quad \tan^2\frac{\eta}{4} = 4\frac{m^2 - \omega\omega'}{(\omega+\omega')^2}$$



Left panel: Suppression exponent dependence on ω/m at different ω': $\omega' = 0$ (purple curve), $\omega' = 0.1m$ (green curve) and $\omega' = 0.5m$ (blue curve). Right panel: Suppression exponent dependence on ω'/m at different ω: $\omega = m$ (purple curve) and $\omega = 5m$ (green curve).

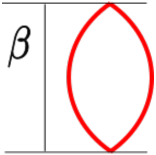
The zoo of Worldline instantons.. Turn on temperature.



The Schwinger effect at finite temperature T

Brown 2015 (arXiv:1512.05716)

- QFT at finite temperature — euclidean time x_0^E is periodic with period $\beta = \frac{1}{T}$. New semiclassical condition: $T \ll m$.
- Small temperatures — the same instanton as at zero temperature until the size of instanton $\frac{2m}{eE}$ is less than β.
- At critical temperature $T_c = \frac{eE}{2m}$ the size of the instanton $=$ the length of the compact dimension x_0^E



- Larger temperatures — a new solution: two arc of circle.

$$\Gamma_T \propto \exp\left(-\frac{2m^2}{eE}\arcsin\left(\frac{T_c}{T}\right) - \frac{m}{T}\sqrt{1-\frac{T_c^2}{T^2}}\right)$$

The limit $T \gg T_c$: $\Gamma \propto e^{-\frac{2m}{T}}$ — Boltzmann exponent. $2m$ – energy of a pair.

Pair production at fixed energy $\mathcal{E}$

Photon decay in th. bath is a non-equilibrium process. Thermal approach does not work.

Thermal bath: Photons with Boltzman energy spectrum
Pair production rate at fixed energy $\mathcal{E}$:

$$\Gamma_T = \int_0^\infty d\mathcal{E}\, e^{-\mathcal{E}/T}\, \Gamma_{\mathcal{E}}$$

$\Gamma_{\mathcal{E}}$ — decay rate of off-shell photon $\gamma_{\mathcal{E}}$ with 4-momentum $(\mathcal{E},0,0,0)$.

Thermal Schwinger effect via integral over fixed energy

$$\Gamma_T = \int_0^\infty d\mathcal{E} \int_0^\infty \frac{ds}{s} \int_{p.b.c.} Dx_\mu\, e^{-m^2 s - \int_0^s d\tau \left(\frac{\dot{x}_\mu^2}{4} + ieA_\mu \dot{x}_\mu\right) - \mathcal{E}(x_0(1/2) - x_0(0)) - \mathcal{E}/T}.$$

Photon decay in ext. field $\quad = \quad \gamma\gamma_{\mathcal{E}} \to e^+ e^-$

effective external 4-momentum $(\omega + \mathcal{E},0,\omega,0)$.

Photon decay in electric field at finite temperature

$T < T_c \;\rightarrow\;$ the same solution as for $T = 0$. $\quad T > T_c \;\rightarrow\;$ new solution.

$$\Gamma \propto e^{-S}.$$

Action on the classical solution, exactly on arbitrary $\frac{2m}{\omega}$ and $\frac{T_c}{T} \leq 1$.

$$S = \frac{4m^2}{eE} \cdot \arctan\left(\frac{2mT_c}{\omega T}\right)\left[1 - \frac{1}{2}\left(\left(\frac{2m}{\omega}\right)^{-2} + \frac{T_c^2}{T^2}\right) + \left(\frac{2m}{\omega}\right)^{-2}\left[1 - \left(\frac{2m}{\omega}\right)^2 \theta^2\right]\right] -$$

$$- \frac{2m^2}{eE} \cdot \left(\frac{2m}{\omega}\right)^{-1} \cdot \frac{T_c}{T} + \frac{4m^2}{eE}\left(\frac{2m}{\omega}\right)^{-1} \frac{T_c}{T}\left[1 - \sqrt{1 - \left(\frac{2m}{\omega}\right)^2 \theta^2}\right], \qquad \theta^2 = 1 - \frac{T_c^2}{T^2}.$$

In the limit $\omega \gg 2m$

$$S = \frac{4m^3}{\omega eE} \cdot \frac{T_c}{T}\left(1 - \frac{T_c^2}{T^2}\right) + \frac{8m^3}{3\omega eE} \cdot \left(\frac{T_c}{T}\right)^3.$$

In the limit $T \gg T_c \quad \rightarrow \quad S = \frac{2m^2}{\omega T}$,
in the limit $T = T_c \quad \rightarrow \quad S = \frac{8m^3}{3\omega eE}$, the same as for $T = 0$.

Photon decay in magnetic field at finite temperature

Critical temperature $T_c = \frac{eH}{2m}$. Semiclassics: $T \ll m$

$T < T_c \;\rightarrow$ the same solution as for $T = 0$. $T > T_c \;\rightarrow$ new solution.

In the limit $\omega \gg 2m$:

$$\Gamma \propto e^{-S}, \qquad S = \frac{4m^3}{\omega eH} \cdot \frac{T_c}{T}\left(1 - \frac{T_c^2}{T^2}\right) + \frac{8m^3}{3\omega eH} \cdot \left(\frac{T_c}{T}\right)^3.$$

In the limit $T \gg T_c \quad \rightarrow \quad S = \frac{2m^2}{\omega T}$,
in the limit $T = T_c \quad \rightarrow \quad S = \frac{8m^3}{3\omega eH}$, the same as for $T = 0$.

Semiclassics: $T \ll m \;\rightarrow$ only sub-Schwinger magnetic field $H \ll m/e \sim 10^{13}$ G.

Conclusions

- The exponential part of the cross-section of Breit-Wheeler process in external electric field has been computed via Worldline instantons far from the threshold, in a deep semiclassical regime.
- The cross-section of pair production is enhanced significantly if 3 components of electromagnetic field presented (strong constant field **E**, and 2 photons.
- Worldline instanton method may be also applied to photon decay in external electromagnetic field at zero and nonzero temperature in the regime of exponential suppression.
- Astrophysical applications of photon decay in thermal magnetized plasma for sub-Schwinger magnetic field?
- Possible generalization to nontrivial chemical potential?
- ...

Thank you for your attention!