Evolution of a domain wall in expanding universe: inflation and after it

S.I. Godunov
in collaboration with A.D. Dolgov and A.S. Rudenko

based on
JCAP 1610 (2016) №10, 026
arXiv:1711.04704

Quarks-2018
XXth International Seminar on High Energy Physics

May 27 — June 2, 2018 Valday, Russia

It is well known that a symmetry, which is broken in vacuum, at high temperatures tends to be restored. But in general, the situation is not that simple and straightforward. It is also possible that a symmetry is broken only in a particular range of temperatures, i.e. it is restored at the highest as well as at the lowest temperatures. This is just what is needed for a matter-antimatter domain generation without domain wall problem.

A.D. Dolgov, S.I. Godunov, A.S. Rudenko, I.I. Tkachev, JCAP 1510, 027 (2015)

- The model with spontaneous CP violation is suggested.
- CP violation appears due to interaction of additional scalar field with inflaton.
- BAU is generated just after inflation due to interaction of introduced scalar field with quarks and leptons.
- This scenario leads to the generation of matter-antimatter domains in the Early Universe.
- To avoid annihilation at the domain boundary, the distance between the domains should grow exponentially fast during inflation.

Inspiration

It is well known that a symmetry, which is broken in vacuum, at high temperatures tends to be restored. But in general, the situation is not that simple and straightforward. It is also possible that a symmetry is broken only in a particular range of temperatures, i.e. it is restored at the highest as well as at the lowest temperatures. This is just what is needed for a matter-antimatter domain generation without domain wall problem.

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How fast the domain wall width can grow in the Early Universe?

Domain wall evolution in the de Sitter space-time

Metric:

$$ds^2 = dt^2 - e^{2Ht}\left(dx^2 + dy^2 + dz^2\right).$$

Scalar field:

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \, \partial_\nu \varphi - \frac{\lambda}{2}\left(\varphi^2 - \eta^2\right)^2.$$

Equations of motion:

$$\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi\right) = -2\lambda\varphi\left(\varphi^2 - \eta^2\right).$$

$H = 0$ (static universe), 1d case ($\varphi = \varphi(z)$):

$$\frac{d^2\varphi}{dz^2} = 2\lambda\varphi\left(\varphi^2 - \eta^2\right).$$

Solution (wall at $z = 0$):

$$\varphi(z) = \eta \tanh \frac{z}{\delta_0},$$

where $\delta_0 = 1/(\sqrt{\lambda}\eta)$ is the wall width.

Stationary solutions for $H > 0$

Basu, Vilenkin, Phys. Rev. D 50 (1994) 7150

Ansatz for stationary solutions (φ depends only on $a(t) \cdot z$):

$$\varphi = \eta \cdot f(u), \qquad \text{where} \quad u = Hze^{Ht}.$$

Equations of motion:

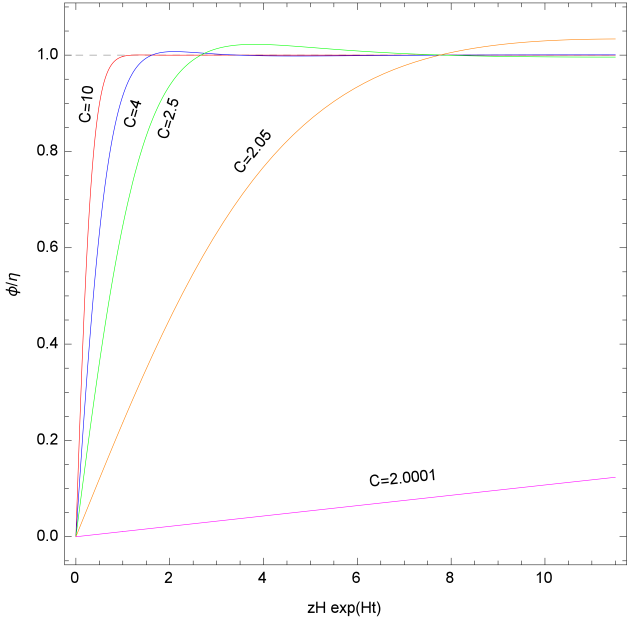
$$\left(1 - u^2\right) f'' - 4uf' = -\, 2Cf\left(1 - f^2\right),$$

$$\text{where } C = \frac{1}{(H\delta_0)^2} = \frac{\lambda\eta^2}{H^2} > 0$$

Boundary conditions:

$$f(0) = 0,$$
$$f(\pm\infty) = \pm 1.$$

Stationary solutions



Beyond stationary limit

$$\frac{\partial^2 \varphi}{\partial t^2} + 3H\frac{\partial \varphi}{\partial t} - e^{-2Ht}\frac{\partial^2 \varphi}{\partial z^2} = -2\lambda\varphi\left(\varphi^2 - \eta^2\right).$$

Introducing dimensionless parameters $\tau = Ht$, $\zeta = Hz$, $f(\zeta,\tau) = \varphi(z,t)/\eta$, we get

$$\frac{\partial^2 f}{\partial \tau^2} + 3\frac{\partial f}{\partial \tau} - e^{-2\tau}\frac{\partial^2 f}{\partial \zeta^2} = 2Cf\left(1 - f^2\right),$$

where $C = \lambda\eta^2/H^2 = 1/(H\delta_0)^2 > 0$.

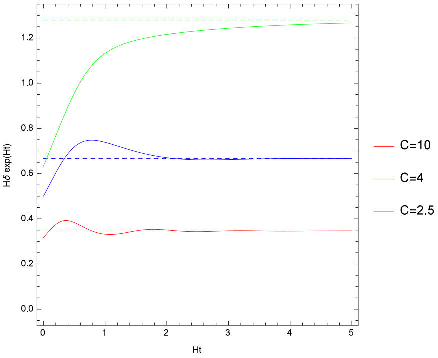
Boundary conditions:

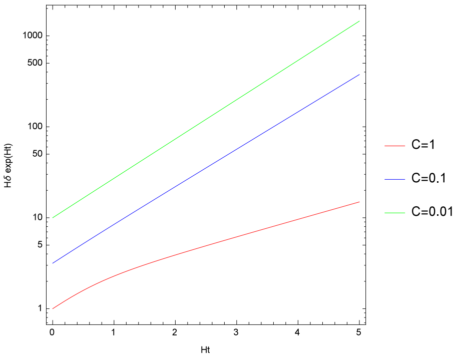
$$f(0,\tau) = 0, \qquad f(\pm\infty,\tau) = \pm 1,$$

Initial configuration:

$$f(\zeta,0) = \tanh\frac{z}{\delta_0} = \tanh\sqrt{C}\zeta, \qquad \left.\frac{\partial f(\zeta,\tau)}{\partial \tau}\right|_{\tau=0} = 0.$$

Wall width





Domain wall during inflation

Let us consider a simple model of inflation with quadratic inflaton potential $U = m^2\Phi^2/2$, then the Hubble parameter is

$$H = \sqrt{\frac{8\pi\rho}{3m_{pl}^2}} \approx \sqrt{\frac{8\pi}{3m_{pl}^2}\frac{m^2\Phi^2}{2}} = \sqrt{\frac{4\pi}{3}}\frac{m}{m_{pl}}\Phi,$$

and the equation of motion of the inflaton in the slow-roll regime is the following:

$$\dot{\Phi} \approx -\frac{m^2\Phi}{3H} \approx -\frac{m_{pl}m}{\sqrt{12\pi}},$$

where m_{pl} is the Planck mass, m is the inflaton mass.

$$\Phi(t) = \Phi_i - \frac{m_{pl}m}{\sqrt{12\pi}}t,$$

where Φ_i is the initial value of inflaton field.

The Hubble parameter and the scale factor can also be easily found:

$$H(t) = \sqrt{\frac{4\pi}{3}} \frac{m}{m_{pl}} \Phi_i - \frac{1}{3} m^2 t,$$
$$a(t) = a_0 \cdot \exp\left(\sqrt{\frac{4\pi}{3}} \frac{m}{m_{pl}} \Phi_i t - \frac{1}{6} m^2 t^2 \right).$$

These formulas are valid only till the end of inflation, $t < t_e \equiv \frac{\sqrt{12\pi}\Phi_i}{m_{pl}} m^{-1}$.
it is convenient to use $1/m$ units in equation of motion:

$$\frac{\partial^2 f}{\partial (t \cdot m)^2} + m (t_e - t) \frac{\partial f}{\partial (t \cdot m)} - \frac{1}{a^2(t)} \frac{\partial^2 f}{\partial (z \cdot m)^2} = \frac{2}{(m \cdot \delta_0)^2} f \left(1 - f^2\right).$$

In numerical calculations the following parameters were used:

$$\Phi_i = 2\, m_{pl},\ t_i = 0,\ a_0 = 1.$$

Inflation: $C(t)$ dependence

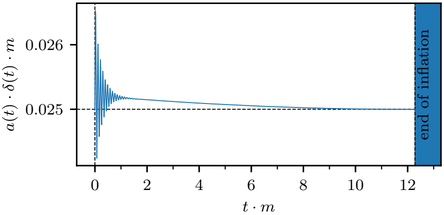
$$C(t) = \frac{1}{(H(t)\delta_0)^2}.$$

Time t_C at which $C(t_C) = 2$:

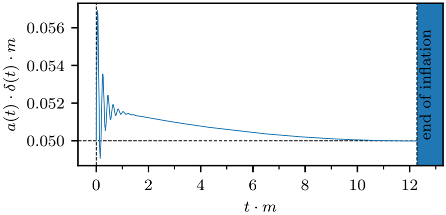
$$m \cdot t_C = m \cdot t_e - \frac{3\sqrt{2}}{2m\delta_0}.$$

Parameter $C(t)$ can be equal 2 only if $t_C \geq 0$:

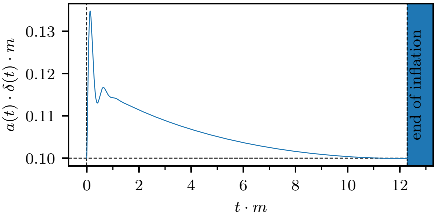
$$m \cdot \delta_0 \geq \frac{3\sqrt{2}}{2mt_e} = \frac{\sqrt{3}m_{pl}}{2\sqrt{2\pi}\Phi_i} \approx 0.173.$$



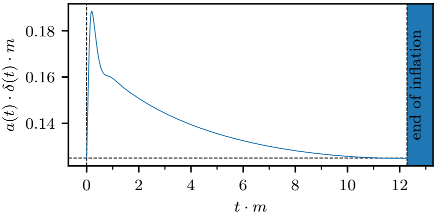
$\delta_0 = 0.025 \cdot m^{-1}$.



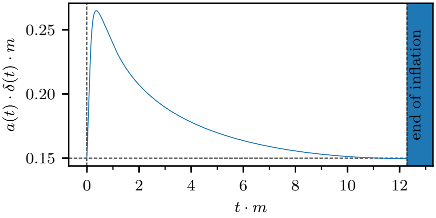
$\delta_0 = 0.05 \cdot m^{-1}$.



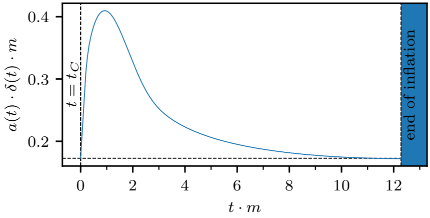
$\delta_0 = 0.1 \cdot m^{-1}$.



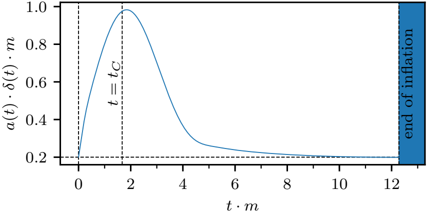
$\delta_0 = 0.125 \cdot m^{-1}$.



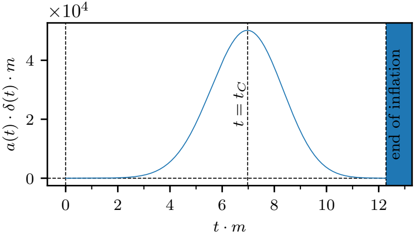
$\delta_0 = 0.15 \cdot m^{-1}.$



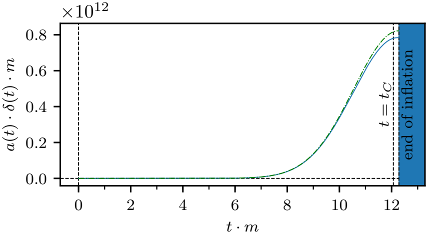
$\delta_0 \approx 0.173 \cdot m^{-1}.$



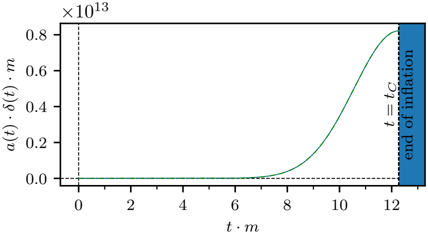
$\delta_0 = 0.2 \cdot m^{-1}.$



$\delta_0 = 0.4 \cdot m^{-1}.$



$\delta_0 = 10 \cdot m^{-1}.$



$\delta_0 = 100 \cdot m^{-1}.$

$$a(t) = a_0 \cdot \left(\frac{t}{t_i}\right)^{\alpha},$$

$$H(t) = \frac{\dot{a}}{a} = \frac{\alpha}{t}, \text{ where } \alpha = \frac{2}{3(1+w)} > 0,$$

The values $w = 0$ $(\alpha = 2/3)$ and $w = 1/3$ $(\alpha = 1/2)$ correspond to the matter-dominated and radiation-dominated universe, respectively.

The equation of motion

$$\frac{\partial^2 f}{\partial t^2} + 3H(t)\frac{\partial f}{\partial t} - \frac{1}{a^2(t)}\frac{\partial^2 f}{\partial z^2} = \frac{2}{\delta_0^2} f\left(1 - f^2\right),$$

where $f(z,t) = \varphi(z,t)/\eta$.

Feature of the $p = w\rho$ universe

Since

$$H(t)\delta_0 = H\left(t/\delta_0\right),$$

after the substitution $\tau = t/\delta_0$, $\zeta = z/\delta_0$ we get:

$$\frac{\partial^2 \tilde{f}}{\partial \tau^2} + \frac{3}{\sqrt{C\left(\tau\right)}} \frac{\partial \tilde{f}}{\partial \tau} - \frac{1}{\tilde{a}^2(\tau)} \frac{\partial^2 \tilde{f}}{\partial \zeta^2} = 2\tilde{f}\left(1 - \tilde{f}^2\right),$$

where $\tilde{f}(\zeta, \tau) = f(\zeta \cdot \delta_0, \tau \cdot \delta_0)$, $\tilde{a}\left(\tau\right) = a\left(\tau \cdot \delta_0\right) = a_0 \cdot \left(\tau/\tau_i\right)^{\alpha}$, and

$$C\left(\tau\right) = \left(H\left(\tau \cdot \delta_0\right) \cdot \delta_0\right)^{-2} = H^{-2}\left(\tau\right).$$

No explicit dependence on δ_0!

The parameter $C(t)$ increases as

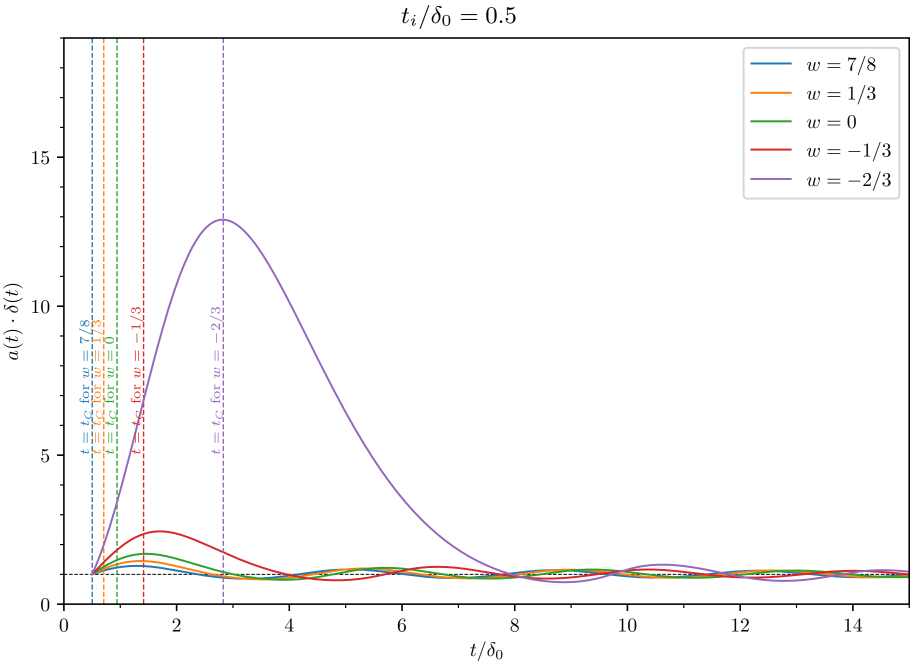
$$C(t) = \frac{1}{(H(t)\delta_0)^2} = \frac{t^2}{(\alpha\delta_0)^2} \propto t^2.$$

The time t_C at which $C(t_C) = 2$:

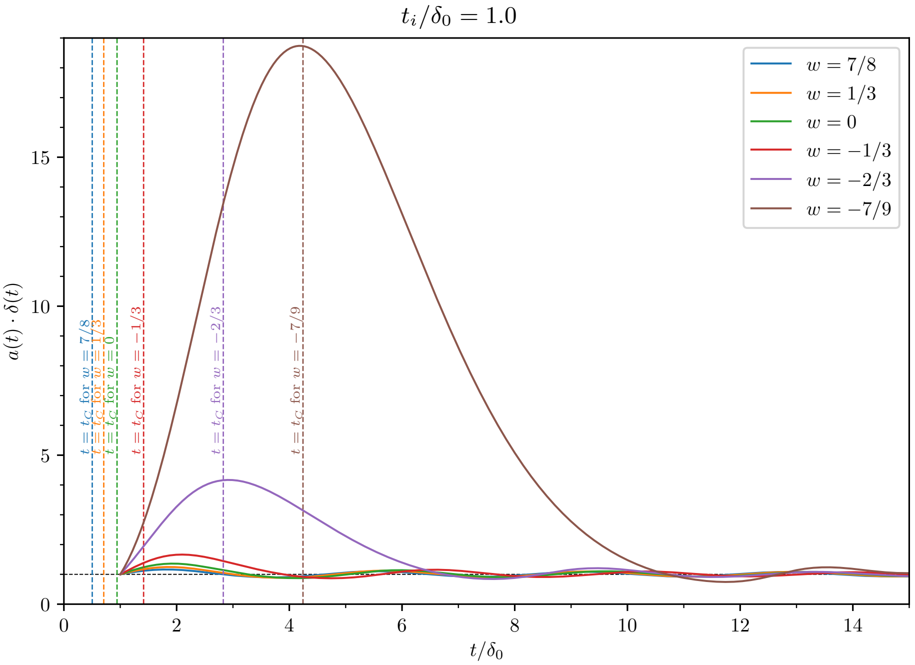
$$\frac{t_C}{\delta_0} = \sqrt{2}\alpha.$$

We obtain that $t_C > t_i$ for

$$w < \frac{2\sqrt{2}}{3}\frac{\delta_0}{t_i} - 1.$$



$t_i/\delta_0 = 0.5$
$w = 7/8$
$w = 1/3$
$w = 0$
$w = -1/3$
$w = -2/3$
$t = t_C$ for $w = 7/8$
$t = t_C$ for $w = 1/3$
$t = t_C$ for $w = 0$
$t = t_C$ for $w = -1/3$
$t = t_C$ for $w = -2/3$
$a(t) \cdot \delta(t)$
t/δ_0
0
5
10
15
0
2
4
6
8
10
12
14



$t_i/\delta_0 = 1.0$
$w = 7/8$
$w = 1/3$
$w = 0$
$w = -1/3$
$w = -2/3$
$w = -7/9$
$t = t_C$ for $w = 7/8$
$t = t_C$ for $w = 1/3$
$t = t_C$ for $w = 0$
$t = t_C$ for $w = -1/3$
$t = t_C$ for $w = -2/3$
$t = t_C$ for $w = -7/9$
$a(t) \cdot \delta(t)$
0
5
10
15
0
2
4
6
8
10
12
14
t/δ_0

Conclusions

The evolution of the domain walls was considered in the following cases:

- de Sitter universe
  - For $C = \lambda\eta^2/H^2 = 1/(H\delta_0)^2 > 2$ the solutions tend to the stationary ones.
  - For $C = \lambda\eta^2/H^2 = 1/(H\delta_0)^2 < 2$ the wall width grows rapidly. For $C \lesssim 0.1$ the growth is practically exponential, so the wall expands with the universe.
- during inflation:
  - For $m \cdot \delta_0 \lesssim 0.173$ the deviation of the wall width from δ_0 is small.
  - For $0.173 \lesssim m \cdot \delta_0 \lesssim 1$ the wall width can reach cosmologically large values, but then it quickly diminishes and reaches δ_0.
  - For $m \cdot \delta_0 \gg 1$ the wall width grows with the scale factor and by the end of inflation it reaches cosmologically large size.
- $p = w\rho$ universe:
  - Domain walls with cosmologically large width can exist only in the beginning of this phase.
  - For $t/\delta_0 \gg \sqrt{2}\alpha$ the wall width is close to δ_0.