Generality of Starobinsky inflation

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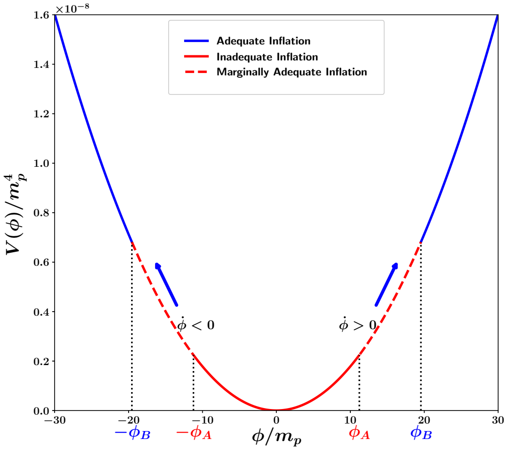
Based on arXiv:1801.04948

Current observations show that inflation, most probably, is one of the following types:

- Minimally coupled scalar field with flat potential
- Higgs field with non-minimal coupling
- R^2 inflation.

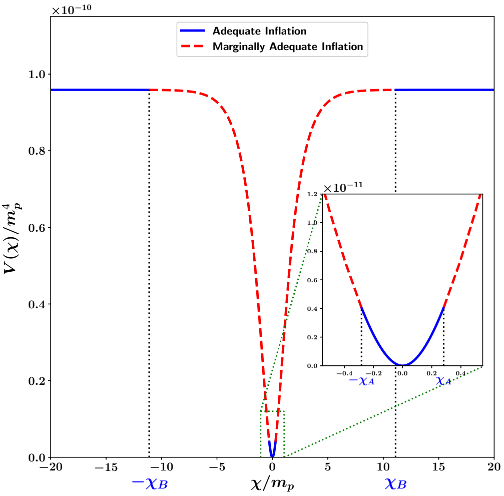
$$3(H^2 + k/a^2) = \dot{\phi}/2 + V(\phi) \qquad (1)$$

When we study generality of inflation it is reasonable to start with some fixed scalar field energy about Planckean one. In this case and if $k = 0$ (flat Universe) the fate of the Universe is fully specified by initial scalar field ϕ and direction of $\dot{\phi}$.



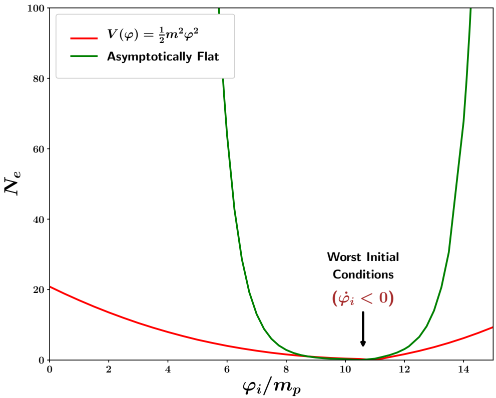
×10⁻⁸
Adequate Inflation
Inadequate Inflation
Marginally Adequate Inflation
$V(\phi)/m_p^4$
$\dot{\phi} < 0$
$\dot{\phi} > 0$
$-\phi_B$
$-\phi_A$
ϕ/m_p
ϕ_A
ϕ_B

Figure:



×10^−10
Adequate Inflation
Marginally Adequate Inflation
1.0
0.8
0.6
0.4
0.2
0.0
V(χ)/m_p^4
−20
−15
−10
−5
0
5
10
15
20
−χ_B
χ/m_p
χ_B
×10^−11
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.4
−0.2
0.0
0.2
0.4
−χ_A
χ_A

Figure:



$V(\varphi) = \frac{1}{2}m^2\varphi^2$
Asymptotically Flat
Worst Initial Conditions
$(\dot{\varphi}_i < 0)$
N_e
φ_i/m_p
100
80
60
40
20
0
0
2
4
6
8
10
12
14

Figure:

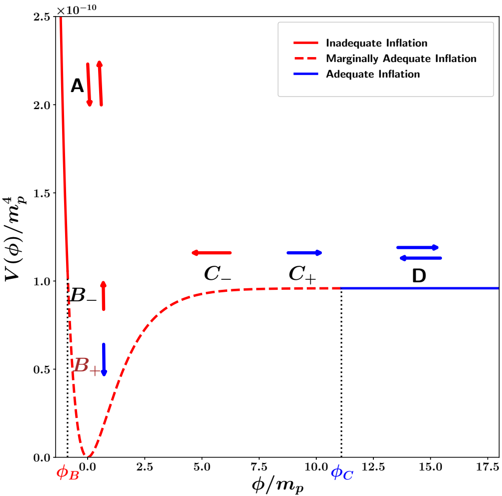

×10⁻¹⁰
Inadequate Inflation
Marginally Adequate Inflation
Adequate Inflation
A
B_-
B_+
C_-
C_+
D
$V(\phi)/m_p^4$
ϕ/m_p
ϕ_B
ϕ_C
0.0
0.5
1.0
1.5
2.0
2.5
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5

Figure:

For a scalar field in a universe with positive spatial curvature, condition for bounce when $H_i = 0$ is given by

$$\frac{V(\varphi)}{m_p^4} \geq \frac{2}{a^2 m_p^2} \tag{2}$$

So bounce happens for $a > a_b$ and collapse happens at $a < a_b$ where a_b is given by

$$a_b m_p = \sqrt{2}\frac{m_p^4}{V(\varphi)} \tag{3}$$

being at the same time

$$a m_p < \sqrt{3}\frac{m_p^4}{V(\varphi)} \tag{4}$$

For power-law potentials or any other monotonically increasing potentials, starting from $H_i = 0$, we can choose a φ_i such that we get bounce. So there is some range of initial φ_i for which inflation becomes possible. This result is well-known, we have also derived it in this work and I am not going to comment more about it here.

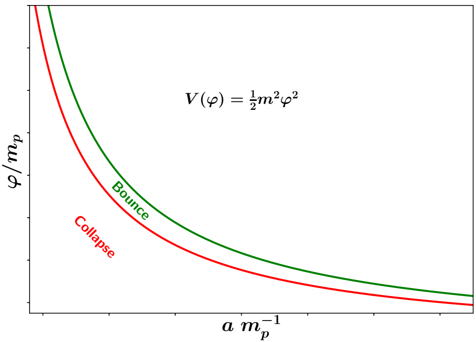


Figure:

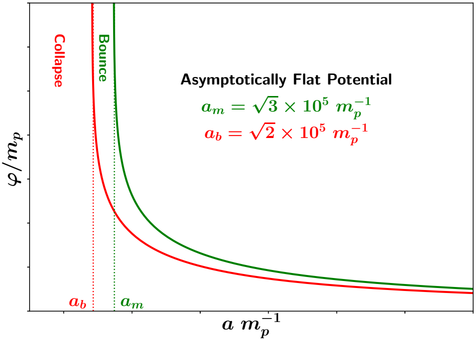


Figure:

Rather I am going to describe our new results on asymptotically flat potentials. Let's think of an asymptotically flat potential like the Higgs potential in the Einstein frame. For large values of φ, $V(\varphi) \longrightarrow V_0$ and observations (CMB normalization for density fluctuations) limit the value of V_0 to be $V_0 \simeq 10^{-10}\ m_p^4$. This is the root of the problem. If we start at Planck scale, initial density $\rho_{\varphi,i} = 1\ m_p^4$ which is hugely dominated by the kinetic term. In this case a_b becomes

$$a_b m_p = \sqrt{2}\frac{m_p^4}{V(\varphi)} \simeq \sqrt{2} \times 10^5 \qquad (5)$$

while for $H_i = 0$, $a_i = \sqrt{3} m_p^{-1} \ll a_b$.

So for $H_i = 0$, we always get collapse and hence no inflation at all (no matter whatever φ_i is). This is in sharp contrast to the case of power-law potentials where even at $H_i = 0$, we do get inflation for some range of values of φ_i.

We know that at higher H_i, we might have a better chance to get inflation. The next important thing to find out is the value of H_i (and hence the corresponding a_i) for which inflation becomes possible. This has to be done numerically. We carried out a numerical study of this for the Higgs potential in the Einstein frame and the central result of our analysis can be represented as the following expressions. The initial value of a_i (and hence the corresponding H_i) for which universe penetrates through the collapsing region and bounces back is given by

$$a_i \geq 1.6 \times 10^3 a_{min} \left(= 2.76 \times 10^3 m_p^{-1} \right) \quad (6)$$

where $a_{min} = \sqrt{3} m_p^{-1}$.

Analysis of asymptotically flat right wing of Starobinsky potential in the Einstein frame is exactly similar to the Higgs potential in the Einstein frame as discussed in the previous section. So we only discuss the initial conditions for the exponential left wing of the Starobinsky potential.

For the flat case $K = 0$, starting at Planckian initial energy density, we get adequate inflation with 60 e-foldings if $|\varphi_i| \geq 4.7m_p$ for the left wing. However for $K = 1$, starting at $\varphi_i = \varphi_{flat} = -4.7m_p$ with

$H_i = 0$ we do not get adequate inflation. Lowering the initial energy also does not help at all. So we do not get adequate inflation for $H_i = 0$ if we start at $\varphi_i = -4.7m_p$.

$$a_b(\varphi_i = -4.7m_p) = 3.17 \times 10^3 \ m_p^{-1}$$

Now we fix $\varphi_i = -4.7m_p$ and Planckian initial energy density and vary H_i ($\neq 0$) to obtain the value of of a_i where inflation becomes possible. This value is given by

$$a_i = 1.61 \times 10^3 \ m_p^{-1}$$

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CONCLUSIONS

In Einstein frame the initial conditions good for inflation for asymptorically flat potentialin the $k = 0$ case have the form different from known for a massive scalar field.

In the case of positive spatial curvature we need to start from $a >\sim 10^3 a_{min}$ to get inflation from Planck energy, otherwise Universe recollapses inevitably.