Holographic study of Wilson loop in the anisotropic background with confinement/deconfinement phase transition

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Motivation

Purpose: Study of the QCD phase diagram as a function of temperature T and chemical potential μ in the anisotropic background.

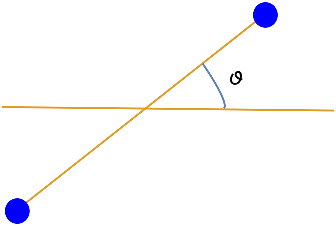
Holographic methods for calculating of the loops (J. Maldacena, D. Gross and others).

Holographic approach to the study of QGP is actively developed. O.Andreev and V.I.Zakharov, [hep-ph/0604204]

Formulation of the problem

It is natural to expect, that the phase transition depends on the orientation of the quark pair relative to the anisotropy axis.

Anisotropy axis in the QGP created in the HIC is defined by the axes of ions collisions.
Confinement/deconfinement phase transition line depends on the angle θ between quarks' line and heavy ions collisions line.



Background

$$ds^2 = G_{\mu\nu} dx^\mu dx^\nu = \frac{b(z)}{z^2}\left[-g(z)dt^2 + dx^2 + z^{2-\frac{2}{\nu}}(dy_1^2 + dy_2^2) + \frac{dz^2}{g(z)}\right],$$

ν – anizotropic parameter, $b(z) = e^{\frac{cz^2}{2}}$ – warp factor, z_h – horizon, AdS radius $L = 1$, blackening function $g(z)$:

$$g(z) = 1 - \frac{z^{2+\frac{2}{\nu}}}{z_h^{2+\frac{2}{\nu}}} \frac{\mathfrak{G}(\frac{3}{4}cz^2)}{\mathfrak{G}(\frac{3}{4}cz_h^2)} - \frac{\mu^2 cz^{2+\frac{2}{\nu}} e^{\frac{cz_h^2}{2}}}{4\left(1 - e^{\frac{cz_h^2}{4}}\right)^2} \mathfrak{G}(cz^2) +$$

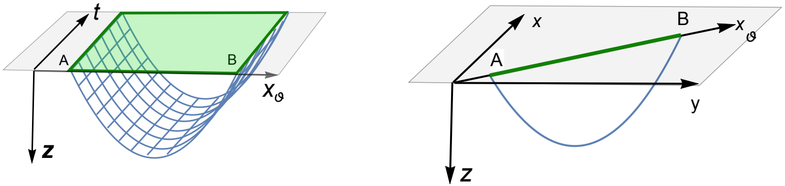
$$+ \frac{\mu^2 cz^{2+\frac{2}{\nu}} e^{\frac{cz_h^2}{2}}}{4\left(1 - e^{\frac{cz_h^2}{4}}\right)^2} \frac{\mathfrak{G}(\frac{3}{4}cz^2)}{\mathfrak{G}(\frac{3}{4}cz_h^2)} \mathfrak{G}(cz_h^2), \qquad \mathfrak{G}(x) \equiv \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!(1 + n + \frac{1}{\nu})}$$

I.Ya.Aref'eva and K.Rannu, arXiv:1802.05652 [hep-th]
I.Ya.Aref'eva, K.Rannu and P.Slepov (in preparation)

Wilson loop

Expectation value of the temporal Wilson loop: $W[C_\vartheta] = e^{-S_{\vartheta,t}}$.
Orientation along vector $\vec{n}$: $n_x = \cos\vartheta, \quad n_y = \sin\vartheta$.
World sheet parametrization:

$$\begin{aligned} X^0 &\equiv t = \xi^0, \\ X^1 &\equiv x = \xi^1 \cos\vartheta, \\ X^2 &\equiv y_1 = \xi^1 \sin\vartheta, \\ X^3 &\equiv y_2 = const, \\ X^4 &\equiv z = z(\xi^1). \end{aligned}$$



Nambu-Goto action

Components of the induced metric: $h_{\alpha\beta} = G_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$
Lagrangian: $\mathcal{L} = \sqrt{-\det h_{\alpha\beta}}$
Born-Infeld-type action:

$$S = -\frac{\tau}{2\pi\alpha'} \int d\xi^1 \; M(z)\sqrt{\mathcal{F}(z,\theta) + z'^2},$$

$$\tau = \int d\xi^0, \quad z = z(\xi^1)$$

$$M(z) = \frac{b(z)}{z^2}$$

$$\mathcal{F}(z,\theta) = g(z)\left(z^{2-\frac{2}{\nu}}\sin^2(\theta) + \cos^2(\theta)\right)$$

Effective potential:

$$\mathcal{V}(z) \equiv M(z)\sqrt{\mathcal{F}(z,\theta)}$$

First integral

First integral of motion:

$$\mathfrak{I} = \frac{M(z)\mathcal{F}(z,\theta)}{\sqrt{\mathcal{F}(z,\theta) + z'^2}}$$

First integral can be expressed as:

$$\mathfrak{I} = M(z_{min})\sqrt{\mathcal{F}(z_{min},\theta)},$$

z_{min} - point at which $z'(\xi) = 0$ and $\mathcal{V}'(z_{min}) = 0$
Character length of string:

$$\ell = \int_0^{z_{min}} \frac{2}{\sqrt{\mathcal{F}(z,\theta)}} \frac{dz}{\sqrt{\left(\frac{\mathcal{V}(z)}{\mathcal{V}(z_{min})}\right)^2 - 1}},$$

$$\mathcal{V}(z) \equiv M(z)\sqrt{\mathcal{F}(z,\theta)}$$

Conditions of the phase transition

We study the character length behaviour at special points $\Rightarrow$ expansion of $\mathcal{V}^2(z)/\mathcal{V}^2(z_{min})$ in the Taylor series at the point z_{min}:

I.Ya.Aref'eva, arXiv:1612.08928 [hep-th]

$$\frac{\mathcal{V}^2(z)}{\mathcal{V}^2(z_{min})} = 1 + \mathcal{V}_1 \cdot (z - z_{min}) + \mathcal{V}_2 \cdot (z - z_{min})^2 + o((z - z_{min})^2)$$

$$\mathcal{V}_1 \equiv \frac{2\mathcal{V}'(z_{min})}{\mathcal{V}(z_{min})}$$

$$\mathcal{V}_2 \equiv \frac{\mathcal{V}''(z_{min})}{\mathcal{V}(z_{min})} + \frac{\mathcal{V}'(z_{min})^2}{\mathcal{V}^2(z_{min})}$$

$$\Downarrow$$

$$\ell = 2\int_0^{z_{min}} \frac{dz}{\sqrt{\mathcal{F}(z,\theta)(\mathcal{V}_1 \cdot (z - z_{min}) + \mathcal{V}_2 \cdot (z - z_{min})^2)}}$$

For the confinement/deconfinement phase transition the character length ℓ must be infinite for $z \to z_{min} \Longrightarrow$ two different cases: $\mathcal{V}'(z) = 0$ and $\mathcal{V}'(z) \neq 0$.

Conditions of the phase transition

1) $\mathcal{V}'(z) \neq 0 \Longrightarrow$ we can consider the first order only:

$$\ell = 2\int_0^{z_{min}} \frac{dz}{\sqrt{\mathcal{F}(z,\theta)\mathcal{V}_1(z - z_{min})}} \sim \sqrt{\frac{z - z_{min}}{\mathcal{F}(z_{min},\theta)\mathcal{V}_1}}$$

$\Rightarrow \ell \to 0$ as $z \to z_{min} - 0$

2) $\mathcal{V}'(z) = 0 \Longrightarrow$ we have to use the second order:

$$\ell = 2\int_0^{z_{min}} \frac{dz}{\sqrt{\mathcal{F}(z,\theta)\,\mathcal{V}_2} \cdot (z_{min} - z)} \sim \sqrt{\frac{\mathcal{V}}{\mathcal{V}''(z_{min})\mathcal{F}(z_{min},\theta)}} \log(z_{min} - z)$$

$\Rightarrow \ell \to \infty$ as $z \to z_{min} - 0$

Potential with the dilaton field:

$$\mathcal{V}_\theta(z) = \frac{b(z)e^{\sqrt{\frac{2}{3}}\phi(z)}}{z^2}\sqrt{g(z)\left(z^{2-\frac{2}{\nu}}\sin^2(\theta) + \cos^2(\theta)\right)}$$

Conditions of the phase transition

So we solve the equation $\mathcal{V}'(z) = 0$:

$$\mathcal{DW}_\theta \equiv cz + \frac{1}{\nu z}\sqrt{\frac{2}{3}}\sqrt{3c\,\nu^2 z^2\left(\frac{cz^2}{2}-3\right)+4\nu-4}+\frac{g'}{2g} - \left. -\frac{2}{z}+\frac{\left(1-\frac{1}{\nu}\right)z^{1-\frac{2}{\nu}}\sin^2(\theta)}{\cos^2(\theta)+z^{2-\frac{2}{\nu}}\sin^2(\theta)}\right|_{z=z_{DW\theta}} = 0$$

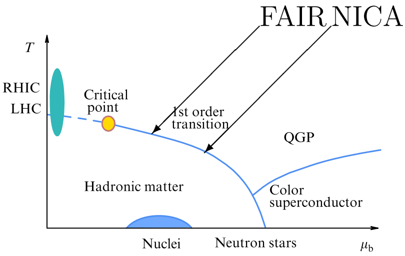
This expression leads to the particular cases $\theta = 0^0,\ 90^0$:

$$\mathcal{DW}_x \equiv cz + \frac{1}{\nu z}\sqrt{\frac{2}{3}}\sqrt{3c\,\nu^2 z^2\left(\frac{cz^2}{2}-3\right)+4\nu-4}+\left.\frac{g'}{2g}-\frac{2}{z}\right|_{z=z_{DWx}} = 0,$$

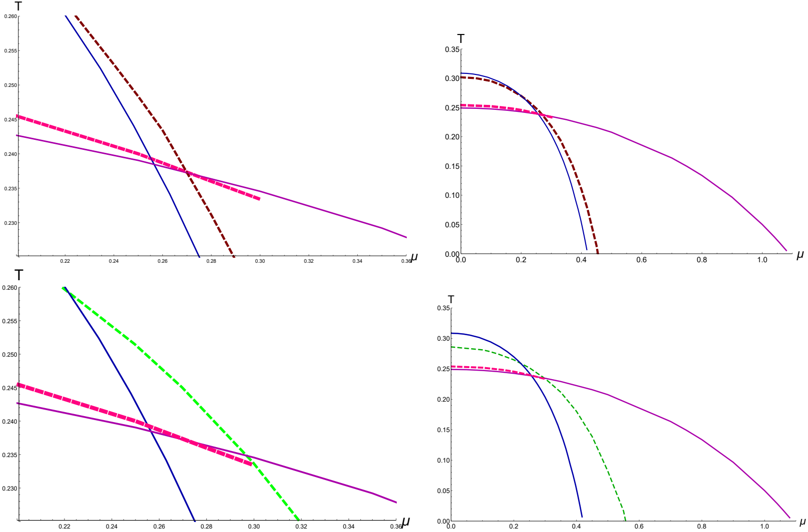
$$\mathcal{DW}_y \equiv cz + \frac{1}{\nu z}\sqrt{\frac{2}{3}}\sqrt{3c\,\nu^2 z^2\left(\frac{cz^2}{2}-3\right)+4\nu-4}+\left.\frac{g'}{2g}-\frac{\nu+1}{\nu z}\right|_{z=z_{DWy}} = 0.$$

Phase diagram

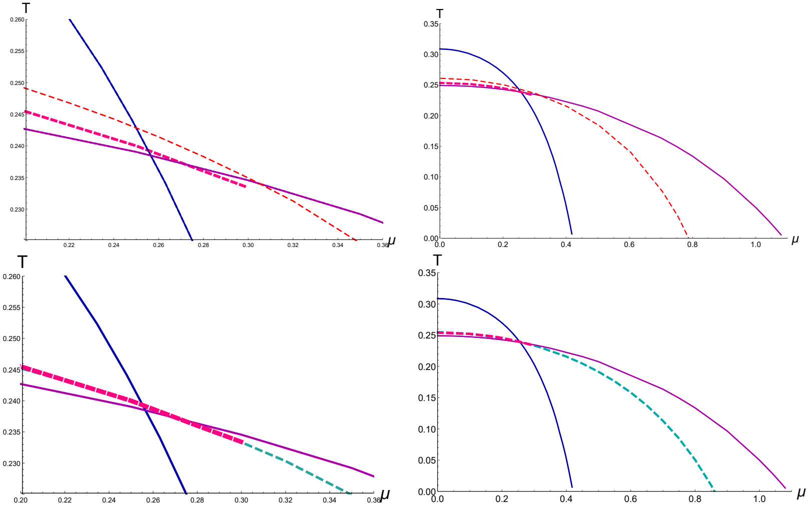
$$T(z_h,\mu,c,\nu)=\frac{g'(z_h)}{4\pi}=\frac{e^{-\frac{3cz_h^2}{4}}}{2\pi z_h}\left|\frac{1}{\mathfrak{G}\left(\frac{3}{4}cz_h^2\right)}+\frac{\mu^2 cz_h^{2+\frac{2}{\nu}}e^{\frac{cz_h^2}{4}}}{4\left(1-e^{\frac{cz_h^2}{4}}\right)^2}\left(1-e^{\frac{cz_h^2}{4}}\frac{\mathfrak{G}(cz_h^2)}{\mathfrak{G}\left(\frac{3}{4}cz_h^2\right)}\right)\right|$$



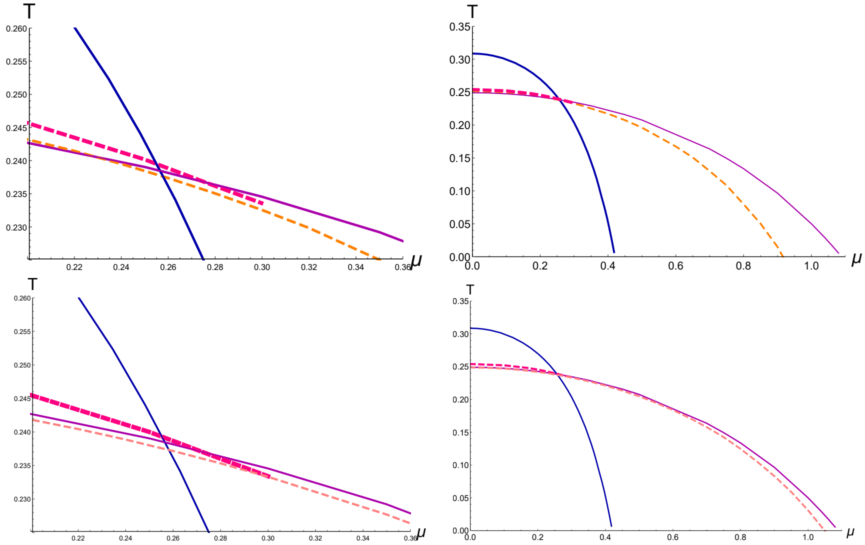
Angles $\theta = 10^0$ (brown), $\theta_{cr1} = 22^0$ (green)



Angles $\theta = 45^0$ (red), $\theta_{cr2} = 54^0$ (cyan)



Angles $\theta = 60^0$ (orange), $\theta_{cr3} = 78^0$ (pink)



Results

1)The dependence of the confinement/deconfinement phase transition on the orientation of the timelike Wilson loop is studied.

2)The expansions for the arbitrary angle are generalized.

3) The critical angles values ($\theta_{cr1} = 22^0$, $\theta_{cr2} = 54^0$, $\theta_{cr3} = 78^0$) are obtained. At $\theta_{cr2} = 54^0$, the Hawking-Page phase transition of the background metric coincides with the confinement/deconfinement phase transition. For angles $\theta < \theta_{cr2}$ the first order phase transition takes place at the top of the phase diagram.

Thank you for your attention!