Holographic RG flows at zero and finite temperatures

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based on work with
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arXiv:1803.06764

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Outline

The domain wall/ QFT correspondence

- All regular p-brane: AdS-geometry in the near horizon limit;

    Duff, Gibbons, Townsend' hep-th/9405124.

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- AdS + a dilaton $\Leftrightarrow$ Domain Wall spacetime

$$ds^2 = e^{2\mathcal{A}(r)}\eta_{ij}dx^i dx^j + dr^2, \quad \phi = \phi(r)$$

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DW/CFT dualities Itzhaki et. al.'98, Boonstra et. al.'98;Skenderis'99

- $AdS \Leftrightarrow DW$, CFT$\Leftrightarrow$ QFT,

- AdS isometry group $\Leftrightarrow$ Poincaré isometry group of DW (the dilaton breaks the scale invariance)

- non-trivial RG flow; a restoration of the conformal symmetry only at fixed points (in the UV and/or IR)

Holographic dictionary, review

$$S = M_p^{d-1} \int d^d x \int dr \sqrt{-g} \left[R - \frac{1}{2}(\partial \phi)^2 - V(\phi) \right] + S_{YH}.$$

- The scale factor $e^{\mathcal{A}}$ – measures the field theory energy scale
- The scalar field ϕ – the running coupling
-

$$\beta = \frac{d\lambda}{d \log E} = \frac{d\phi}{d\mathcal{A}}$$

The auxiliary scalar function $W(\phi)$ (aka superpotential)

$$W(\phi(u)) = -2(d-1)\frac{d\mathcal{A}}{dr}, \quad -\frac{d}{4(d-1)}W^2 + \frac{1}{2}(\partial_\phi W)^2 = V.$$

The potential can be tuned to reproduce β-function.
Gursoy,Kiritsis' 0707.1324, Gursoy et. al.'0707.1349, Gubser'0804.0434

For asymptotically AdS UV　$\lambda \to 0$　$V(\lambda) = V_0 + v_1 \lambda + v_2 \lambda^2 + \dots$

For confinement in the IR　$\lambda \to \infty$　$V(\lambda) \sim \lambda^Q (\log \lambda)^P$

Gursoy,Jarvinnen, Policastro'15 $V = -V_0 e^{\kappa \phi}$, $\kappa > 0$.

Outline

The action

$$S = \frac{1}{2\kappa^2} \int d^4x \int du \sqrt{-g} \left(R - \frac{4}{3}(\partial\phi)^2 + V(\phi) \right) - \frac{1}{\kappa^2} \int\limits_{\partial} d^4x \sqrt{-\gamma},$$

$V(\phi) = C_1 e^{2k_1\phi} + C_2 e^{2k_2\phi}$, C_s, k_s, $s = 1, 2$ are some constants.
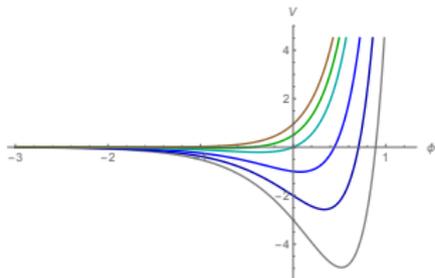

Figure: The behaviour of the potential $V(\phi)$ for $C_1 < 0$, $C_2 > 0$.

The ansatz for the metric

$$ds^2 = -e^{2A(u)}dt^2 + e^{2B(u)}d\vec{y}^{\,2} + e^{2C(u)}du^2, \quad \vec{y} = (y_1, y_2, y_3).$$

The gauge $\quad\quad C = A + 3B$ and

$$k_1 k_2 = \frac{16}{9}, \quad k_1 = k, \quad k_2 = \frac{16}{9k}, \quad 0 < k < 4/3.$$

The solution for the metric and the dilaton

$$
\begin{aligned}
ds^2 &= F_1^{\frac{8}{9k^2-16}} F_2^{\frac{9k^2}{2(16-9k^2)}} \left(-e^{2\alpha^1 u} dt^2 + e^{-\frac{2}{3}\alpha^1 u} d\vec{y}\,^2 \right) + F_1^{\frac{32}{9k^2-16}} F_2^{\frac{18k^2}{16-9k^2}} du^2, \\
\phi &= -\frac{9k}{9k^2-16} \ln F_1 + \frac{9k}{9k^2-16} \ln F_2,
\end{aligned}
$$

$$
F_s(u - u_{0s}) = \begin{cases}
\sqrt{\dfrac{|C_s|}{2|E_s|}} \sinh\left[\mu_s(u - u_{0s})\right], & \text{if } \eta_{ss}C_s > 0, \eta_{ss}E_s > 0, \\[2ex]
\sqrt{\dfrac{|C_s|}{2|E_s|}} \sin\left[\mu_s(u - u_{0s})\right], & \text{if } \eta_{ss}C_s > 0, \eta_{ss}E_s < 0, \\[2ex]
\sqrt{\dfrac{C_s}{2}}|\mu_s(u - u_{0s})|, & \text{if } \eta_{ss}C_s > 0, E_s = 0, \\[2ex]
\sqrt{\dfrac{|C_s|}{2|E_s|}} \cosh\left[\mu_s(u - u_{0s})\right], & \text{if } \eta_{ss}C_s < 0, \eta_{ss}E_s > 0,
\end{cases}
$$

$$
s = 1, 2, \quad \mu_1 = \sqrt{\left|\frac{3E_1}{2}\left(k^2 - \frac{16}{9}\right)\right|}, \quad \mu_2 = \sqrt{\left|\frac{3E_2}{2}\left(\left(\frac{16}{9}\right)^2 \frac{1}{k^2} - \frac{16}{9}\right)\right|}.
$$

Constraints

$$E_1 + E_2 + \frac{2(\alpha^1)^2}{3} = 0.$$

1. $\alpha^1 = 0$ –Vacuum solutions, Poincaré invariant, $|E_1| = |E_2|$
2. $\alpha^1 \neq 0$. – Non-vacuum solutions, no Poincaré invariance, $|E_1| \neq |E_2|$

- Conditions from the potential
$$C_1 < 0, \quad C_2 > 0, \quad 0 < k < 4/3.$$

- Constants of integration $u_{01} > 0, u_{02} < u_{01}$, 3-branch solutions

$$\begin{array}{ll} \text{left:} & u < u_{02} \\ \text{middle:} & u_{02} < u < u_{01} \\ \text{right:} & u > u_{01} \end{array}$$

- The degenerate case with $u_{01} = u_{02} = u_0$, 2-branch solutions

$$\begin{array}{ll} \text{left:} & u < u_0 \\ \text{right:} & u > u_0. \end{array}$$

Behaviour of solutions $u_{01} \neq u_{02} \neq 0$

$$ds^2 = F_1^{\frac{8}{9k^2-16}} F_2^{\frac{9k^2}{2(16-9k^2)}} (-dt^2 + dy_1^2 + dy_2^2 + dy_3^2) + F_1^{\frac{32}{9k^2-16}} F_2^{\frac{18k^2}{16-9k^2}} du^2,$$

$$
\begin{aligned}
F_1 &= \sqrt{\left|\frac{C_1}{2E_1}\right|} \sinh(\mu_1 |u - u_{01}|), \quad F_2 = \sqrt{\left|\frac{C_2}{2E_2}\right|} \sinh(\mu_2 |u - u_{02}|), \\
E_1 &= -E_2, \quad E_1 < 0, \quad E_2 > 0, \quad \mu_2 = \frac{4}{3k}\mu_1.
\end{aligned}
$$

The dilaton

$$\phi = \frac{9k}{9k^2 - 16} \log \frac{F_2}{F_1}$$

and its potential

$$V = C_1 e^{2k\phi} + C_2 e^{32\phi/(9k)} = C_1 \left(\frac{F_2}{F_1}\right)^{\frac{18k^2}{9k^2 16}} + C_2 \left(\frac{F_2}{F_1}\right)^{\frac{32}{9k^2-16}}.$$

Fixed points $u_{01} \neq u_{02} \neq 0$

- The left solution $u < u_{02}$ (the conformally flat spacetimes)
  - $u \to -\infty$ $ds^2 \sim z^{2/3} \left(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2\right)$, $z \sim e^{-\frac{3\mu_1 u}{4+3k}}$
    $\phi \sim \frac{9k}{16-9k^2}(\mu_2 - \mu_1)u \sim \log z \to -\infty$
  - $u \to u_{02} - \epsilon$ $ds^2 \sim z^{\frac{18k^2}{64-9k^2}} \left(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2\right)$,
    $z \sim \frac{64-9k^2}{4(16-9k^2)}(u - u_{02})^{\frac{64-9k^2}{4(16-9k^2)}}$,
    $\phi \sim -\frac{36k}{64-9k^2} \log z \to +\infty$.

- The middle solution $u_{02} < u < u_{01}$ (the conformally flat spacetimes)

  - $u \to u_{02} + \epsilon$ the same as for the left solution at $u \to u_{02} - \epsilon$
  - $u \to u_{01} - \epsilon$ $ds^2 \sim z^{\frac{8}{9k^2-4}} \left(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2\right)$,
    $\phi \sim \frac{9k}{4-9k^2} \log z \to -\infty$, $z \sim \frac{16-9k^2}{9k^2-4}(u - u_{01})^{\frac{4-9k^2}{16-9k^2}}$.

- The right solution $u > u_{01}$ (the conformally flat spacetimes)
  - $u \to u_{01} + \epsilon$ the same as for the middle solution at $u \to u_{01} + \epsilon$
  - $u \to +\infty$ $ds^2 \sim z^{2/3} \left(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2\right)$,
    $\phi \sim \log z \to -\infty$

Fixed points: $u_{01} = u_{02} = 0$

- In the UV $u \to u_0$ we obtain the *AdS*-spacetime

$$ds^2 \sim \frac{1}{z^2}(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2), \quad z \sim 4u^{1/4}.$$
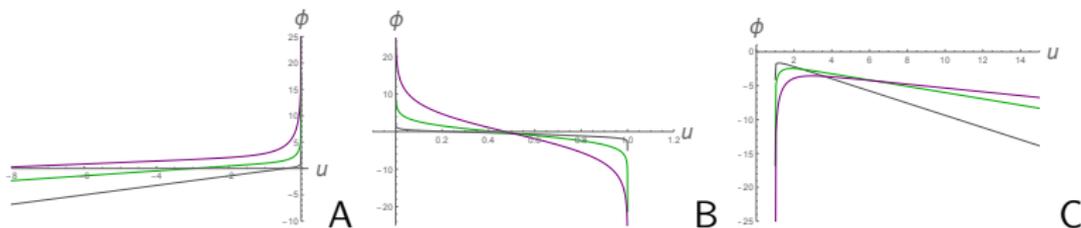
The dilaton is constant in the UV

$$\phi = \frac{9k}{16 - 9k^2} \log \frac{3k}{4} + \frac{9k}{2(16 - 9k^2)} \log \left| \frac{C_1}{C_2} \right|.$$

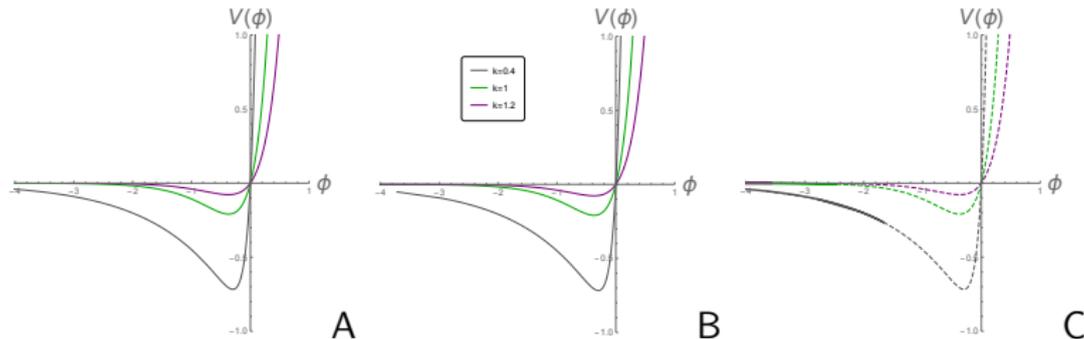- In the IR $u \to +\infty$ we obtain the conformally flat spacetime

$$ds^2 \sim z^{2/3}(-dt^2 + dy_1^2 + dy_2^2 + dy_3^2 + dz^2), \quad z \sim e^{-\frac{3\mu_1 u}{4+3k}}.$$

The dilaton in the IR

$$\phi \sim \log z \to -\infty$$

Introduction
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Holographic dual and solutions
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RG equations at $T = 0$

RG-flow at finite T

Outlook

Figure: Dilaton solutions as functions of u: A) the dilaton for $u < u_{02}$, $u_{02} = 0$ B) the dilaton for $u_{02} < u < u_{01}$, $u_{01} = 1$, $u_{02} = 0$ C) the dilaton for $u > u_{01}$, $u_{01} = 1$. For all $E_1 = E_2 = -1$, $C_1 = -C_2 = -1$, $k = 0.4, 1, 1.2$.

Figure: The dilaton potential plotted on the left, middle and right solutions for ϕ.

Dilaton and its potential $T=0$ $u_{01}=u_{02}$
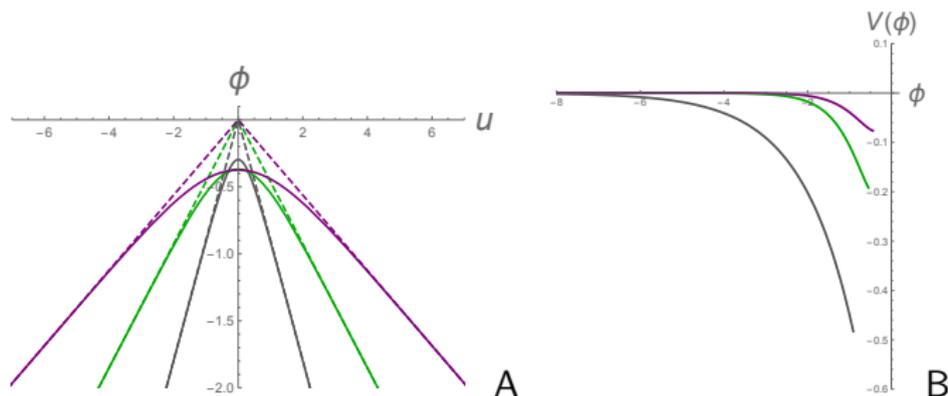
Figure: A) The behaviour of the dilaton (solid lines) and its asymptotics at infinity (dashed lines) for $u_{01}=u_{02}=0$, $C_1=-C_2=-1$, $E_1=-E_2=-1$ and different values of k. From bottom to top $k=0.4,1,1.2$. B) The dilaton potential as a function of ϕ for $u>0$.

Non-vacuum solutions

The metric reads

$$ds^2 = F_1^{\frac{8}{9k^2-16}} F_2^{\frac{9k^2}{2(16-9k^2)}} (-e^{2\alpha^1 u} dt^2 + e^{-\frac{2}{3}\alpha^1 u} \sum_{i=1}^3 dy_i^2) + F_1^{\frac{32}{9k^2-16}} F_2^{\frac{18k^2}{16-9k^2}} du^2.$$

The dilaton reads

$$\phi = -\frac{9k}{9k^2 - 16} \ln F_1 + \frac{9k}{9k^2 - 16} \ln F_2,$$

where $F_1 = \sqrt{\left|\frac{C_1}{2E_1}\right|} \sinh(\mu_1 |u - u_{01}|)$, $F_2 = \sqrt{\left|\frac{C_2}{2E_2}\right|} \sinh(\mu_2 |u - u_{02}|)$,

$$\mu_1 = \sqrt{\left|\frac{3E_1}{2}\right|} \sqrt{\frac{16}{9} - k^2}, \quad \mu_2 = \sqrt{\left|\frac{3E_2}{2}\right|} \frac{4}{3k} \sqrt{\frac{16}{9} - k^2} = \frac{4}{3k} \sqrt{\frac{E_2}{E_1}} \mu_1.$$

$$E_1 + E_2 + \frac{2}{3}(\alpha^1)^2 = 0.$$

Fixed points $u_{01} \neq u_{02} \neq 0$

- The left solution $u < u_{02}$
  - $u \to -\infty$ $\phi_{u \to -\infty} \sim \frac{9k}{16-9k^2}\left[(\mu_2 - \mu_1)\, u + \frac{1}{2}\log|\frac{C_2 E_1}{C_1 E_2}|\right]$
  - $u \to u_{02} - \epsilon$
    $\phi_{u \to u_{02}-\epsilon} \sim -\frac{9k}{16-9k^2}\log\left[\sqrt{\frac{C_2 E_1}{C_1 E_2}}\,\frac{\mu_2\,\epsilon}{\sinh(\mu_1\,(u_{01}-u_{02}))}\right] \to +\infty.$

- The middle solution $u_{02} < u < u_{01}$
  - $u \to u_{02} + \epsilon$ the same as for the left solution at $u \to u_{02} - \epsilon$
  - $u \to u_{01} - \epsilon$
    $\phi_{u \to u_{01}-\epsilon} \sim -\frac{9k}{16-9k^2}\log\left[\sqrt{\frac{C_2 E_1}{C_1 E_2}}\,\frac{\sinh(\mu_2\,(u_{01}-u_{02}))}{\mu_1\,\epsilon}\right] \to -\infty.$

- The right solution $u > u_{01}$
  - $u \to u_{01} + \epsilon$ the same as for the middle solution at $u \to u_{01} + \epsilon$
  - $u \to +\infty$ $\phi_{u \to \infty} \sim -\frac{9k}{16-9k^2}\left[(\mu_2 - \mu_1)\, u + \frac{1}{2}\log|\frac{C_2 E_1}{C_1 E_2}|\right].$

- $\mu_1 = \mu_2, \quad E_2 = \dfrac{6k^2(\alpha^1)^2}{16 - 9k^2}.$
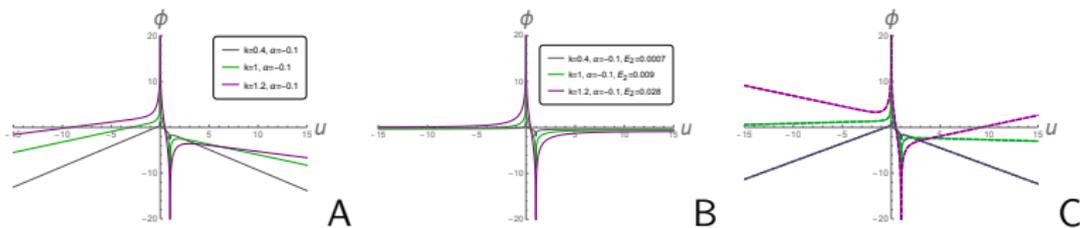- $\mu_1 > \mu_2$

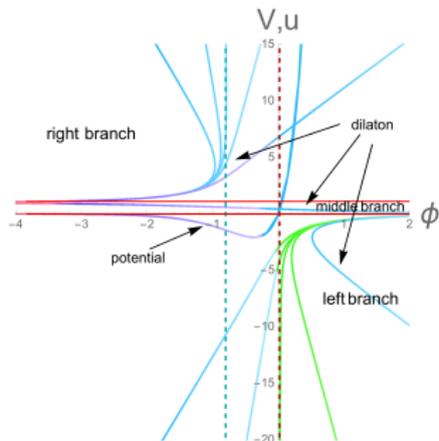Figure: The dilaton non-vacuum solutions with $u_{01} = 1, u_{02} = 0$

Figure: $V = V(\phi)$ on non-vacuum solutions $\phi = \phi(u)$ and $u = u(\phi)$.

Outline

Holographic RG equations

The solution in the domain wall coordinates

$$ds^2 = dw^2 + e^{2\mathcal{A}(w)} \left(-dt^2 + \eta_{ij} dx^i dx^j \right).$$

$\phi(w)$, $\lambda = e^{\phi}$ – the running coupling.
The β-function

$$\beta(\lambda) = \frac{d\lambda_{QFT}}{d \log E} = \frac{d\lambda}{d\mathcal{A}}$$

The β-function satisfies the holographic RG eqs.

$$\frac{dX}{d\phi} = -\frac{4}{3} \left(1 - X^2 \right) \left(1 + \frac{3}{8X} \frac{d \log V}{d\phi} \right),$$

where $X(\phi)$ is related with the β-function

$$X(\phi) = \frac{\beta(\lambda)}{3\lambda}$$

The enegy scale

$$\mathsf{A} = e^{\mathcal{A}}$$

RG equations at $T = 0$

The domain wall coordinates $dw = F_1^{\frac{16}{9k^2-16}} F_2^{\frac{9k^2}{16-9k^2}} \, du$.
The running coupling

$$\lambda = e^\phi = \left(\frac{F_2}{F_1}\right)^{\frac{9k}{9k^2-16}}.$$
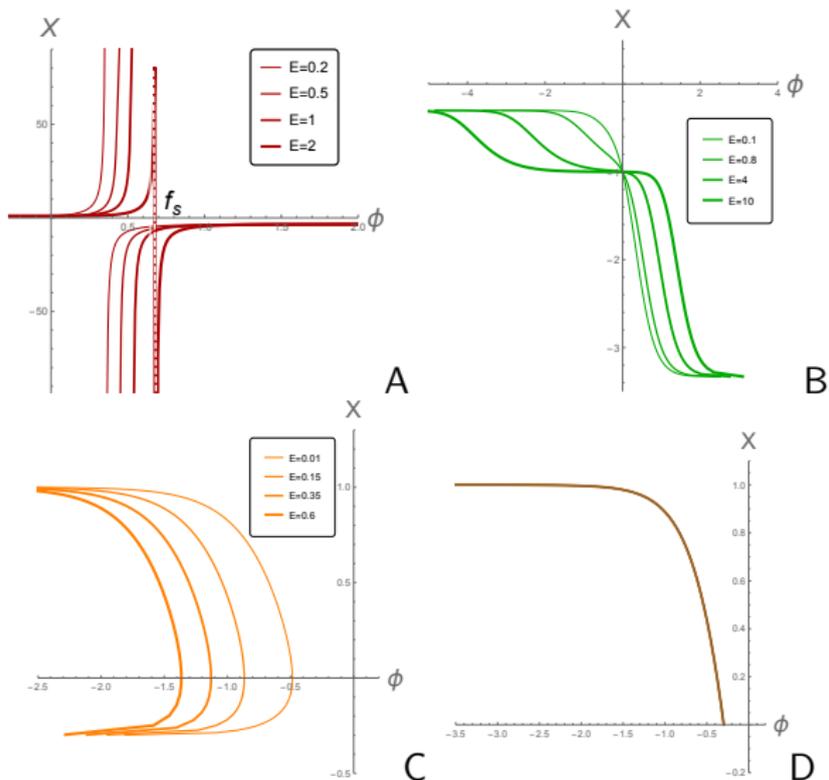
The scale factor of the domain wall

$$\mathcal{A} = \frac{4}{9k^2-16} \log F_1 + \frac{9k^2}{4(16-9k^2)} \log F_2$$

The energy scale

$$\mathsf{A} = e^{\mathcal{A}} = F_1^{\frac{4}{9k^2-16}} F^{\frac{9k^2}{4(16-9k^2)}}.$$

The X-function

$$X = \frac{1}{3}\left(\frac{F_2}{F_1}\right)^{\frac{9k}{16-9k^2}} \frac{\lambda'}{\mathcal{A}'}.$$

Figure: The behaviour of the X-function with the dependence on the dilaton plotted using the solutions for $\mathcal{A}$. A)left B) middle C)right D) $u_{01} = u_{02}$
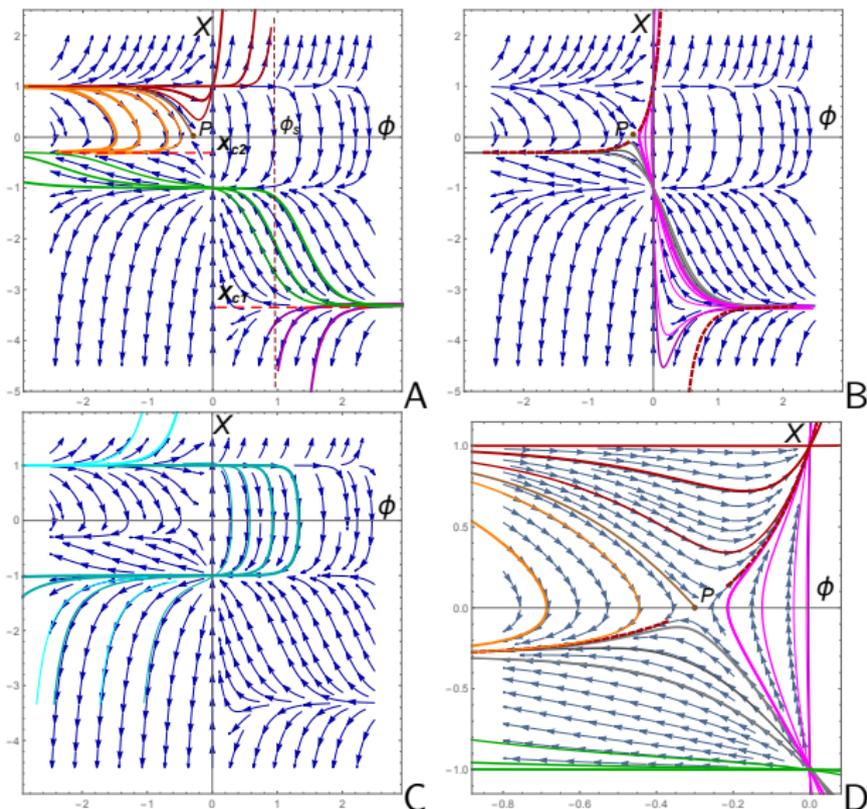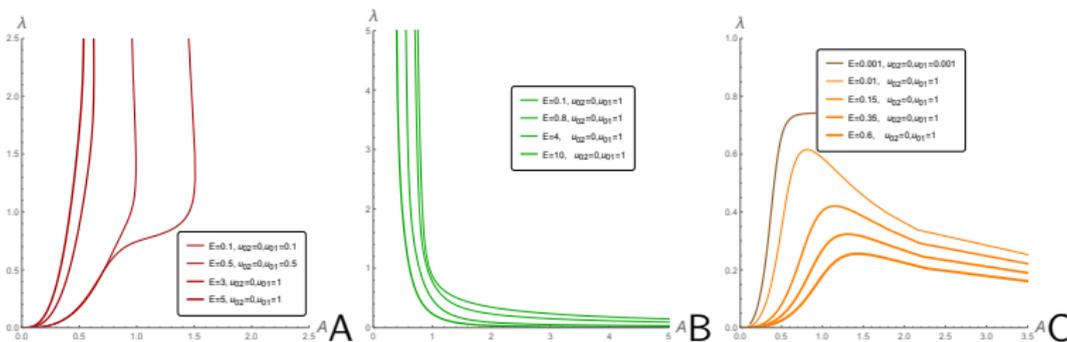
Figure: All solutions X with potential fixed as $C_1 = -C_2 = -2$ and $k = 0.4$

The behaviour of the running coupling λ on the energy scale

Figure: λ on the energy A on the dilaton plotted using the solutions for $\mathcal{A}$ and ϕ. A) the left branch with $u_{02} > u$, B) the middle branch $u_{02} < u < u_{01}$; C) the right branch $u > u_{01}$. For all plots $k = 0.4$, $C_1 = -2$, $C_2 = 2$, different curves on the same plot corresponds to the different values of $|E_1| = |E_2|$, labeled as E on the legends and different u_{01} and u_{01} also indicated on the legends.

Introduction
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Holographic dual and solutions
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RG equations at $T = 0$
RG-flow at finite T
Outlook

Outline

1. Introduction
   - Some facts about holographic RG-flow
   - Holographic dictionary

2. Holographic dual and solutions
   - Vacuum solutions
   - Non-vacuum solutions

3. RG equations at $T = 0$

4. RG-flow at finite T

5. Outlook

RG flow at finite temperature

The domain wall solution

$$ds^2 = e^{2\mathcal{A}(r)}\eta_{ij}dx^i dx^j + dr^2, \quad \phi = \phi(r).$$

The black brane

$$ds^2 = e^{2\mathcal{A}(r)}\left(-f(r)dt^2 + \delta_{ij}dx^i dx^j\right) + \frac{dr^2}{f(r)},$$

Example The Chamblin-Reall solution $f = 1 - C_2\lambda^{-\frac{4(1-X^2)}{3X}}$, $\lambda = e^\phi$.
Gursoy, Kiritsis, Mazzantini, Nitti'09, Gursoy, Jarvinen, Policastro'16
The Y-variable is defined through the function f

$$
\begin{aligned}
Y(\phi) &= \frac{1}{4}\frac{g'}{\mathcal{A}'}, \quad g = \log f, \\
\frac{dX}{d\phi} &= -\frac{4}{3}\left(1 - X^2 + Y\right)\left(1 + \frac{3}{8X}\frac{d\log V}{d\phi}\right), \\
\frac{dY}{d\phi} &= -\frac{4}{3}\left(1 - X^2 + Y\right)\frac{Y}{X}.
\end{aligned}
$$

Black brane solution

$$
\begin{aligned}
ds^2 &= \mathcal{C} \, \mathcal{X} \Big(- e^{\frac{8}{3}\alpha^1 u} dt^2 + d\vec{y}^2 \Big) + \mathcal{C}^4 \, \mathcal{X}(u)^4 e^{\frac{8}{3}\alpha^1 u} du^2, \\
\mathcal{C} &= \Big(\frac{1}{2} \sqrt{\Big| \frac{C_1}{2E_1} \Big|} e^{-\mu_1 u_{01}} \Big)^{\frac{8}{9k^2-16}} \Big(\frac{1}{2} \sqrt{\Big| \frac{C_2}{2E_2} \Big|} e^{-\mu_2 u_{02}} \Big)^{\frac{9k^2}{4(16-9k^2)}} \\
\mathcal{X}(u) &= (1 - e^{-2\mu_1(u-u_{01})})^{-\frac{8}{16-9k^2}} (1 - e^{-2\mu_2(u-u_{02})})^{\frac{9k^2}{2(16-9k^2)}}
\end{aligned}
$$

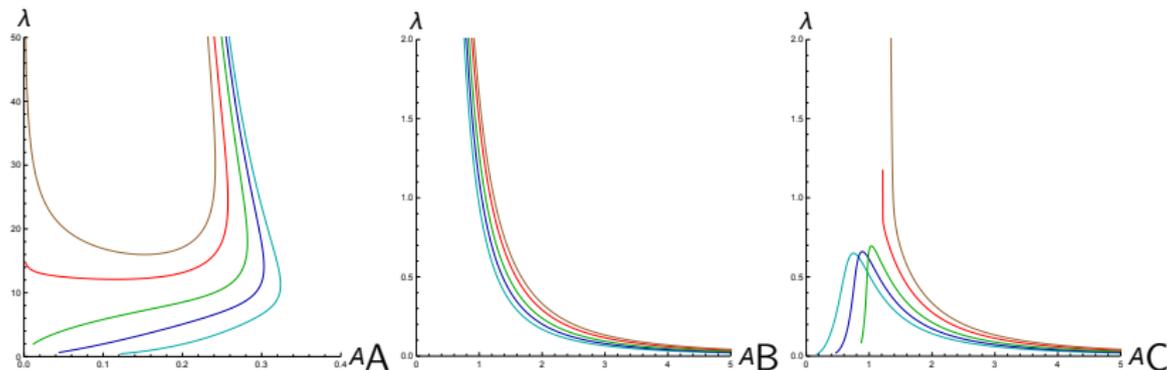Null geodesics $ds^2 = 0$, i.e. for the light moving in the radial direction

$$
t - t_0 = \int\limits_{u_0}^{u} d\bar{u} \, \mathcal{C}^{3/2} \Big(1 + ... \Big) \underset{u \to \infty}{\to} \infty.
$$

This calculation confirms that we have at $u = +\infty$ the horizon.

Hawking temperature

$$
T = \frac{2}{3\pi} \frac{|\alpha^1|}{\mathcal{C}^{3/2}}
$$

The running coupling on the energy scale ($u_{01} \neq 0, u_{02} \neq 0$)

Figure: The dependence of λ on the energy scale A$= e^{\mathcal{A}}$ at the left solution A), the middle solution B) and the right one C). $\alpha^1 = 0$ (cyan), $\alpha^1 = -0.25$ (blue), $\alpha^1 = -0.5$ (green), $\alpha^1 = -0.8$ (red), $\alpha^1 = -1$ (brown).

Outline

The bottom line

Done

- Vacuum and non-vacuum holographic RG-flows were constructed
- Holographic RG flows can have AdS fixed point in the UV and hyperscaling violating boundary in the IR region.
- The holographic running coupling can mimic QCD behaviour at zero T.
- Exact solutions allow not to deal with superpotential W

The bottom line

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- The holographic running coupling can mimic QCD behaviour at zero T.
- Exact solutions allow not to deal with superpotential W

?

- Analysis of the non-vacuum solutions ($T \neq 0$).
- Careful studies of the behaviour $\lambda = e^{\phi}$ on the energy scale at $T \neq 0$.
- Analysis of confinement-deconfinement phase transition.

Thank you for attention!