Metal or insulator? Dirac operator spectrum in holographic QCD

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Motivation

- How the QCD ground state in confinement and deconfinement phases looks like?
- Spectrum of the Dirac operator- important probe of the ground state structure
- Tool - to look at the QCD via methods of disordered systems. Spectral correlators-more refined object. Anderson transition?
- Surprizes from lattice QCD in deconfined phase
- How it matches with holographic picture with BH background ?

Spectral properties of Dirac operator in low-energy QCD

– Casher-Banks relation(82)

$$D\psi_n = i\lambda_n\psi_n$$

$$\rho(\lambda) = < \sum_n \delta(\lambda - \lambda_n) >_{QCD}$$

$$< \bar{\Psi}\Psi >= \Sigma = \frac{\pi\rho(0)}{V}$$

– Smilga-Stern linear correction(93)

$$\frac{\rho'(0)}{\rho(0)} = \frac{(N_f - 2)(N_f + 1)\Sigma}{16\pi N_f F_\pi^2}$$

--Leutwyler-Smilga sum rules in finite volume (90)

--Microscopic and macroscopic spectral densities and correlators from two different matrix models (Verbaarschot et al 94-....).

Spectral curve for QCD

Verbaarschoot et al, 94

$$g^3 - 2zg^2 + g(z^2 - t^2 + 1/\Sigma^2) - z/\Sigma^2 = 0\,.$$

$$g = G/\Sigma^2$$

$$G(z) = \frac{1}{N}\,\mathrm{Tr}\left\langle \frac{1}{z-\mathcal{D}} \right\rangle\,,$$

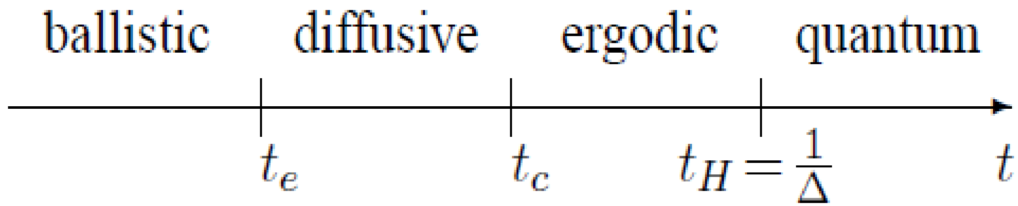
The spectral correlator is derived from the analytic properties of the correlator of two resolvents on the spectral curve

This is stardard object in the matrix model framework. There are examples of criticality in such correlators when one point
Is placed at the pinched cycle (c=1 matrix models,for example)

Topological recursion?

QCD as chiral disordered system

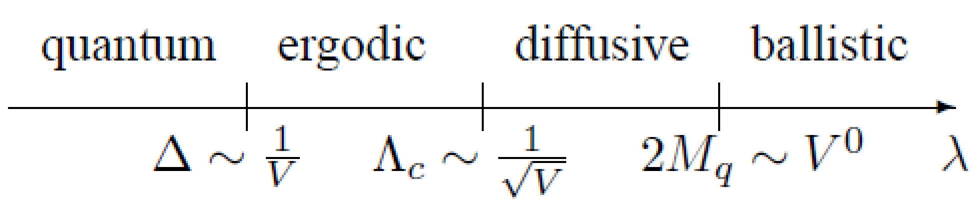
- Consider the QCD Dirac operator in the Euclidean 4d space-time as Hamiltonian of disordered system in 4+1 space
- Disorder — vacuum fluctuations of gluon field(instanton-antiinstantons?)
- Hamiltonian in Euclidean 4d is conjugated to the Schwinger proper time variable(Janik, Nowak,Zahed — 98, Osborn -Verbaarchoot 98)
- The low-energy QCD can be described in terms of chiral diffusion at some energy scale



Hierarchy of the time-scales in diffusion in disordered system

$E_c = 1/t_c = D/L^2.$ – Thouless energy

$D = F_\pi^2/|<\bar{q}q>|$ --- diffusion coefficient in low-energy QCD
(Stern-98, JNZ- 98, OV- 98)



Hierarchy of energies
In low-energy QCD

Localization-delocalization

- The standard question for disordered systems- the wave functions are localized or delocalized? Localized — insulator, delocalized — metal.
- If there are no topological terms the wave functions are localized in $d<3$ and there can be mobility edge at $d=3$ and $d>3$ at strong disorder
- Mobility edge separates energies when corresponding states are localized and delocalized. Critical behaviour at mobility edge
- Famous Anderson transition('58)- no order parameter. Non-Anderson transition ('13-18)

Localization-delocalization in QCD

- The Dirac operator spectrum is delocalized in the confinement phase(metal!). Conjectured Parisi-'82,confirmed at lattice(Garsia-Garsia,Osborn '06)
- Big surprise — there is mobility edge in the deconfinement phase! Low-energy modes in deconfined phase are localized (GGO '06). New scale in deconfined phase
- Confirmed by several groups. There is strong indication that Polyakov loops are the source of disorder. Temperature dependence!

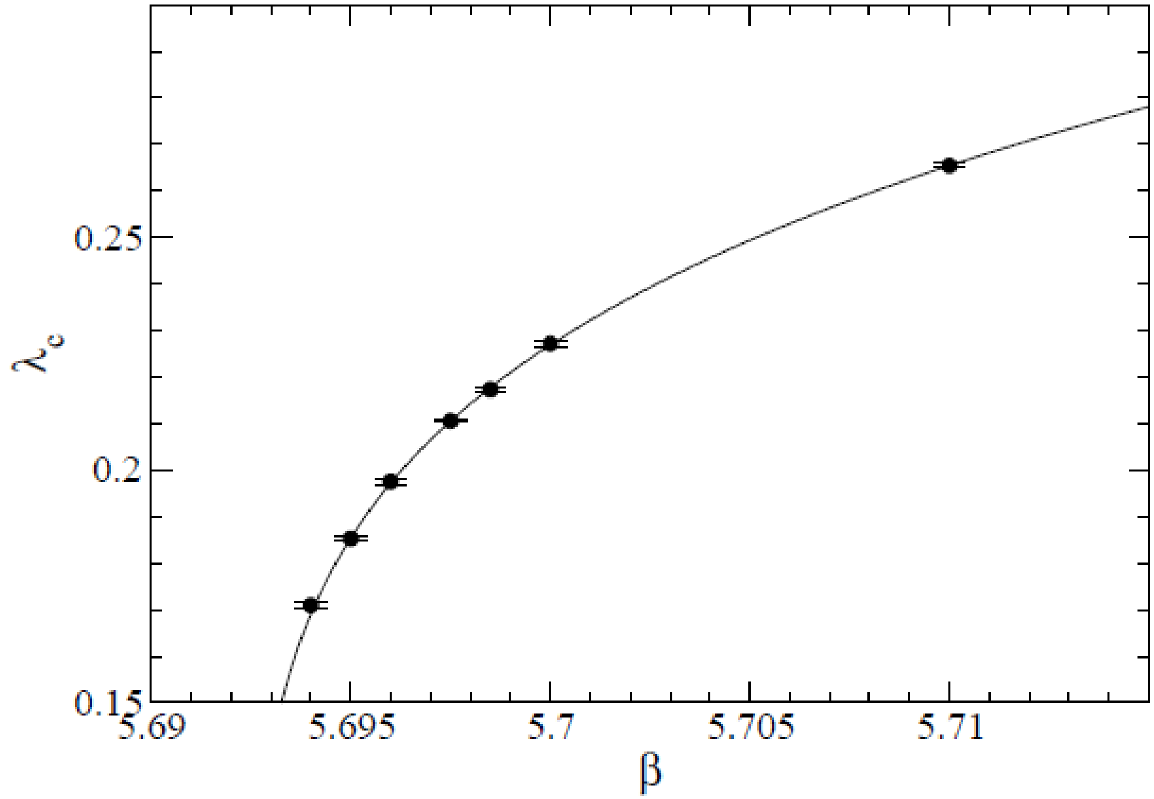


FIG. 3. The mobility edge λ_c as a function of the inverse gauge coupling for temporal lattice size $N_t = 4$. The solid

N_t	$\beta_c{}^{\text{deconf}}$	$\beta_c{}^{\text{loc}}$
4	5.69254(24)	5.69245(17)
6	5.8941(5)	5.8935(16)

TABLE II. The critical gauge couplings where the localization and the deconfining transition happen for $N_t = 4$ and $N_t = 6$ temporal lattice sizes.

The temperature of deconfinement phase transition equals with high accuracy to the temperature when mobility edge appears(Kovacs et al,'17)

Digression. Tools for diagnostics of localization transition

– Two-point spectral correlator and spectral formfactor

$$\rho(\lambda) = Im < Tr\frac{1}{H-\lambda} >$$

$$R(\lambda_1, \lambda_2) = < \rho(\lambda_1)\rho(\lambda_2) > - < \rho(\lambda_1 >< \rho(\lambda_2) >$$

$$K(E,t) = \frac{1}{2\pi\hbar}\int d\lambda R(E+\lambda, E-\lambda)e^{\frac{i\lambda t}{\hbar}}$$

In our case t — RG scale

– Level spacing distribution

$$\begin{cases} P_{deloc}(s) = A\, s\, e^{-Bs^2} & \text{below mobility edge, } \lambda_m \\ P_{loc}(s) = e^{-s} & \text{above mobility edge, } \lambda_m \end{cases}$$

Tools for diagnostics

-The level number variance

$$< (\delta N)^2 >= \int (< N > -s) R(s) ds \propto \int\int d\lambda_1 d\lambda_2 R(\lambda_1 - \lambda_2)$$

It behaves differently in metal phase, insulator phase and at criticality

– Violation of the sum rules(anomalous UV contribution)

$$R(\lambda) \propto \lambda^{-1+\frac{D_2}{d}}$$

Anomalous UV behavior
If one eigenvalue is fixed
At criticality

$$\int ds R(s) = 1$$

broken

$$\sum_{r,n} < |\Psi_n(r)|^{2p} \delta(E - E_n) > \propto L^{-D_p(p-1)}$$

Critical matrix models for the localization transition

- One-matrix model with the shallow-confinement potential (Mirlin et al '94). Analogue of the matrix model for Chern-Simons theory
- «Gaussian model» with disorder external field. Analogue of the non-singlet sectors(Kravtsov et al, '95)
- Matrix model with constraints for the distribution of the matrix elements (Muttalib et al '95) .Such model used in polymer physics.
- All matrix models have some parameter which yields the fractality dimension.

Critical matrix models

1. $P(H) \propto \exp(-\beta Tr H^2 - \beta b Tr([\Omega, H][\Omega, H]^\dagger)$ $\Omega = diag(\exp(\frac{2\pi i k}{N}))$. $b = \mu N^2$ at $N \to \infty$

First critical model («non-singlet» contribution)

2. $V(x) \to c \log^2 x, \quad x \to \infty$ $P(H) \propto \exp(-\beta Tr V(H))$

Matrix model which can be solved in terms of q-Hermit. Used in description of CS theory.

3. $< H_{ij} >= 0, \quad < (H_{ij}^2 >= \beta^{-1}[1 + \frac{(i-j)^2}{B^2}]^{-1}$

Polymer matrix model B- parameter

Back to QCD

---- Non-Anderson transition at mobility edge if ! (Fradkin 86-87, Gurarie,Radzikhovsky Syzranov 13-18)

$$d > 2\chi, \qquad E \propto k^{\chi}$$

In QCD d=4 and linear dispersion relation. Non-Anderson. New features — possible order parameter and complicated Interplay between order parameter and the localization critical point. Importance of Griffiths effects.

--- Meaning of the spectral correlators

Dirac operator is dual to the Schwinger proper time. On the other hand Schwinger proper time = radial coordinate in AdS(Gopakumar 03, Lysov A.G 04, Furuuchi 05). Eigenvalue of the Dirac operator — dual radial AdS coordinate. Spectral correlator measures the correlation of the quark wave-functions at two different RG scales.

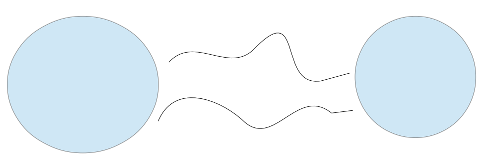
Back to QCD. Remarks

$$K(E,t) \propto \frac{|t|}{(2\pi\hbar)^2}\frac{d\Omega}{dE}p(t)$$

Return probability to the same point at time t with the energy E. Semiclassical formulae.

$$K(x,y) = \frac{1}{2}(xy)^{N_f+1/2}\frac{Z_{N_f+2}(m_1 = ix, m_2 = iy)}{Z_{N_f}}$$

Spectral correlator at two eigenvalues x,y
Is equal to the partition function with two additional flavors with imaginary masses. The masses can be made imaginary by switching on theta term

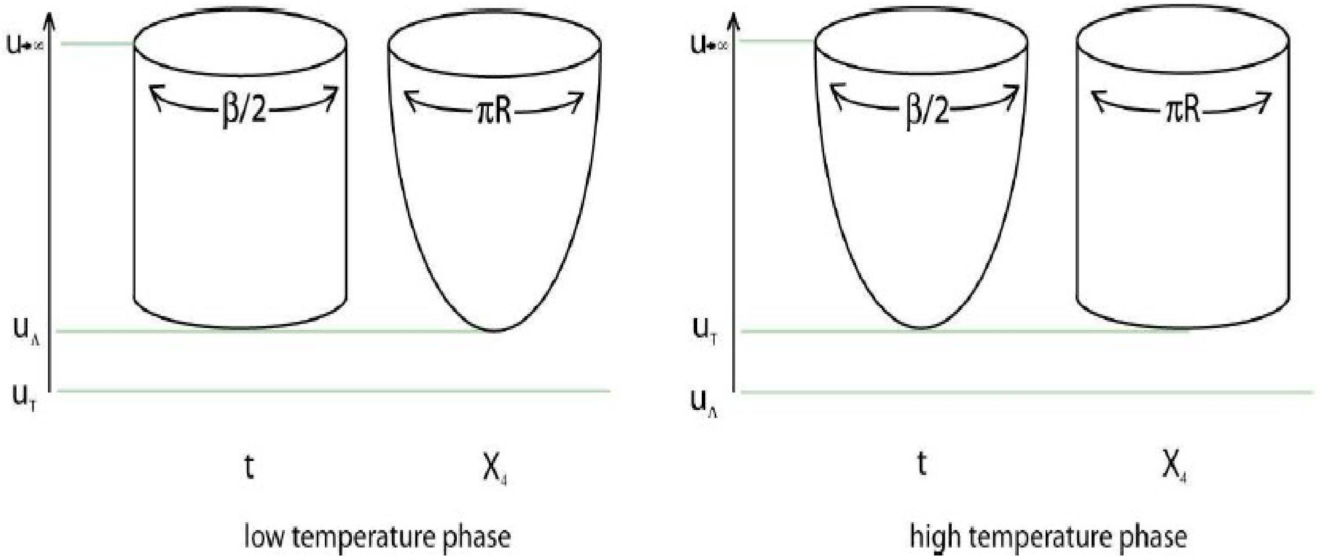


Roughly — spectral correlator in QCD is the correlator of two fermionic determinants with two different imaginary masses.

Equivalently — weighted sum of the correlators of two Wilson loops in fundamental averaged over the disorder and over the Wilson loop shapes.

Deconfinement phase transition in holography

$$ds^2 = (r/R)^{3/2} f(r) d\phi^2 + (R/r)^{3/2} \frac{dr^2}{f(r)} \qquad f(r) = 1 - (\frac{r_{kk}}{r})^3$$



Thermal and KK circles get interchanged at phase transition.
The high temperature phase background involves black hole.

QCD at deconfinement phase

Using Eq. (3), one can construct the general free energy density of two-flavor QCD invariant under $SU(2)_R \times SU(2)_L$ but *not* under $U(1)_A$, as

$$f = f_0 - f_2 \mathrm{tr} M^\dagger M - f_A (\det M + \det M^\dagger) + O(M^4)\,.$$

↗ anomaly

$$f(\theta) = \tilde{f} - 2 f_A m_u m_d \cos\theta + O(m^4),$$

$$\chi \equiv -\frac{1}{V_4} \frac{\partial^2 \ln Z_{\mathrm{QCD}}}{\partial \theta^2}\bigg|_{\theta=0} = 2 f_A m_u m_d + O(m^4).$$

$$\chi = -\frac{m}{N_f^2}\langle \bar{\psi}_f \psi_f \rangle + \frac{m^2}{N_f^2} \int d^4x \, \langle \bar{\psi}_f \gamma_5 \psi_f(x) \bar{\psi}_g \gamma_5 \psi_g(0) \rangle\,,$$

Veneziano-Witten relation
Is fulfilled

$$Z_{\mathrm{QCD}} = e^{-V_4 \tilde{f}} \sum_{N_+=0}^{\infty} \frac{(V_4 \lambda e^{i\theta})^{N_+}}{N_+!} \sum_{N_-=0}^{\infty} \frac{(V_4 \lambda e^{-i\theta})^{N_-}}{N_-!}\,.$$

Formally rewritten in this form
Kanazawa'15

Conjecture and evidences

- Mobility edge scale is close to the BH horizon
- Anomaly matching.Conductance jumps at the BH horizon
- Critical matrix models reveals the BH interpretation
- Kanazawa relations
- Brane picture. Long fundamental string stretched to horizon. Vortex condensation at the string worldsheet near BH.
- Temperature dependence

Anomaly matching and conductivity jump

At the mobility edge the conductivity has to vanish or at least jumps

According to the analogy with the disordered matter
F_π
the conductivity in QCD is proportional to the pion decay constant.

F_π

Can we recognize the energy scale where the jump of the pion decay constant occurs?

Anomaly matching and conductance jump

$$< V^{a\perp}_\mu(q) V^{b\perp}_\nu(-q) >= \delta^{ab}\Pi^\perp_{\mu\nu}(q) q^2 \Pi_V(q)$$

$$< A^{a\perp}_\mu(q) A^{b\perp}_\nu(-q) >= \delta^{ab}\Pi^\perp_{\mu\nu}(q) q^2 \Pi_A(q)$$

$$< V^a_\mu(k) V^{\perp b}_\nu(-k-Q) A^{\perp c}_\alpha(Q) >= \frac{Q^2}{4\pi^2}\epsilon_{\mu\nu\alpha\beta} k^\beta w_T(Q)(t^a t^b t^c),\ k \to 0$$

$$\Pi^\perp_{\mu\nu}(q) = \eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$

$$\omega_T = \frac{N_c}{Q^2} - \frac{N_c}{F_\pi^2}(\Pi_A - \Pi_V)$$

Son-Yamamoto relation for anomaly matching.
Relates the 2-point and 3-point correlators

$$F_\pi^{-2}(z) = \int_0^z \frac{1}{f^2(z)}$$

Jump occurs at the BH horizon!

SY relation is diagonal with respect to the RG flows (Dubinkin,Milekin,A.G '14)

Temperature dependence of the mobility edge

$$f(r_h) = 0 \qquad f(r) = 1 + z^2 - \frac{M_{BH}}{z^2} \qquad \beta = \frac{4\pi}{f'(r_h)}$$

Near the maximum of the function $T^{-1}(r_h)$ the relation holds

$$(T - T_c) \propto (r - r_h)^2$$

$$\lambda_m(T) \propto \sqrt{T - T_c}$$

Rough consistency with the temperature dependence of the mobility edge

Worldsheet arguments

--- The open string near the 2d black hole has specific behavior. The vortices at the wordsheet become very important and get condensed (Kazakov,Kostov,Kutasov 2001)

– Vortices can be related to the non-singlet sector of the Corresponding matrix models. Non-singlets interacts via Calogero type Hamiltonians (Gross-Klebanov'91)

--Non-singlets take part in the formation of BH horizon? (Maldacena '05)

– We argue that the critical behavior in the spectral correlator can be traced both at small energy difference and in UV\IR regime. This is consistant with open string extended along the radial coordinate.

Quark as the open string. Worldsheet arguments

- Quark in holography — open string connecting the horizon and the flavor brane. Spectral correlator — partition sum with two artificial flavors. Hence we actually investigate the correlator of two open strings ending at different radial distances («quark» masses).Two Wilson loops.
- If we investigate the mobility edge the end of one string is near horizon region
- Spectral correlaror at the boundary is the correlator of two fields on the worldsheet viewpoint

Matrix model evidences. All three critical matrix models yield the same spectral correlator

$$R(E,s) = < \rho(E)\rho(E+s) > = \delta(s) + Y(E,s)$$

$$Y(E,s) \propto \frac{\pi^2\eta^2}{4} \frac{sin^2(\pi s)}{sinh^2(\pi^2 s\eta/2)} \qquad \beta = 2$$

Small s region

$$\chi = 1 + \int_{-\infty}^{+\infty} Y(E,s) ds$$

$$\chi = \frac{d - D_2}{2d}$$

Fractality index 0f the critical point

Mapping to Calogero model

$$< \rho(E)\rho(E') > \rightarrow < \rho(x)\rho(x') >$$

Spectral correlator in the critical model gets mapped into density-density correlator in the Calogero model at finite temperature

$$H_{rat} = \sum_i p_i^2 + \sum_i x_i^2 + (\beta - 1)\beta \sum \frac{1}{(x_i - x_i)^2}$$

↑

Dual systems (Nekrasov'96) . In our case — large N limit

$$H_{trig} = \sum_i p_i^2 + (\beta - 1)\beta \sum \frac{1}{sin(x_i - x_i)^2}$$

Hamiltonian in the first system concides with the momentum in the second system

Scalar field in BH background

$$\rho(x,\tau) = \rho_0 + \frac{\partial_x \Phi(x,\tau)}{\pi} + A\cos(4\pi x + 2\Phi(x,\tau)) + ...$$

Introduce the scalar field- on the worldsheet -effect of vortices

$$G(x - x') = \frac{1}{2\pi} ln(\frac{\pi T}{sinh(\pi T|x - x'|)})$$

Correlator at finite temperature

The spectral correlator at the critical regime at small energy difference

||

Kravtsov et al 2009

Two-point correlator of scalar in the 2d BH background

Particle near BH horizon — Calogero system(Gibbons-Townsend '96)

Arguments from Kanazawa relations

$$\int_0^\infty \mathrm{d}\lambda\, \frac{m^2}{(\lambda^2+m^2)^2} R_1(\lambda) = f_A + \mathcal{O}(m^2)\,.$$

$$\left\langle \sum_k{}' \frac{1}{\lambda_k^4} \right\rangle_Q = \frac{(V_4 f_A)^2}{1+Q}$$

$$\langle \rho^A(\lambda)\rho^A(\lambda')\rangle_{\mathrm{Po}} = \{\delta(\lambda-\lambda')+\delta(\lambda+\lambda')\}\,\langle\rho^A(\lambda)\rangle + \left(1-\frac{1}{N}\right)\langle\rho^A(\lambda)\rangle\,\langle\rho^A(\lambda')\rangle\,,$$

$$T_2(\lambda,\lambda') \equiv R_{\mathrm{C}}^{\mathrm{Po}}(\lambda,\lambda') - R_{\mathrm{C}}(\lambda,\lambda')\,.$$

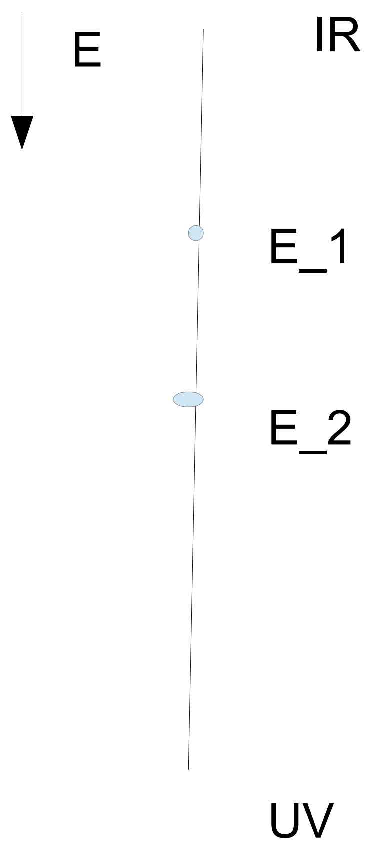
$$\int_{-\infty}^\infty \mathrm{d}\lambda \int_{-\infty}^\infty \mathrm{d}\lambda' \frac{T_2(\lambda,\lambda')}{(i\lambda+m)(i\lambda'+m)} = 2f_A + \mathcal{O}(m^2)\,.$$

$$T_2(0,0) = \frac{2}{\pi^2} f_A$$

Kanazawa '16

Combining Kanazawa relation, lattice data and holographic viewpoint about U_a(1) (Bergmann-Lifshitz'07) we can conclude that this symmetry is not anomalous at low energy which is consistant with holography and our conjecture about mobility edge.

Geometry of spectral correlator

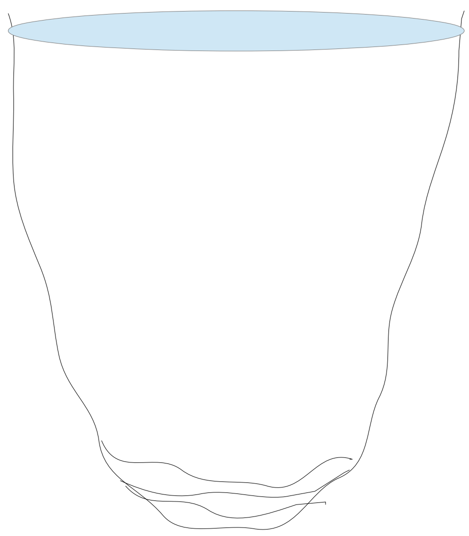


E- coordinate dual to the radial coordinate
In AdS. To get criticality one has to put
E_1 at near horizon region in IR

Disorder versus Polyakov loops

- The Polyakov loops get condensed in the deconfined phase. Geometrically the loops around the cygar shrink to the horizon region.Similar to the shrinking of «instanton» loops in the confinement phase. 3D!
- Condensate of Polyakov loops is inhomogeneous in 3d. Random rod model in condmat! Disorder is align in the euclidean time
- Consistant with the latttice studies. Correlation of strong disorder with the areas with large Polyakov loops(Kovacs et al '17)

Lattice result corresponds to the 3d (not 4d) criticality indexes at localization transition.
Consistant with holography.



$$< \exp \oint A_0 dt > \neq 0$$

Condensate of the Polyakov loops
Near horizon region

D0 branes wrapped around the thermal circle. Compare with D0 branes wrapped KK circle which yield the theta-dependence
In the confinement phase

Comparison with SYK model

- Idea of SYK — use simple boundary model to recognize the quantum physics of 2d BH. Here we have 5d BH and Dirac operator spectrum-probe of BH physics at the boundary
- The same tools- spectral correlators, spectral formfactors, Lyapunov exponents
- New feature- mobility edge in deconfined QCD. Not identified yet in SYK.
- Probably deconfined QCD is more complicated due to different type of disorder but lattice QCD is well developed contrary to SYK.

Conclusion

- New scale at the deconfined phase. Non-Anderson localization transition
- Probably new interesting phase at small energy in deconfinement regime. Mott glass?
- Highly interesting to investigate the critical regime. It could tell us smth about BH horizon.
- How it behaves at finite density? Is there the mobility edge? Horizon of charged BH?
- Probably new possibilities to define order parameters for the confinement.