Tidal charge and its extensions in black hole astrophysics

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Reissner-Nordstrom metric :

$$\Delta(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

where M is the black hole mass, Q is the electric or tidal charge.

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Metric under consideration:

$$\Delta(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{A}{r^3}$$

where M is the black hole mass, Q is the charge, A is the third correction.

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$$ds^2 = \Delta(r)dt^2 - \frac{dr^2}{\Delta(r)} - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Metric under consideration after normalization:

$$\Delta(\hat{r}) = 1 - \frac{2}{\hat{r}} + \frac{q}{\hat{r}^2} + \frac{\alpha}{\hat{r}^3}$$

where ř=r/M, $q=Q^2/M^2$, $\alpha=A/M^3$

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The curves of the changing between states with different numbers of horizons are determined by the system of equations:

$$\Delta(\hat{r}) = 0, \qquad \frac{d\Delta(\hat{r})}{d\hat{r}} = 0.$$

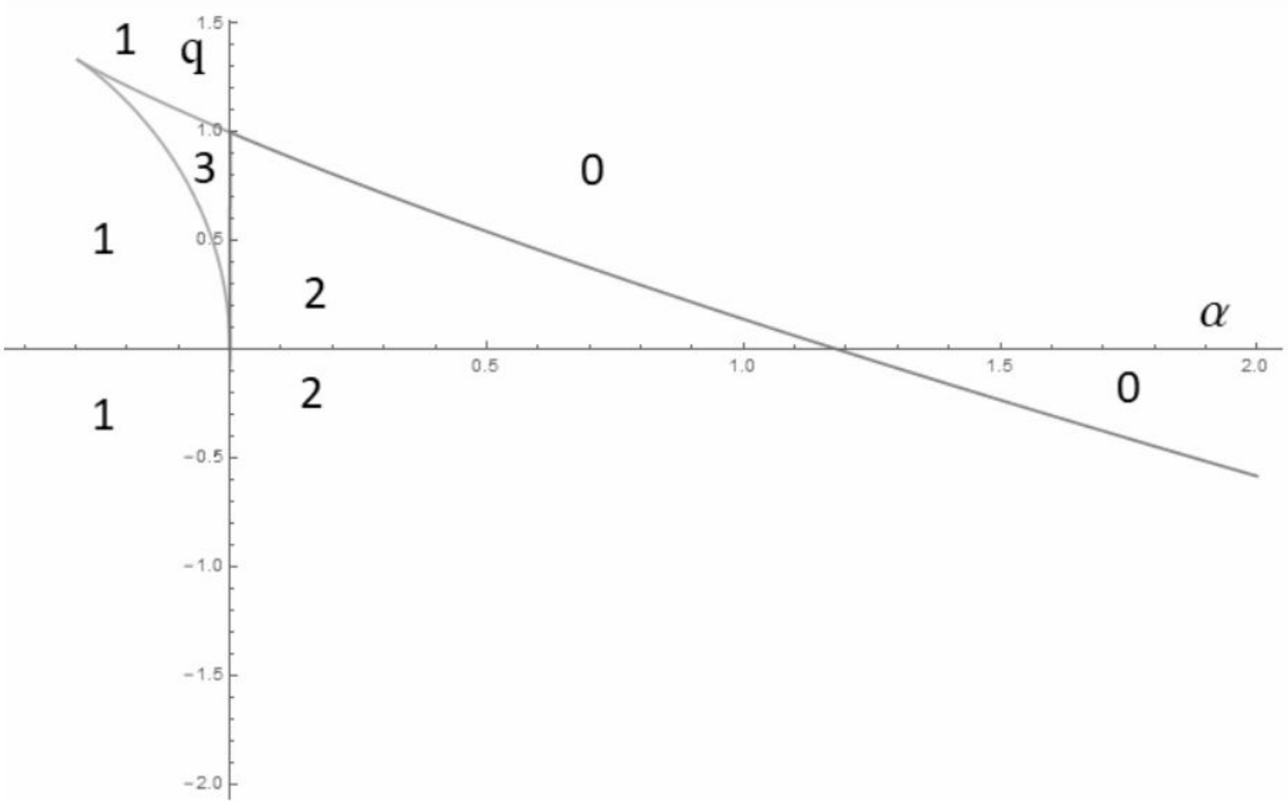

Fig. 1 Configuration space of the model with the third correction divided in areas with various number of horizons.

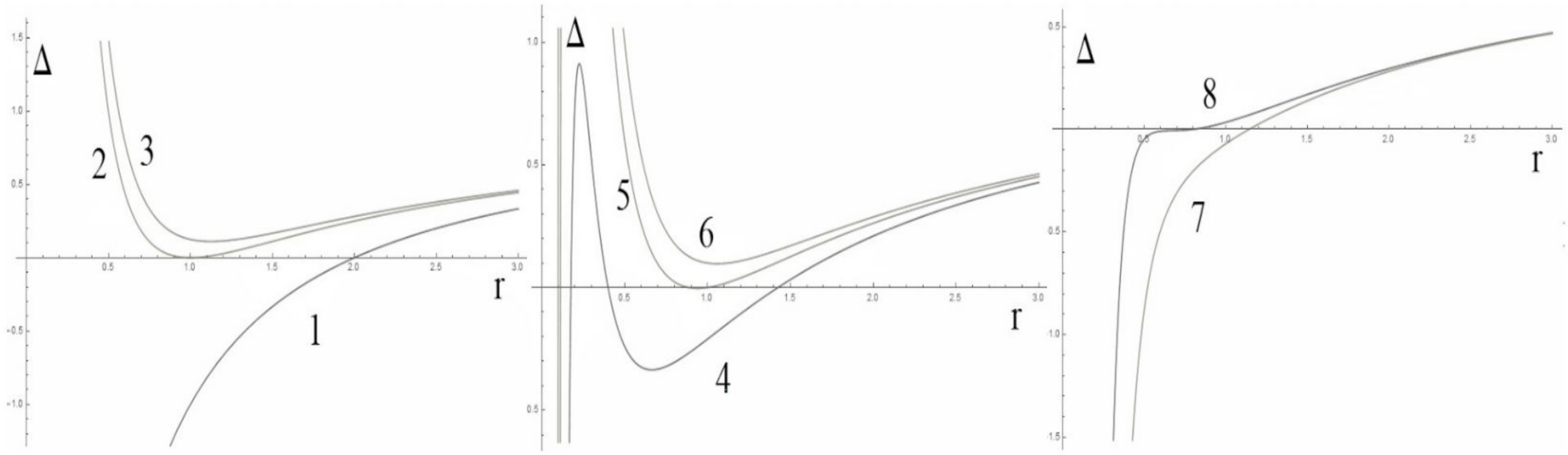

Fig. 2 Plots of the metric function Δ of the generalized Reissner-Nordström ansatz with different values of q and α. 1)$\alpha = 0$, $q = 0$; Schwarzschild BH 2) $\alpha = 0$, $q = 1$; Reissner-Nordström BH with critical charge. 3) $\alpha = 0$, $q = 9/8$; "naked singularity" in Reissner-Nordström ansatz. 4) $\alpha = -0.1$, $q = 0.89$; BH with three horizons. 5) $\alpha = -0.1$, $q = 1.1$; BH with two horizons. 6) $\alpha = -0.1$, $q = 1.2$; BH with one compact horizon. 7) $\alpha = -0.4$, $q = 1.33$; BH with one horizon. 8) $\alpha = -0.296$, $q = 1.33$: end of horizon disappearance curve.

Black hole shadow.

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Following Chandrasekar, photon geodesic equation reads:

$$\left(\frac{d\hat{r}}{d\tau}\right)^2 + \left(1 - \frac{2}{\hat{r}} + \frac{q}{\hat{r}^2} + \frac{\alpha}{\hat{r}^3}\right)\frac{L^2}{\hat{r}^2} = E^2,$$

$$\frac{d\phi}{d\tau} = \frac{L}{\hat{r}^2},$$

where E is the photon energy, L is the angular momentum of a photon beam, and τ is an affine parameter.

Black hole shadow.

We map photons motion by the angle ϕ, so the equation reads:

$$u(r) = \left(\frac{d\hat{r}}{d\phi}\right)^2 = \frac{\hat{r}^4}{D^2} - \hat{r}^2\left(1 - \frac{2}{\hat{r}} + \frac{q}{\hat{r}^2} + \frac{\alpha}{\hat{r}^3}\right)$$

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Such orbits are described by the following equations:

$$u(r) = 0, \quad \frac{du(r)}{dr} = 0.$$

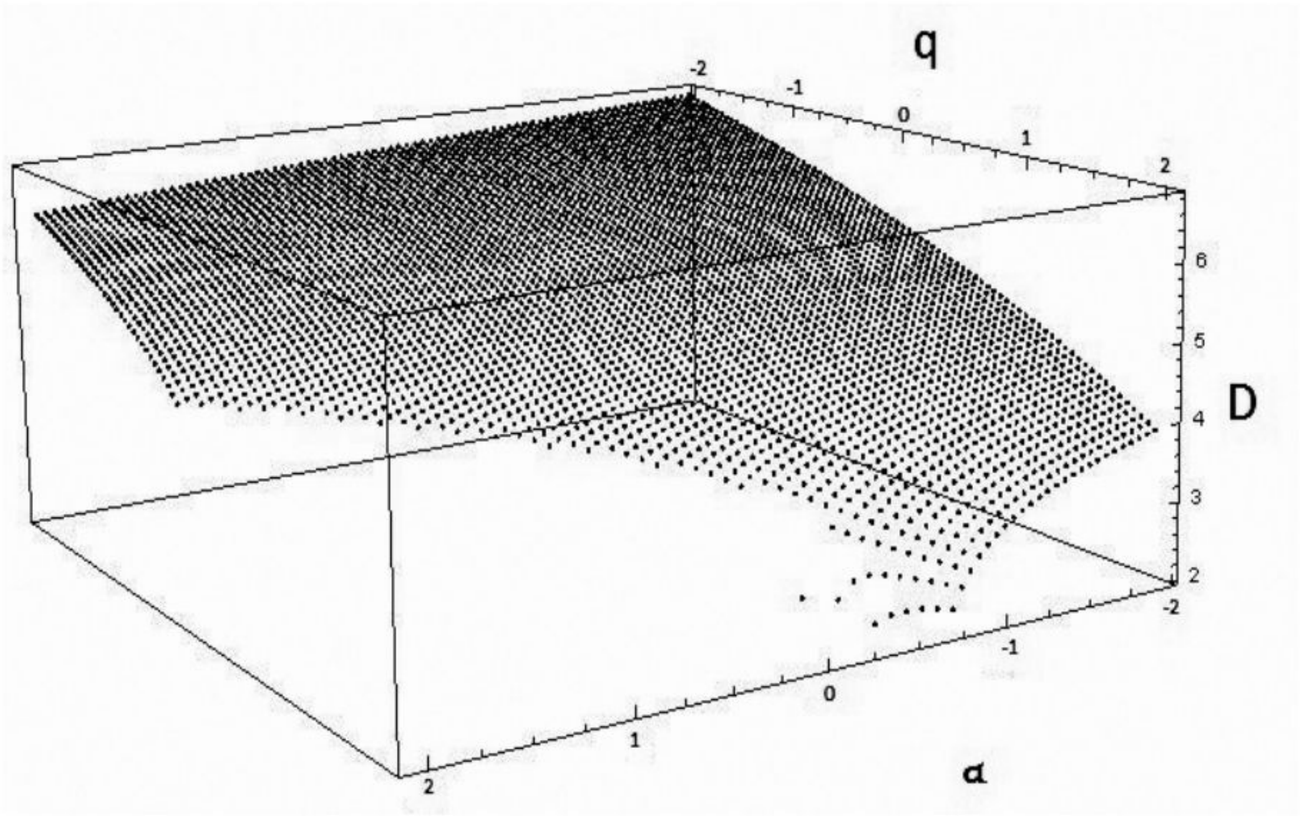

Fig. 3 The dependence of the critical impact parameter D on q and α

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Critical configurations separating the configuration space into areas where the object either has or doesn't have a shadow correspond the following system:

$$u(r) = 0, \quad \frac{du(r)}{dr} = 0, \quad \frac{d^2u(r)}{dr^2} = 0.$$

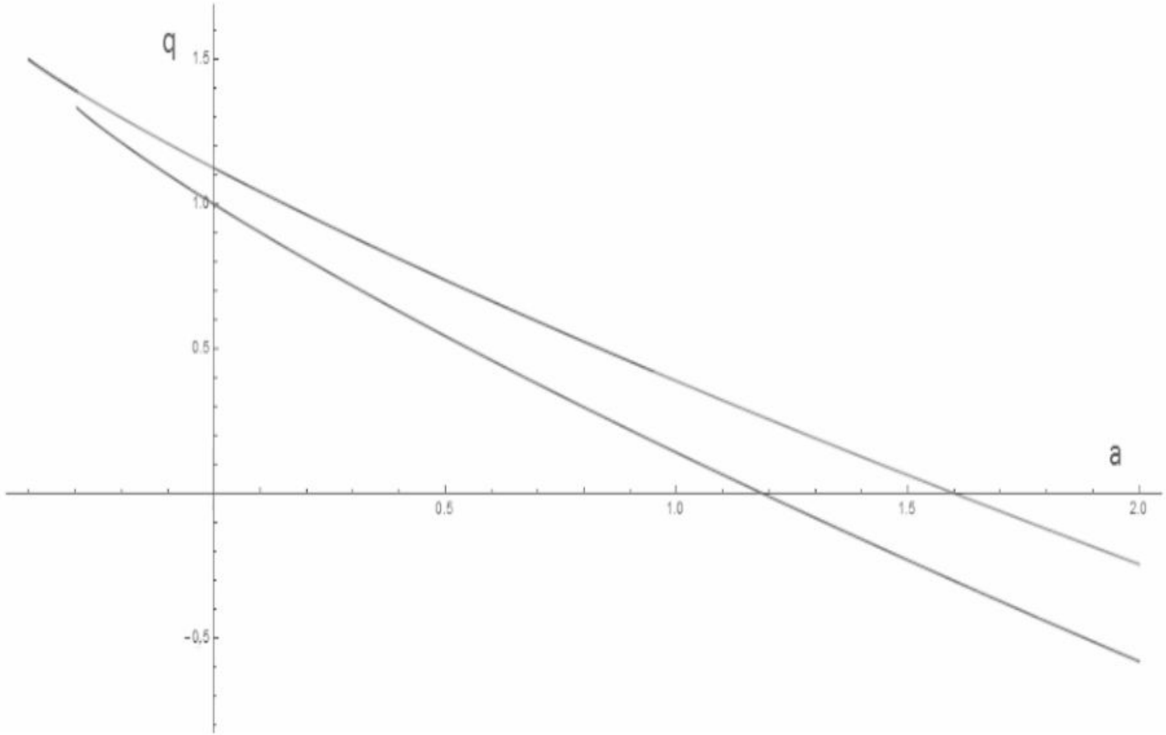

Fig. 4 Critical curves corresponding to finite shift of the horizon radius and to the finite shift of size of the shadow. The upper curve corresponds to the shadow size shift, while the lower one to the horizon radius shift.

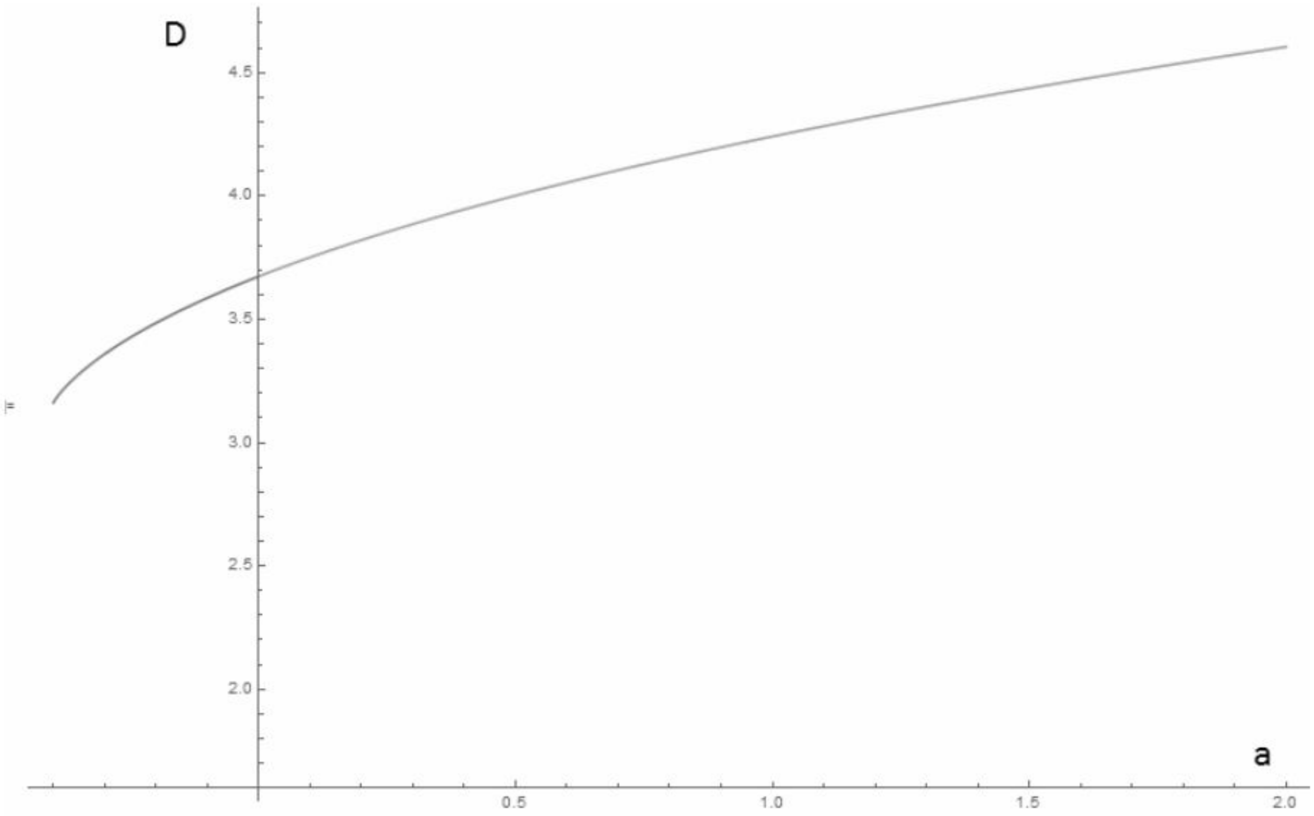

Fig. 5 The dependence of $D(\alpha)$ on $\alpha(q)$ for the case of the critical shadow size.

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As a result two BHs with equal shadow sizes have different value of the parameter α. Therefore one can parameterize configurations with any given shadow size by α.

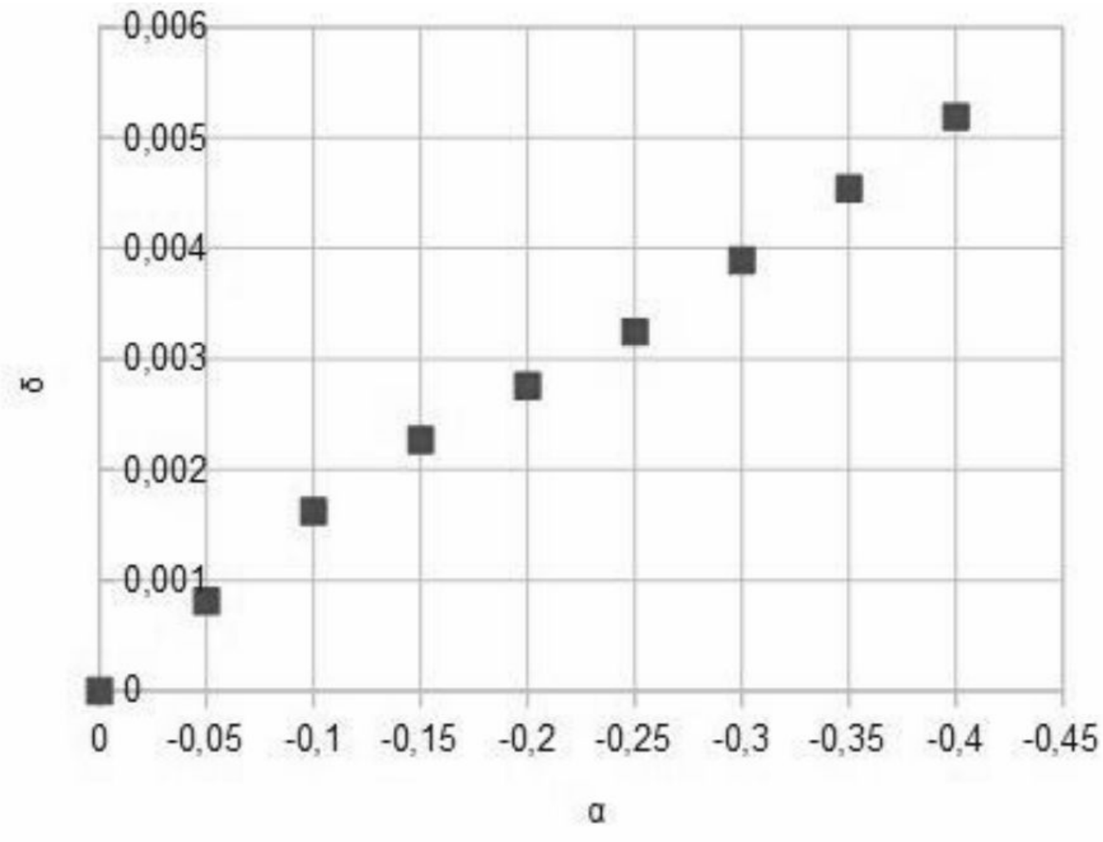

Fig. 6 Deviation parameter $\delta = |D(\alpha) - D(0)|/D(0)$ for the BH with same shadow size corresponding to $D_{sh} = 3\sqrt{3}$.

The last stable orbit.

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Following Chandrasekar,equation of motion of a test particle with unit mass reads:

$$\left(\frac{d\hat{r}}{d\tau}\right)^2 + U = E^2,$$

$$\frac{d\phi}{d\tau} = \frac{L}{\hat{r}^2}$$

$$U = \left(1 - \frac{2}{\hat{r}} + \frac{q}{\hat{r}^2} + \frac{\alpha}{\hat{r}^3}\right)\left(1 + \frac{L^2}{\hat{r}^2}\right).$$

where E is the energy of a particle, L is its angular momentum, and U is the potential energy.

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The last stable orbit is situated at the inflection point of the potential energy, so it can be found from the equation:

$$\frac{dU}{dr} = 0, \qquad \frac{d^2U}{dr^2} = 0.$$

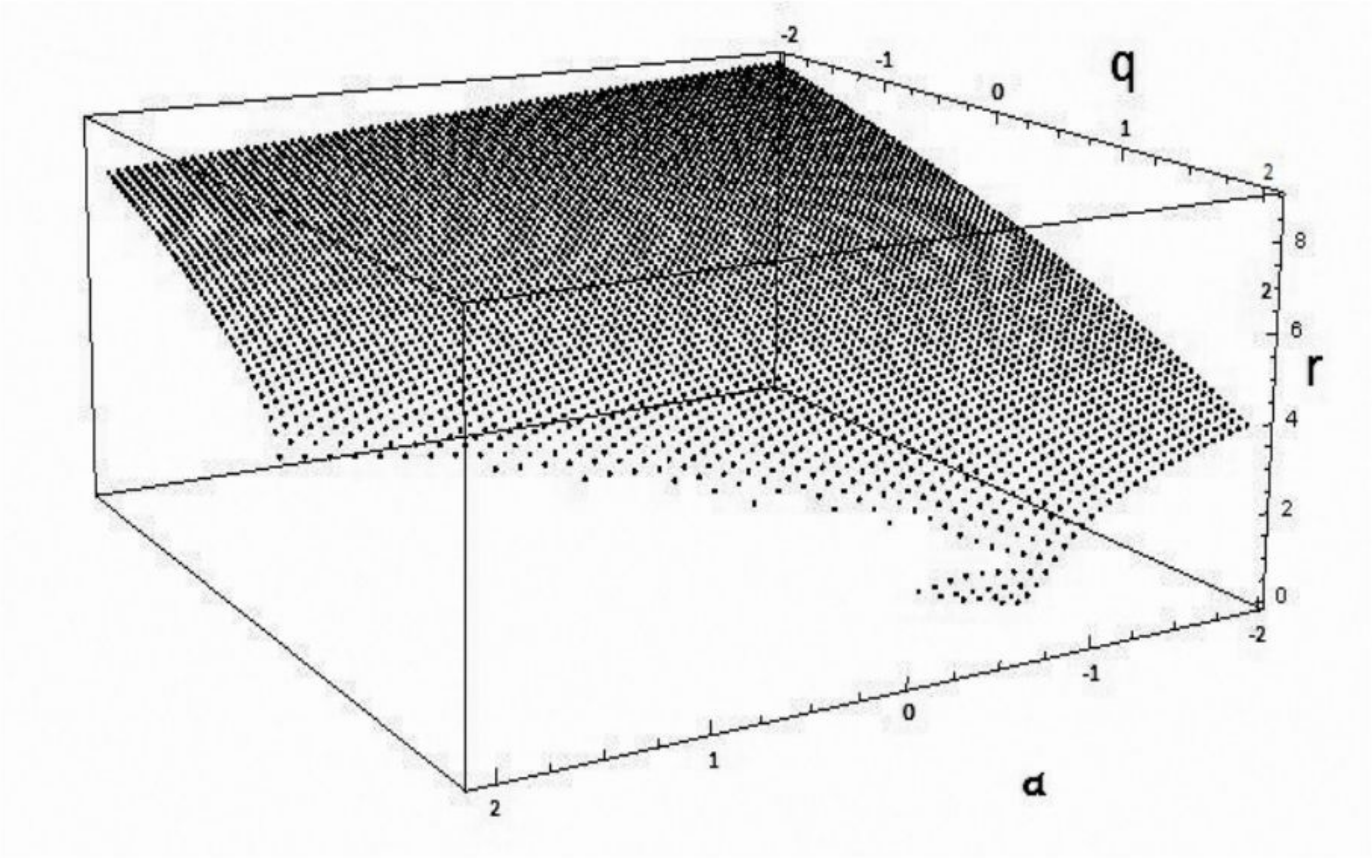

Fig. 7 The size of the last stable orbit $r(q, \alpha)$

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The metric admits exotic solutions with zero orbital momentum:

$$\frac{dU}{dr} = 0, \qquad L = 0.$$

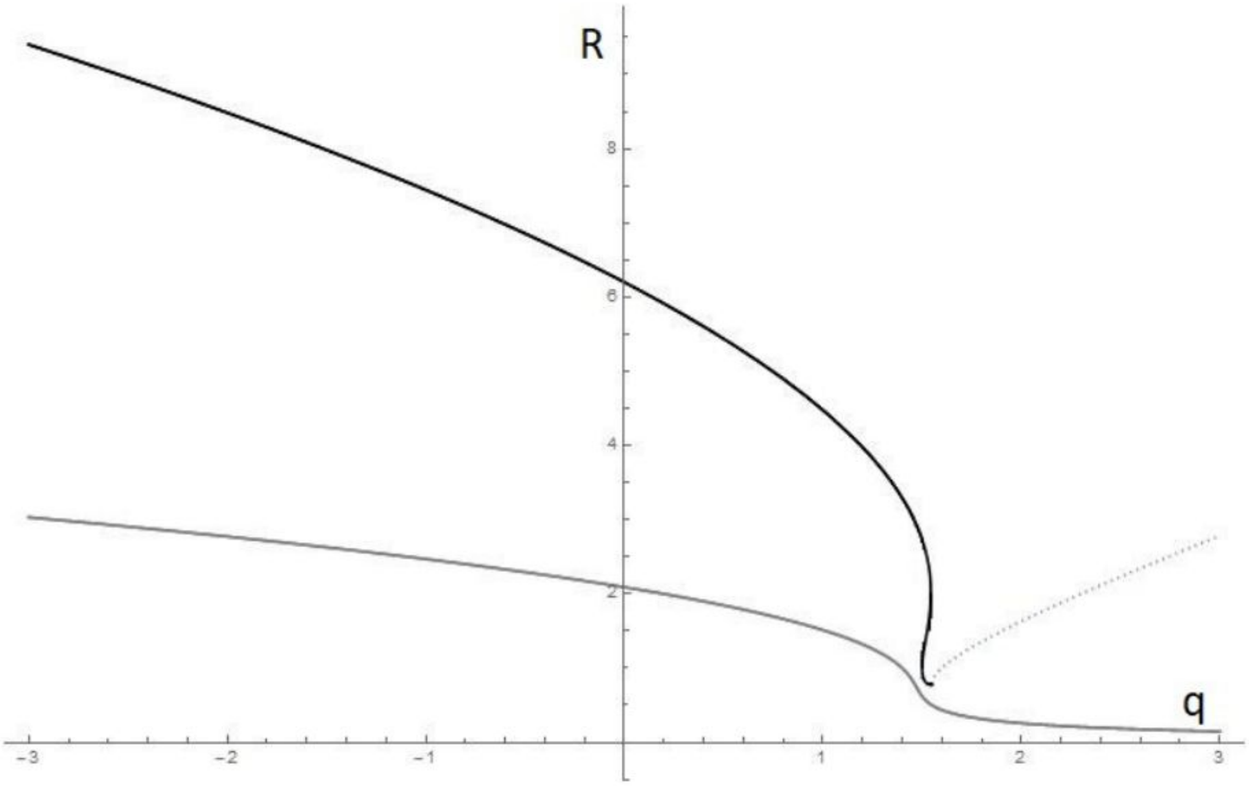

Fig. 8 The size of the last stable orbit (upper curve) for $\alpha = -0.4$. The lower curve corresponds to the size of the horizon. Dashed one shows to the orbits with zero moment.

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Our metric can describe more compact black holes (r <0.4), black holes without normal shadow and black holes with the last stable orbit with zero moment.

Our metric can describe a finit shift of the horizon, shadow size and last stable orbit radius with infinitely small variation of metric parameters q and α.