Three Waves for Quantum Gravity

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QUARKS-2018

Table of Contents

Table of Contents

Quantum General Relativity

- Nonrenormalizable at the two-loop level without matter
- Nonrenormalizable at the one-loop level with matter

† Formally renormalizable model contains a ghost

G. 't Hooft, M.J.G. Veltman, Ann.Inst.H.Poincare Phys.Theor. A20(1974) 69-94

M.H. Goroff, A. Sagnotti, Nucl.Phys. B266 (1986) 709-736.

K.S. Stelle, Phys.Rev. D16 (1977) 953-969; Gen.Rel.Grav. 9 (1978) 353-371.

Model can be treated as an effective one.
To define an effective model one should:

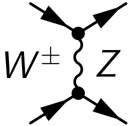
† Define sets of states and operators

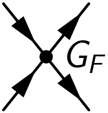
† Define the area of applicability

- C.P. Burgess, Living Rev.Rel. 7 (2004) 5-56.
- J.F. Donoghue, Phys.Rev. D50 (1994) 3874-3888; AIP Conf.Proc. 1483 (2012) 73-94.

Gravity and Effective Field Theory

Fermi model: the simplest example.





Heavy degrees of freedom are integrated out:

$$\exp\left[i\Gamma_{\text{eff}}(l)\right] = \int \mathcal{D}[h] \exp\left[i\mathcal{A}(h,l)\right]$$

Table of Contents

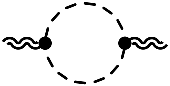
Effective Field Theory for Gravity

Effective Field Theory for Gravity is build by integrating out light degrees of freedom.

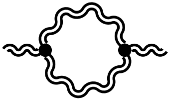
$$\exp\left[i\Gamma_{\text{eff}}(g_{\mu\nu})\right] = \int \mathcal{D}[h_{\mu\nu}] \exp\left[i\mathcal{A}_{\text{QG}}(g_{\mu\nu} + \kappa h_{\mu\nu})\right]$$

Effective Field Theory for Gravity

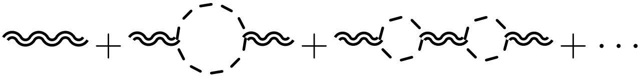
Local Terms



$$\Delta\Gamma \simeq \int d^4x\sqrt{-g}\left[c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu}\right]$$



- G. 't Hooft, M.J.G. Veltman, Ann.Inst.H.Poincare Phys.Theor. A20 (1974) 69-94.
- M.H. Goroff, A. Sagnotti, Nucl.Phys. B266 (1986) 709-736.



$$G_{\mu\nu\alpha\beta} = \frac{C_{\mu\nu\alpha\beta}}{k^2} \to \mathcal{G}_{\mu\nu\alpha\beta} = \frac{C_{\mu\nu\alpha\beta}}{k^2\left(1 - \frac{NGk^2}{120\pi}\ln\left(-\frac{q^2}{\mu^2}\right)\right)}$$

$$N = N_s + 3N_f + 12N_V.$$

T. Han, S. Willenbrock, Phys.Lett. B616 (2005) 215-220 .

$$\Gamma = \int d^4x \sqrt{-g} \left[-\frac{\gamma}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu}^2 \right.$$

$$\left. - b_1 R \ln\left(\frac{\Box}{\mu^2}\right) R - b_2 R_{\mu\nu} \ln\left(\frac{\Box}{\mu^2}\right) R^{\mu\nu} - b_3 R_{\mu\nu\alpha\beta} \ln\left(\frac{\Box}{\mu^2}\right) R^{\mu\nu\alpha\beta} \right]$$

- U. Aydemir, M.M. Anber, J.F. Donoghue, Phys.Rev. D86 (2012) 014025.
- X. Calmet, D. Croon, C. Fritz, Eur.Phys.J. C75 (2015) no.12, 605.
- S.O. Alexeyev, X. Calmet, B.N. Latosh, Phys.Lett. B776 (2018) 111-114.

Model contains

- ◇ Massless spin-2 mode
- ◇ Massive spin-2 “ghost” mode
- ◇ Massive spin-0 mode

- X. Calmet, B. Latosh, Eur.Phys.J. C78 (2018) no.3, 205;

$$G_{\mu\nu\alpha\beta} = \frac{(P^{(2)}_{m=0})_{\mu\nu\alpha\beta}}{k^2} - \frac{(P^{(2)}_{m\neq 0})_{\mu\nu\alpha\beta}}{k^2 - \mathcal{M}_2^2} + \frac{P^{(0)}_{\mu\nu\alpha\beta}}{k^2 - \mathcal{M}_0^2}$$

$$\begin{cases} \mathcal{M}_2^2 = \dfrac{2}{\kappa^2\left(-c_2 + (b_2 + 4b_3)\ln\left(-\dfrac{k^2}{\mu^2}\right)\right)} \\ \mathcal{M}_0^2 = \dfrac{1}{\kappa^2\left(3c_1 + c_2 - (3b_1 + b_2 + b_3)\ln\left(-\dfrac{k^2}{\mu^2}\right)\right)} \end{cases}$$

Classical "ghost" is reduced to repulsive gravity

$$\Gamma = \int d^4x \left[\mathcal{L}_{\mathrm{FP}}(h_{\mu\nu}) + \mathcal{L}_{\mathrm{FP}}(k_{\mu\nu}) - \frac{m_2^2}{2}\left(k_{\mu\nu}k^{\mu\nu} - k^2\right) \right.$$

$$\left. \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{m_0^2}{2}\sigma^2 - \kappa\left(h_{\mu\nu} - k_{\mu\nu} + \frac{1}{\sqrt{3}}\sigma\eta_{\mu\nu}\right) T^{\mu\nu}\right] + \mathcal{O}(h^2)$$

$$\mathcal{L}_{\mathrm{FP}}(h) = -\frac{1}{2}h_{\mu\nu}\Box h^{\mu\nu} + \frac{1}{2}h\Box h - h^{\mu\nu}\partial_\mu\partial_\nu h + h^{\mu\nu}\partial^\rho\partial_\nu h_{\mu\rho}$$

Table of Contents

Modes of effective gravity are coupled to the energy-momentum in a linear way, so each mode is generated independently.

$$\left(h_{\mu\nu} - k_{\mu\nu} + \frac{1}{\sqrt{3}}\eta_{\mu\nu}\ \sigma\right) T^{\mu\nu} \rightarrow \begin{cases} \Box h_{\mu\nu} = P_{\mu\nu\alpha\beta} 16\pi G T^{\alpha\beta} \\ \\ (\Box + m_2^2) k_{\mu\nu} = P^{(2)}_{\mu\nu\alpha\beta} 16\pi G T^{\alpha\beta} \\ \\ (\Box + m_0^2)\sigma = \dfrac{16\pi G}{\sqrt{3}} T \end{cases}$$

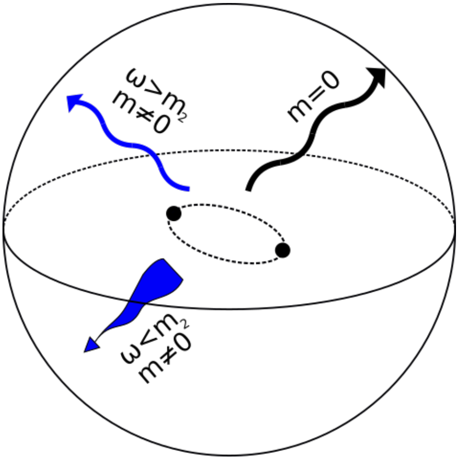
Application to Binary Systems

Modes production

- Spin-0 mode is not produced
- Both spin-2 modes are generated in the same way
- Massive spin-2 has an IR production cutoff m_2

$$\boxed{k_{\mu\nu}(\omega,\vec{x})|_{\text{rad}} = \frac{4G}{|\vec{x}|}e^{-i|\vec{x}|\sqrt{\omega^2-m_2^2}}\int d^3\vec{y}\; P^{(2)}_{\mu\nu\alpha\beta}T^{\alpha\beta}(\omega,\vec{y})}$$

Application to Binary Systems



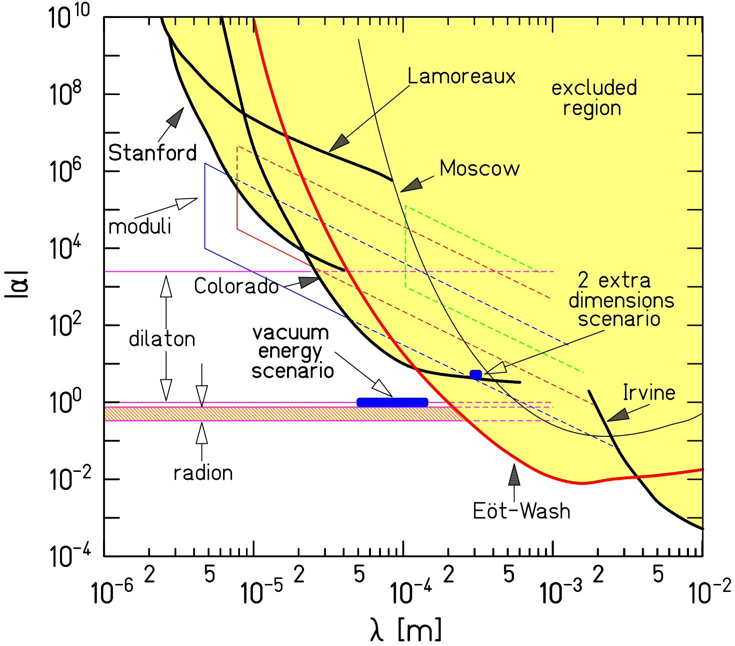
$$\boxed{k_{\mu\nu}(\omega,\vec{x})\big|_{\mathrm{rad}} = \frac{4G}{|\vec{x}|} e^{-i|\vec{x}|\sqrt{\omega^2-m_2^2}} \int d^3\vec{y}\; P^{(2)}_{\mu\nu\alpha\beta}\, T^{\alpha\beta}(\omega,\vec{y})}$$

Total system energy loss:

$$\frac{dE}{d\omega} = \frac{G}{45}\omega^6 \langle Q_{ij} Q^{ij} \rangle \left(1 + \theta(\omega - m_2)\right)$$

Orbital frequency:

$$\omega^2 = \frac{G(m_A + m_B)}{d^3}$$



10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
$|\alpha|$
10^{-6} 2 5 10^{-5} 2 5 10^{-4} 2 5 10^{-3} 2 5 10^{-2}
λ [m]
Lamoreaux
excluded region
Stanford
Moscow
moduli
Colorado
2 extra dimensions scenario
dilaton
vacuum energy scenario
Irvine
radion
Eöt-Wash

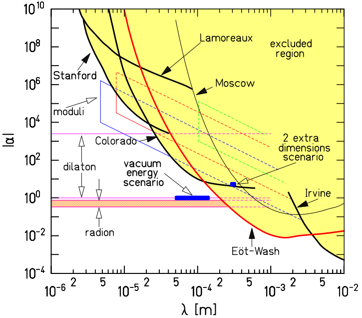
Application to Binary Systems

Eöt-Wash experiment constraints

$$m_2 > (0.03\text{cm})^{-1} = 6.6 \times 10^{-13}\text{GeV}.$$

$$\omega^2 = \frac{G(m_A + m_B)}{d^3}$$

$$\begin{cases} m_A = 36 M_\odot \\ m_B = 29 M_\odot \\ \omega = (0.03\text{cm})^{-1} \end{cases} \rightarrow \boxed{d \simeq 16\text{cm}}$$



- C.D. Hoyle, D.J. Kapner, B.R. Heckel, E.G. Adelberger, J.H. Gundlach, U. Schmidt, H.E. Swanson, Phys.Rev. D70 (2004) 042004;
- LIGO Scientific and Virgo Collaborations, Phys.Rev.Lett. 116 (2016) no.6, 061102;

Table of Contents

Summary

- † Spin-2 "ghost" issue is avoided
- † Binary system doesn't produce spin-0 mode
- † Spin-2 modes generated is the same way
- † Massive spin-2 mode has an IR cutoff m_2
- † Current empirical constraints on m_2 exclude significant influence of new modes on binary system gravitational radiation production

Summary

- † Spin-2 “ghost” issue is avoided
- † Binary system doesn’t produce spin-0 mode
- † Spin-2 modes generated is the same way
- † Massive spin-2 mode has an IR cutoff m_2
- † Current empirical constraints on m_2 exclude significant influence of new modes on binary system gravitational radiation production

Thank you for Attention!

Used Literature

‡ X. Calmet, B. Latosh, Eur.Phys.J. C78 (2018) no.3, 205.

‡ X. Calmet, D. Croon, C. Fritz, Eur.Phys.J. C75 (2015) no.12, 605.

‡ C.P. Burgess, Living Rev.Rel. 7 (2004) 5-56.

‡ J.F. Donoghue, Phys.Rev. D50 (1994) 3874-3888; AIP Conf.Proc. 1483 (2012) 73-94.

‡ K.S. Stelle, Gen.Rel.Grav. 9 (1978) 353-371.

‡ A. Accioly, S. Ragusa, H. Mukai, E.C. de Rey Neto, Int.J.Theor.Phys. 39 (2000) 1599-1608.

$$\Gamma = \int \frac{d^4k}{(2\pi)^4}\ \frac{1}{2}k^2 h^{\mu\nu}\left[P^1_{\mu\nu\alpha\beta} + \left(1 - \frac{k^2}{\mathcal{M}_0^2}\right)P^2_{\mu\nu\alpha\beta} + \left(2\frac{k^2}{\mathcal{M}_0^2} - \frac{1}{2}\right)P^0_{\mu\nu\alpha\beta}\right.$$
$$\left. + \frac{1}{2}\overline{P}^0_{\mu\nu\alpha\beta} - \frac{1}{2}\overline{\overline{P}}^0_{\mu\nu\alpha\beta}\right] h^{\alpha\beta} \text{ in Feynman gauge}$$

$$G_{\mu\nu\alpha\beta} = \frac{1}{2}\frac{\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu\alpha\beta}}{k^2} - \frac{P^2_{\mu\nu\alpha\beta}}{k^2 - \mathcal{M}_2^2}$$
$$+ \frac{1}{2}\frac{P^0_{\mu\nu\alpha\beta} + 3\overline{P}^0_{\mu\nu\alpha\beta} + \overline{\overline{P}}^0_{\mu\nu\alpha\beta}}{k^2 - \mathcal{M}_0^2}$$

$$P^1_{\mu\nu\alpha\beta} = \frac{1}{2}(\Theta_{\mu\alpha}\omega_{\nu\beta} + \Theta_{\mu\beta}\omega_{\nu\alpha} + \Theta_{\nu\alpha}\omega_{\mu\beta} + \Theta_{\nu\beta}\omega_{\mu\alpha})$$

$$P^2_{\mu\nu\alpha\beta} = \frac{1}{2}(\Theta_{\mu\alpha}\Theta_{\nu\beta} + \Theta_{\mu\beta}\Theta_{\nu\alpha}) - \frac{1}{3}\Theta_{\mu\nu}\Theta_{\alpha\beta}$$

$$P^0_{\mu\nu\alpha\beta} = \frac{1}{4}\Theta_{\mu\nu}\Theta_{\alpha\beta}$$

$$\overline{P}^0_{\mu\nu\alpha\beta} = \omega_{\mu\nu}\omega_{\alpha\beta}$$

$$\overline{\overline{P}}^0_{\mu\nu\alpha\beta} = \Theta_{\mu\nu}\omega_{\alpha\beta} + \omega_{\mu\nu}\Theta_{\alpha\beta}$$

$$\Theta_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2},$$

$$\omega_{\mu\nu} = \frac{k_\mu k_\nu}{k^2}$$

$$\mathcal{M}_2^2 = \frac{2}{\kappa^2\left(-c_2 + (b_2 + 4b_3)\ln\left(-\frac{k^2}{\mu^2}\right)\right)}$$

$$\mathcal{M}_0^2 = \frac{1}{\kappa^2\left(3c_1 + c_2 - (3b_1 + b_2 + b_3)\ln\left(-\frac{k^2}{\mu^2}\right)\right)}$$

	b_1	b_2	b_3
real scalar	$\frac{5(6\zeta-1)^2}{11520\pi^2}$	$-\frac{2}{11520\pi^2}$	$\frac{2}{11520\pi^2}$
fermion	$-\frac{5}{11520\pi^2}$	$\frac{8}{11520\pi^2}$	$\frac{7}{11520\pi^2}$
vector	$-\frac{50}{11520\pi^2}$	$\frac{176}{11520\pi^2}$	$-\frac{26}{11520\pi^2}$
graviton	$\frac{430}{11520\pi^2}$	$\frac{-1444}{11520\pi^2}$	$\frac{424}{11520\pi^2}$

J.F. Donoghue, B.K.El-Menoufi, Phys.Rev. D89 (2014) no.10, 104062

$T^{\mu\nu}$ approximations

- System is far away from the observer
- Slow motion approximation

$$\begin{aligned}\gamma_{\mu\nu}(t,x) &= 4G\int d^3y\,\frac{T_{\mu\nu}(t-|\vec{x}-\vec{y}|\,,\vec{x}-\vec{y})}{|\vec{x}-\vec{y}|}\\ &\to \frac{4G}{|\vec{x}|}\int d^3y T_{\mu\nu}(t-|\vec{x}|\,,\vec{y})\end{aligned}$$

N. Straumann, "General relativity : with applications to astrophysics"
Berlin : Springer – Verlag 2004.

$T^{\mu\nu}$ approximations

$$T^{ij} = \frac{1}{2}\frac{\partial^2}{\partial t^2}\int T^{00} x^i x^j \; dV \qquad T^{0i} \sim 0 \qquad T^{00} \sim \rho$$

$$\ddot{\vec{d}} = 0 \qquad Q_{ij} = \int dV \; (3x_i x_j - r^2 \delta_{ij})$$

$$L_{\mathrm{GW}} = \frac{G}{45}\langle \dddot{Q}_{ij} \dddot{Q}_{ij} \rangle$$

Massive Gravity Proector

$$(\Box + m^2)h_{\mu\nu} = \kappa \left[T_{\mu\nu} - \frac{1}{D-1} \left(\eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{m^2} \right) T \right]$$