Massive scalar field theory in the presence of moving mirrors

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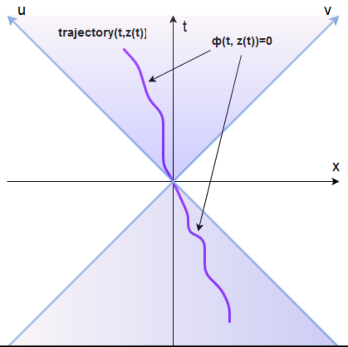
Ideal mirror

K-G equation and boundary condition

$$(\partial_t^2 - \partial_x^2 + m^2)\ \phi(t,x) = 0,\ \phi\big(t,z(t)\big) = 0$$



$\big(t,z(t)\big)$-trajectory of the ideal mirror

Set up of the problem

- Expand field ϕ(t,x) in terms of space-time harmonics according to the mirror trajectory z(t)
- check the commutation relations of the field operator and its conjugate momentum
- obtain the free Hamiltonian via the creation and annihilation operators
- derive the expectation value of the stress–energy tensor

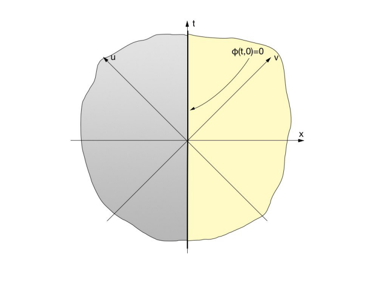
Motivation

This model system evidently can provide insight into more sophisticated process,such as particle production in cosmological models and exploding blalck holes

Mirror at rest

Field and boundary condition

$$\hat{\phi}(t,x) = i \int_0^\infty \frac{dk}{2\pi} \sqrt{\frac{2}{\omega}} \sin(kx) \left[\hat{a}_k e^{-iwt} - \hat{a}_k^\dagger e^{iwt}\right], \hat{\phi}(t,0) = 0$$



$$[\hat{a}_k, \hat{a}_{k'}^\dagger] = 2\pi\delta(k - k')$$

$$[\hat{\phi}(t,x), \hat{\pi}(t,y)] = i\left[\delta(x-y) - \delta(x+y)\right]$$

where $\hat{\pi}(t,y) = \partial_t \hat{\phi}(t,y)$ - canonical momentum

Mirror at rest

The stress-energy tensor

$$T_{\mu\nu} = \frac{1}{2}\left(\partial_\mu\phi\,\partial_\nu\phi + \partial_\nu\phi\,\partial_\mu\phi\right) - \frac{1}{2}g_{\mu\nu}\left(\partial_\alpha\phi\,\partial^\alpha\phi + m^2\phi^2\right), \partial^\mu T_{\mu\nu} = 0$$

$$H = \int_0^\infty T_{tt}\,dx = \int_0^{+\infty} \frac{dk}{2\pi}\,\frac{\omega}{2}(\hat{a}_k\hat{a}_k^\dagger + \hat{a}_k^\dagger\hat{a}_k)$$

$$\langle : T_{\mu\nu} : \rangle = -\frac{1}{2\pi}m^2K_0(2mx)\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}.$$

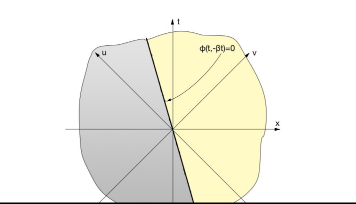
Mirror moving with constant velocity

In this case consider the velocity $0 < \beta < 1$, $\phi(\mathrm{t}, -\beta \mathrm{t}) = 0$

Field

$$\hat{\phi}(\mathrm{t,x}) = \mathrm{i} \int_{\gamma\beta\mathrm{m}}^{+\infty} \frac{\mathrm{dk}}{2\pi} \frac{1}{\sqrt{2\omega}} \hat{\mathrm{a}}_\mathrm{k} (\mathrm{e}^{-\mathrm{i}\omega\mathrm{t} - \mathrm{ikx}} - \mathrm{e}^{-\mathrm{i}\omega_\mathrm{r}\mathrm{t} + \mathrm{ik_r x}}) + \mathrm{h.c.}$$

$$[\hat{\phi}(\mathrm{t,x}), \hat{\pi}(\mathrm{t,y})] = \mathrm{i} \Big[\delta(\mathrm{x} - \mathrm{y}) - \tfrac{1}{2}\delta\big[2\gamma^2\beta(1-\beta)\mathrm{t} + (1-\beta)^2\gamma^2\mathrm{x} + \mathrm{y}\big] - \tfrac{1}{2}\delta\big[2\gamma^2\beta(1+\beta)\mathrm{t} + (1+\beta)^2\gamma^2\mathrm{x} + \mathrm{y}\big] \Big]$$



$$\omega_\mathrm{r} = (1 + \beta^2)\gamma^2\omega - 2\beta\gamma^2\mathrm{k}$$

$$\mathrm{k_r} = 2\beta\gamma^2\omega + (1 + \beta^2)\gamma^2\mathrm{k}$$

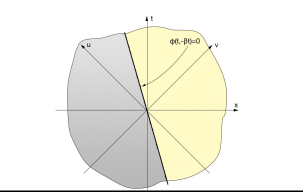
Mirror moving with constant velocity

$$\mathrm{H} = \int_{-\beta \mathrm{t}}^{\infty} \mathrm{T}_{\mathrm{tt}}\ \mathrm{dx} = \frac{1}{2}\int_{-\beta \mathrm{t}}^{+\infty}[(\partial_{\mathrm{t}}\phi)^2 - \phi\partial_{\mathrm{t}}^2\phi]\mathrm{dx},\ \mathrm{P} = \int_{-\beta \mathrm{t}}^{\infty} \mathrm{dx}\ \mathrm{T}_{\mathrm{tx}}$$

Translation operator

$$\mathrm{H} - \beta \mathrm{P} = \int_{\gamma\beta \mathrm{m}}^{\infty} \frac{\mathrm{dk}}{2\pi}\frac{\gamma^2(\omega-\beta \mathrm{k})(\omega-\beta \mathrm{k}-\beta(1-\beta)\omega)}{2\omega_{\mathrm{r}}}\left[\hat{\mathrm{a}}_{\mathrm{k}}\hat{\mathrm{a}}_{\mathrm{k}}^{\dagger} + \hat{\mathrm{a}}_{\mathrm{k}}^{\dagger}\hat{\mathrm{a}}_{\mathrm{k}}\right]$$

Operator of the translations along the mirror is diagonal unlike the Hamiltonian and Momentum separately



For massless field

$$\mathrm{H} - \beta \mathrm{P} =$$

$$(1-\beta)\int_0^{\infty}\frac{\mathrm{dk}}{2\pi}\frac{\mathrm{k}}{2}\left[\mathrm{a}_{\mathrm{k}}\mathrm{a}_{\mathrm{k}}^{\dagger} + \mathrm{a}_{\mathrm{k}}^{\dagger}\mathrm{a}_{\mathrm{k}}\right]$$

Mirror moving with constant velocity

$$\langle T_{tx}\rangle = \lim_{\varepsilon\to 0}\frac{1}{2}\left\langle \partial_t\phi(t,x)\partial_x\phi(t+i\varepsilon,x)+\partial_x\phi(t,x)\partial_t\phi(t+i\varepsilon,x)\right\rangle$$

Vacuum average of tx-component

$$\langle T_{tx}\rangle = -\frac{1}{2\pi}\gamma^2\beta m^2 K_0(2m\gamma(x+\beta t))$$

No flux

For each fixed x, as $t\to+\infty$, $\langle T_{tx}\rangle \to 0$

Boost the mirror at rest

$$\langle T_{tx}\rangle = \beta\gamma^2\Big(\langle :T_{t't'}:\rangle + \langle :T_{x'x'}:\rangle\Big) = -\frac{1}{2\pi}m^2\beta\gamma^2 K_0(2mx')$$

Non-ideal mirror

$$\left(\partial_t^2 - \partial_x^2 + m^2 + \alpha\delta(x)\right)h(t,x) = 0, \quad \alpha > 0.$$

$$h(t,+0) = h(t,-0);\ \partial_x h(t,+0) - \partial_x h(t,-0) = \alpha h(t,0).$$

Modes

$$h_{k>0}(t,x) = \begin{cases} \frac{e^{-i\omega t}}{\sqrt{2\omega}}\left(e^{-ikx} - \frac{\alpha}{2ik+\alpha}e^{ikx}\right) & x < 0 \\ \frac{e^{-i\omega t}}{\sqrt{2\omega}}\frac{2ik}{2ik+\alpha}e^{-ikx} & x > 0; \end{cases}$$

$$h_{k<0}(t,x) = \begin{cases} \frac{e^{-i\omega t}}{\sqrt{2\omega}}\left(e^{-ikx} + \frac{\alpha}{2ik-\alpha}e^{ikx}\right) & x > 0 \\ \frac{e^{-i\omega t}}{\sqrt{2\omega}}\frac{2ik}{2ik-\alpha}e^{-ikx} & x < 0, \end{cases}$$

Non-ideal mirror

The quantized field

$$\hat{\phi}(t,x) = \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \frac{\hat{a}_k h(x) e^{i\omega t} + \hat{a}_k^{\dagger} h^*(x) e^{-i\omega t}}{\sqrt{2\omega}}$$

Commutation relation

$$[\hat{\phi}(t,x), \partial_t \hat{\phi}(t,y)] = i\delta(x-y)$$

Non-ideal mirror

The Hamiltonian

$$\hat{\mathrm{H}} = \frac{1}{2}\int_{-\infty}^{+\infty} \mathrm{dx}\left[(\partial_{\mathrm{t}}\hat{\phi})^2 + (\partial_{\mathrm{x}}\hat{\phi})^2 + \mathrm{m}^2\hat{\phi}^2 + \alpha\hat{\phi}^2(\mathrm{t},0)\right] =$$

$$= \int_{-\infty}^{+\infty} \frac{\mathrm{dk}}{2\pi}\frac{\omega}{2}\left(\hat{\mathrm{a}}_{\mathrm{k}}\hat{\mathrm{a}}_{\mathrm{k}}^{\dagger} + \hat{\mathrm{a}}_{\mathrm{k}}^{\dagger}\hat{\mathrm{a}}_{\mathrm{k}}\right)$$

Non-ideal mirror

The vacuum expectation value of the stress-energy tensor

$$\langle T_{\mu\nu}\rangle = -\frac{1}{4\pi}\eta_{\mu\nu}\left[M^2\log\Lambda - 2\alpha\delta(x)\frac{\arctan(\sqrt{\frac{4m^2}{\alpha^2}-1})}{\sqrt{\frac{4m^2}{\alpha^2}-1}}\right]+$$

$$+F(m,M,x)\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}.$$

$$F(m,M,x) = m^2\int_{-\infty}^{+\infty}\left[\theta(-x)\theta(k)\left(\tfrac{\alpha}{2ik-\alpha}e^{-2ikx}-\tfrac{\alpha}{2ik+\alpha}e^{2ikx}\right)+\right.$$

$$\left.+\theta(x)\theta(-k)\left(\tfrac{\alpha}{2ik-\alpha}e^{2ikx}-\tfrac{\alpha}{2ik+\alpha}e^{-2ikx}\right)\right]\frac{dk}{4\pi\omega}-(m\to M).$$

Conclusions

- In case of the ideal mirror, the field operator and its conjugate momentum do not obey the canonical commutation relations
- in the presence of moving mirrors the diagonal form has the $\hat{\mathrm{H}} - \beta\hat{\mathrm{P}}$ operator rather than $\hat{\mathrm{H}}$ itself
- In the presence of a mirror moving with constant velocity the expectation value of the stress–energy tensor has a non–diagonal contribution
- In the case of non–ideal mirror the commutation relations of the field operator and its conjugate momentum have their canonical form, as it should be in proper physical situations

Thank you for your attention!