Casimir repulsion and attraction due to presence of Chern-Simons layers at the surfaces of dielectrics and metals

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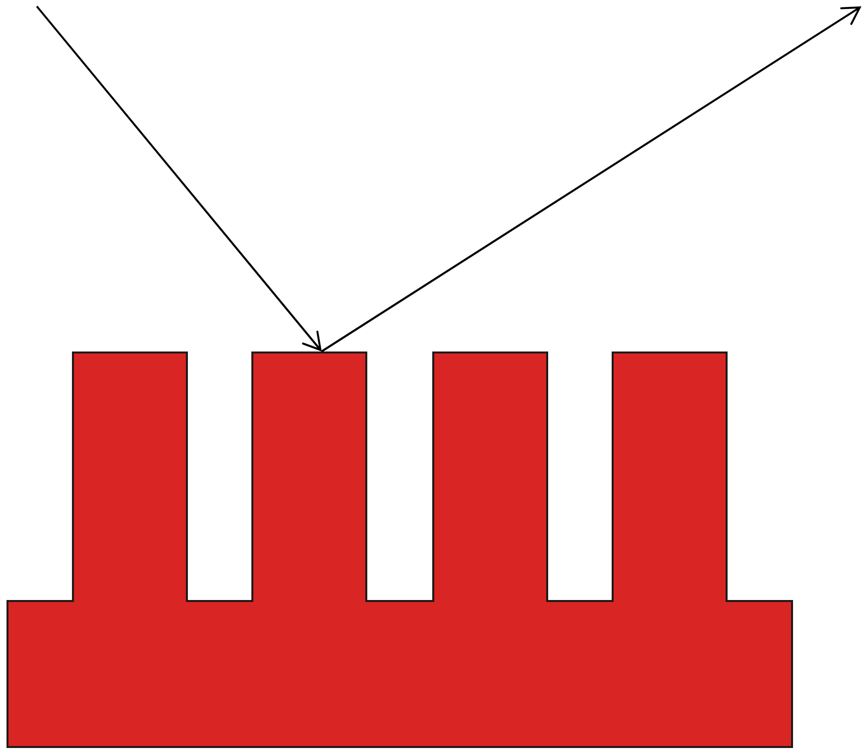
In this talk

1. The solution of grating diffraction problem. Casimir energy of two gratings. Scattering method.
2. The solution of a diffraction problem for Chern-Simons layer in vacuum and at the surface of a dielectric.
3. Casimir energy of two Chern-Simons layers in vacuum.
4. Casimir energy of two dielectrics with Chern-Simons layers at their surfaces.
5. Appearance of the minimum in the Casimir energy due to presence of Chern-Simons layers at the surfaces of dielectrics/metals.



Rayleigh decomposition for 1d gratings.

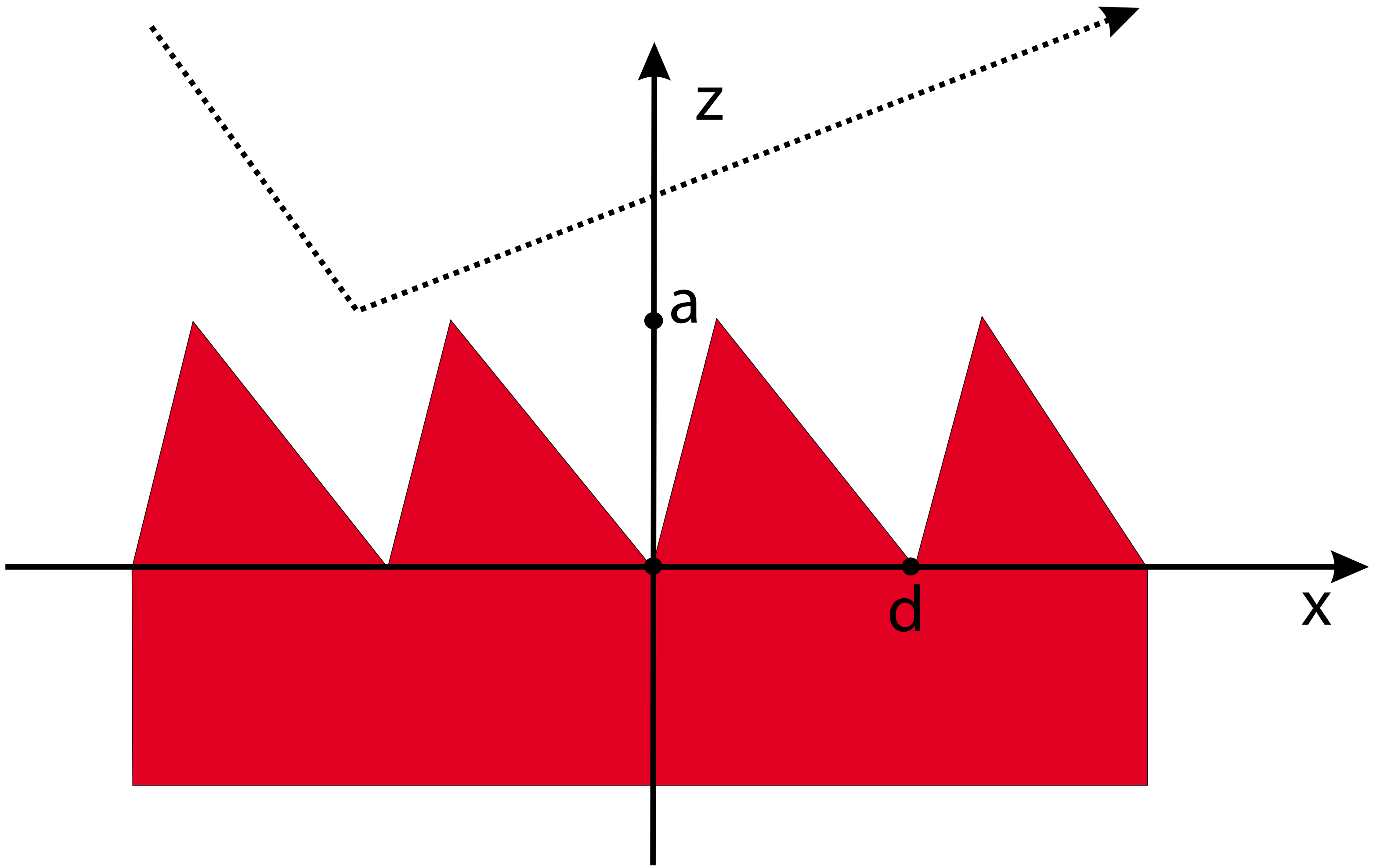
Rayleigh expansion for an incident electromagnetic wave on a single grating

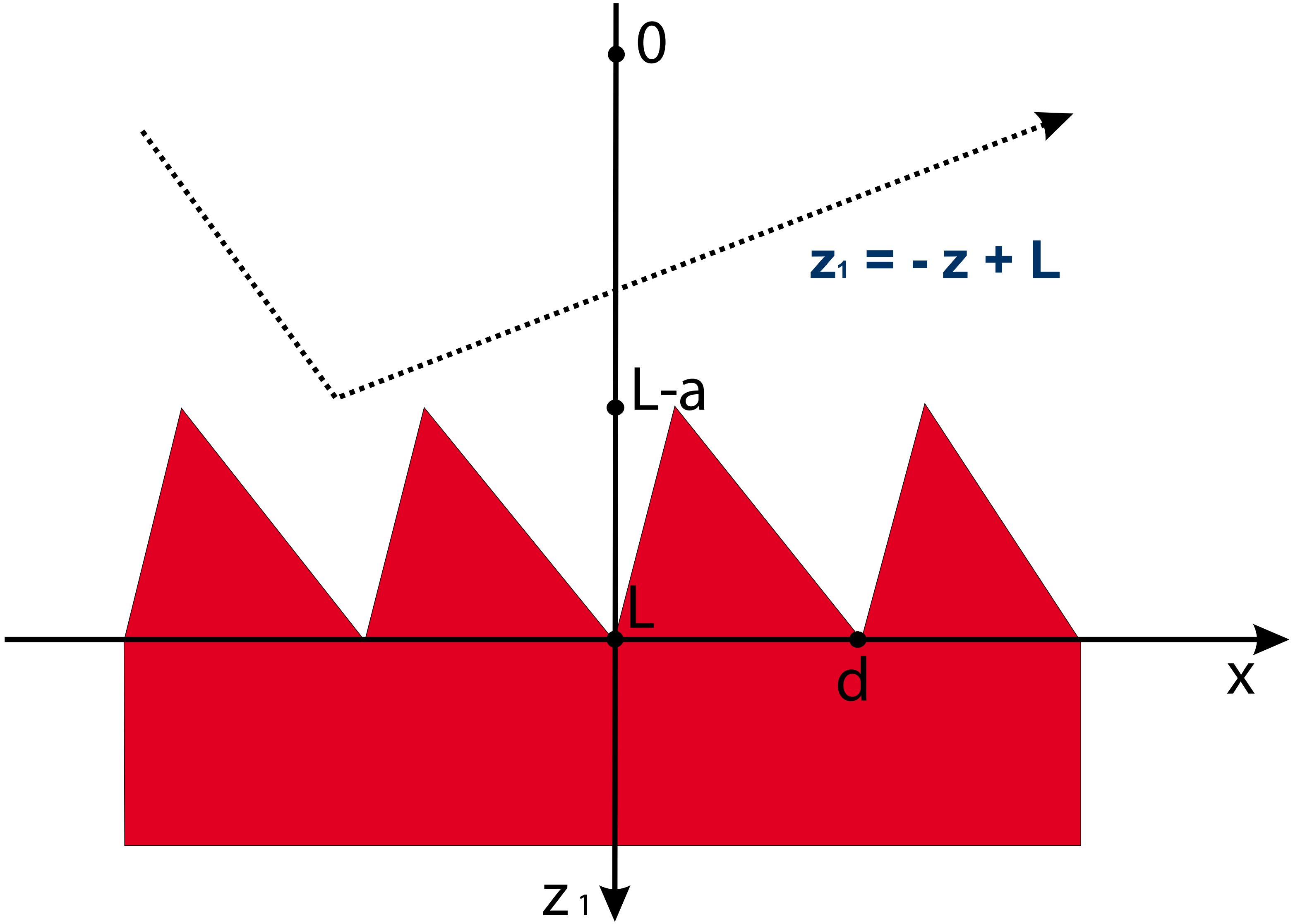
$$
\begin{aligned}
E_y(x,z) &= I_p^{(e)} \exp(i\alpha_p x - i\beta_p^{(1)} z) + \sum_{n=-\infty}^{+\infty} R_{np}^{(e)} \exp(i\alpha_n x + i\beta_n^{(1)} z), \\
H_y(x,z) &= I_p^{(h)} \exp(i\alpha_p x - i\beta_p^{(1)} z) + \sum_{n=-\infty}^{+\infty} R_{np}^{(h)} \exp(i\alpha_n x + i\beta_n^{(1)} z).
\end{aligned}
$$

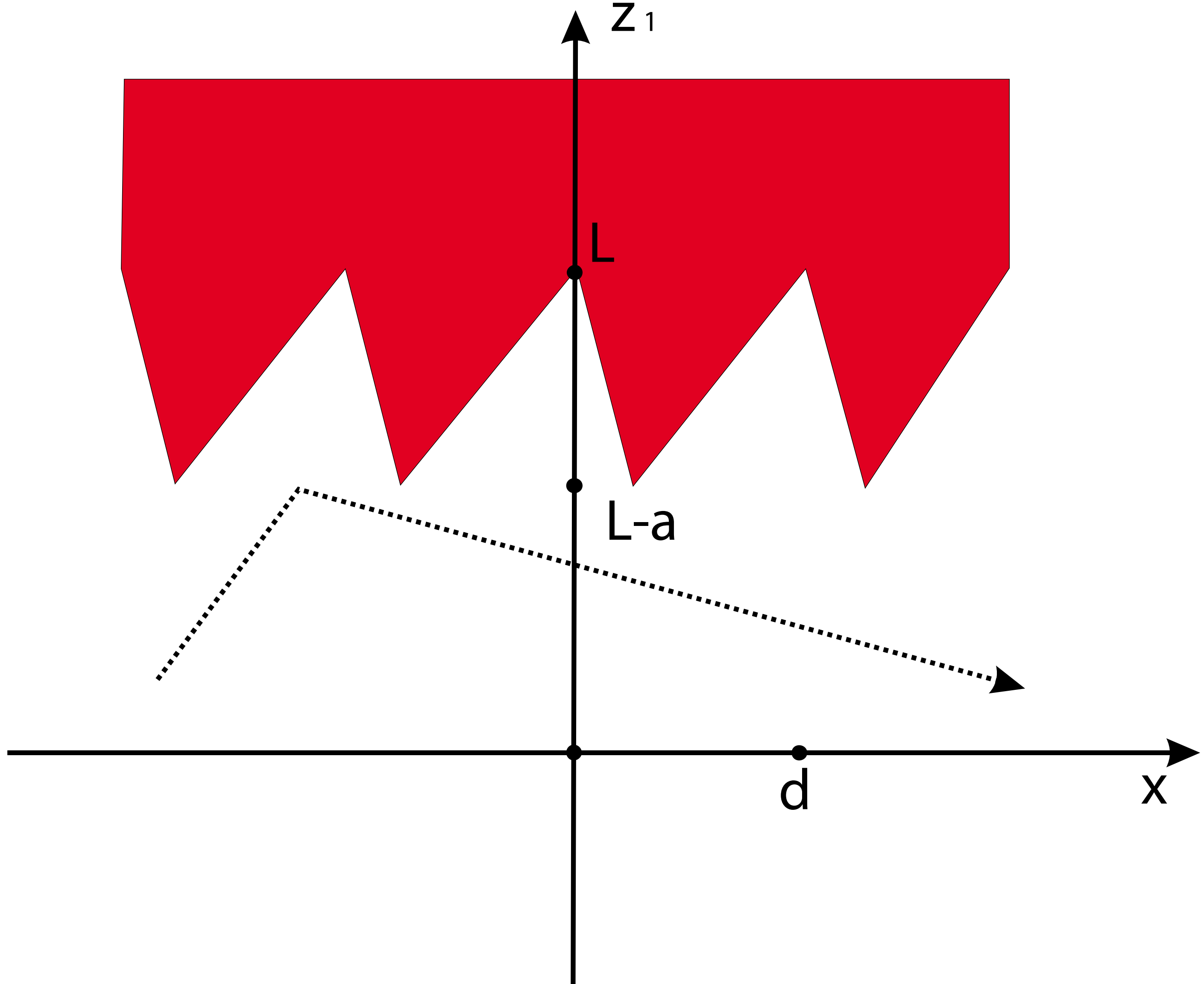
Here $\alpha_p = k_x + 2\pi p/d$ and $\beta_p^{(1)2} = \omega^2 - k_y^2 - \alpha_p^2$.
The reflection matrix is constructed as follows:

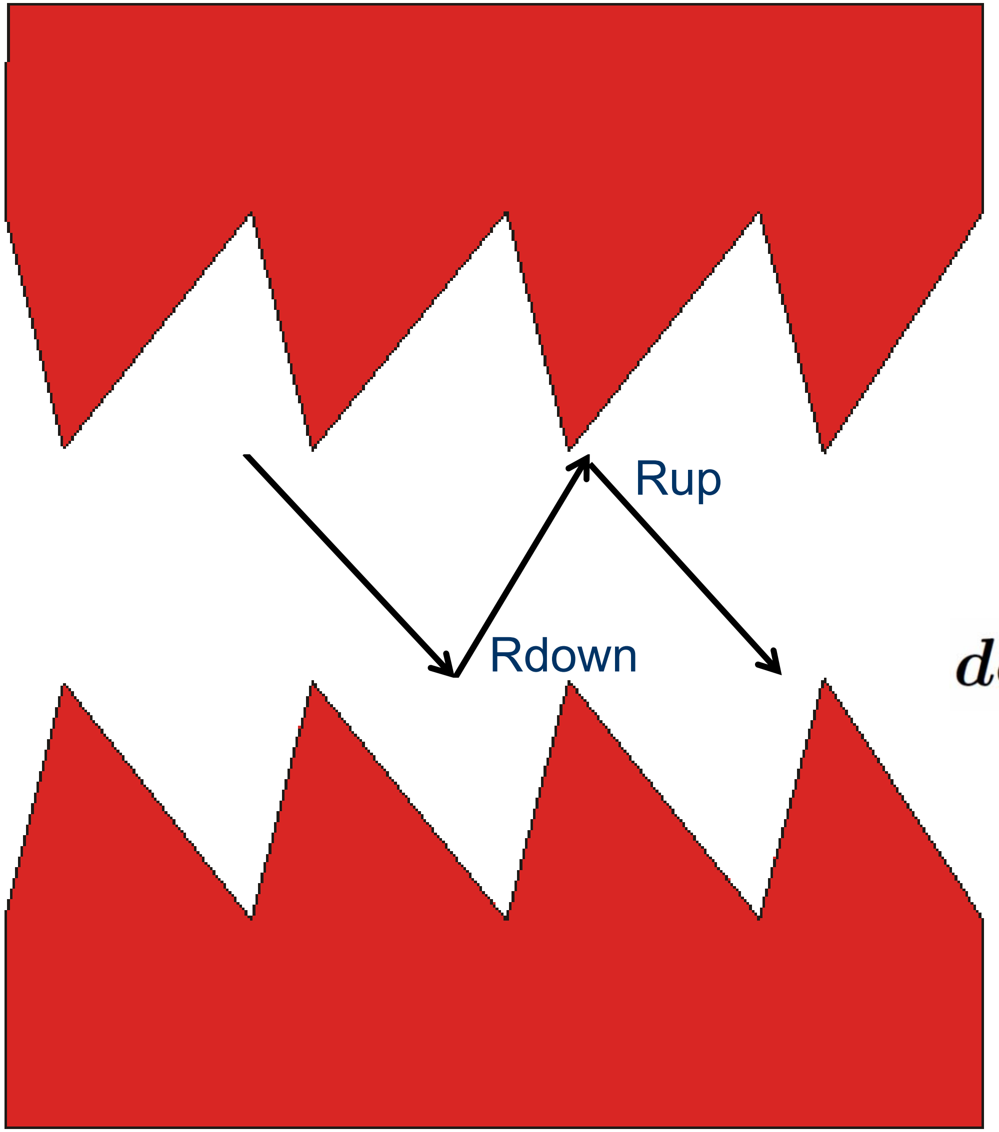
$$
R_1(\omega) = \begin{pmatrix} R_{n_1 l_1}^{(e)}(I_p^{(e)} = \delta_{pl_1}, I_p^{(h)} = 0) & R_{n_2 l_2}^{(e)}(I_p^{(e)} = 0, I_p^{(h)} = \delta_{pl_2}) \\ R_{n_3 l_3}^{(h)}(I_p^{(e)} = \delta_{pl_3}, I_p^{(h)} = 0) & R_{n_4 l_4}^{(h)}(I_p^{(e)} = 0, I_p^{(h)} = \delta_{pl_4}) \end{pmatrix}.
$$

Rayleigh expansion is exact outside gratings. The unknown coefficients can be determined from the exact solution of Maxwell equations.


z
a
d
x


0
$z_1 = - z + L$
L-a
L
d
x
z_1


z 1
L
L-a
d
x



$$\boldsymbol{R_{down} R_{up} \psi_i = \psi_i}$$

$\downarrow$

$$\boldsymbol{det(I - R_{down} R_{up}) = 0}$$

$\downarrow$

argument principle

Argument principle

$$\frac{1}{2\pi i}\oint \phi(\omega)\frac{d}{d\omega}\ln f(\omega)d\omega = \sum \phi(\omega_0) - \sum \phi(\omega_\infty) \qquad (1)$$

$$\phi(\omega) = \hbar\omega/2$$

$$f(\omega) = \det(I - R_{down}(\omega)R_{up}(\omega))$$

Casimir energy of two gratings

$$E = \frac{\hbar c}{(2\pi)^3} \int_0^{+\infty} d\omega \int_{-\infty}^{+\infty} dk_y \int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} dk_x \quad \ln \det\Big(I - R_{down}(i\omega) R_{up}(i\omega)\Big) \tag{2}$$

$$R_{up}(i\omega) = Q^* K(i\omega) R(i\omega) K(i\omega) Q, \tag{3}$$

$$K(i\omega) = \begin{pmatrix} G_1 & 0 \\ 0 & G_1 \end{pmatrix}, \tag{4}$$

with matrix elements $e^{-L\sqrt{\omega^2 + k_y^2 + (k_x + \frac{2\pi p}{d})^2}}, p = -N \dots N$ on the main diagonal of a matrix G_1,

$$Q = \begin{pmatrix} G_2 & 0 \\ 0 & G_2 \end{pmatrix}, \tag{5}$$

with matrix elements $e^{2\pi i m s/d}, p = -N \dots N$ on the main diagonal of a matrix G_2. (A.Lambrecht and V.N.Marachevsky, PRL **101**, 160403 (2008)).

Chern-Simons Casimir effect

Green's function derivation of the Casimir energy of two Chern-Simons plane layers in vacuum, Casimir repulsion is found: V. N. Markov and Yu. M. Pis'mak, J. Phys. A: Math. Gen. **39**, 6525 (2006). Casimir-Polder potential of a neutral atom in front of Chern-Simons plane layer: V. N. Marachevsky and Yu. M. Pis'mak, Phys.Rev.D **81**, 065005 (2010).

This talk is based on:

V.N.Marachevsky, *Chern-Simons layers in the vacuum* , Theor.Math.Phys. **190**(2), 315 (2017);
V.N.Marachevsky, *Chern-Simons layers on dielectrics and metals* , arXiv: cond-mat 1802.06523.

Chern-Simons layer on a dielectric semispace

V.N.Marachevsky, *Chern-Simons layers in the vacuum* , Theor.Math.Phys. **190**(2), 315 (2017);
V.N.Marachevsky, *Chern-Simons layers on dielectrics and metals* , arXiv: cond-mat 1802.06523.

The action with Chern-Simons layer at $z = 0$ has the form:

$$S = \frac{a}{2} \int \varepsilon^{z\nu\rho\sigma} A_\nu F_{\rho\sigma} dtdxdy. \tag{6}$$

Equations of electromagnetic field in the presence of Chern-Simons action (6) can be written as follows:

$$\partial_\mu F^{\mu\nu} + a\, \varepsilon^{z\nu\rho\sigma} F_{\rho\sigma} \delta(z) = 0. \tag{7}$$

Consider a flat Chern-Simons layer put at $z = 0$ on a dielectric semispace $z < 0$ characterized by a frequency dependent dielectric permittivity $\varepsilon(\omega)$, the magnetic permeability $\mu = 1$. Boundary conditions on the components of the electromagnetic field follow:

$$E_z|_{z=0^+} - \varepsilon(\omega) E_z|_{z=0^-} = -2aH_z|_{z=0}, \tag{8}$$

$$H_x|_{z=0^+} - H_x|_{z=0^-} = 2aE_x|_{z=0}, \tag{9}$$

$$H_y|_{z=0^+} - H_y|_{z=0^-} = 2aE_y|_{z=0}. \tag{10}$$

Diffraction problem

Consider TE (s-polarized) electromagnetic plane wave diffracting from a Chern-Simons layer located at $z = 0$ on a dielectric semispace ($z < 0$) defined by a dielectric permittivity $\varepsilon(\omega)$ (the factor $\exp(i\omega t + ik_y y)$ is dropped for simplicity of notations):

$$E_x = \exp(-ik_z z) + r_s \exp(ik_z z), z > 0 \tag{11}$$

$$E_x = t_s \exp(-ik_z^{(2)} z), z < 0 \tag{12}$$

$$H_x = r_{s\to p} \exp(ik_z z), z > 0 \tag{13}$$

$$H_x = t_{s\to p} \exp(-ik_z^{(2)} z), z < 0. \tag{14}$$

Here $k_z = \sqrt{\omega^2 - k_y^2}, k_z^{(2)} = \sqrt{\epsilon(\omega)\omega^2 - k_y^2}$.

From the condition $H_x|_{z=0^+} - H_x|_{z=0^-} = 2aE_x|_{z=0}$ it follows

$$r_{s\to p} - t_{s\to p} = 2a\, t_s. \tag{15}$$

From $E_x|_{z=0^+} = E_x|_{z=0^-}$ we obtain

$$1 + r_s = t_s. \tag{16}$$

From the condition $E_y|_{z=0^+} = E_y|_{z=0^-}$ and Maxwell equation $E_y = -\frac{1}{i\omega\varepsilon(\omega)}\partial_z H_x$ it follows that

$$r_{s\to p} k_z = -\frac{k_z^{(2)}}{\varepsilon(\omega)} t_{s\to p}. \tag{17}$$

From the condition (10) and Maxwell equation $H_y = \frac{1}{i\omega}\partial_z E_x$ we get

$$k_z(-1 + r_s) + k_z^{(2)} t_s = 2a\, \frac{k_z^{(2)}}{\varepsilon(\omega)} t_{s\to p}. \tag{18}$$

Solving equations (15)-(18) we find reflection and transmission coefficients for TE plane wave:

$$r_s = \frac{r_s^f - a^2 T}{1 + a^2 T}, \qquad t_s = \frac{t_s^f}{1 + a^2 T},$$
$$r_{s\to p} = \frac{aT}{1 + a^2 T}, \qquad t_{s\to p} = -\frac{aT}{1 + a^2 T}\frac{\varepsilon(\omega)k_z}{k_z^{(2)}}, \tag{19}$$

where

$$T = \frac{4k_z k_z^{(2)}}{(k_z + k_z^{(2)})(\varepsilon(\omega)k_z + k_z^{(2)})} \tag{20}$$

and

$$r_s^f = \frac{k_z - k_z^{(2)}}{k_z + k_z^{(2)}}, \quad t_s^f = \frac{2k_z}{k_z + k_z^{(2)}} \tag{21}$$

are TE Fresnel coefficients for diffraction on a flat dielectric semispace.

Consider TM (p-polarized) electromagnetic plane wave diffracting from a Chern-Simons layer located at $z = 0$ on a dielectric semispace ($z < 0$) defined by a frequency dependent dielectric permittivity $\varepsilon(\omega)$:

$$H_x = \exp(-ik_z z) + r_p \exp(ik_z z), z > 0 \quad (22)$$

$$H_x = t_p \exp(-ik_z^{(2)} z), z < 0 \quad (23)$$

$$E_x = r_{p \to s} \exp(ik_z z), z > 0 \quad (24)$$

$$E_x = t_{p \to s} \exp(-ik_z^{(2)} z), z < 0. \quad (25)$$

From the condition $E_x|_{z=0^+} = E_x|_{z=0^-}$ it follows

$$r_{p\to s} = t_{p\to s}. \tag{26}$$

From the condition $E_y|_{z=0^+} = E_y|_{z=0^-}$ and equation $E_y = -\frac{1}{i\omega\varepsilon(\omega)}\partial_z H_x$ we get

$$k_z(1 - r_p) = \frac{k_z^{(2)}}{\varepsilon(\omega)} t_p. \tag{27}$$

From (9)

$$1 + r_p - t_p = 2a\, r_{p\to s}. \tag{28}$$

From the condition (10) and Maxwell equations $H_y = \frac{1}{i\omega}\partial_z E_x$, $E_y = -\frac{1}{i\omega\varepsilon(\omega)}\partial_z H_x$ we obtain

$$k_z r_{p\to s} + k_z^{(2)} t_{p\to s} = 2a\, t_p \frac{k_z^{(2)}}{\varepsilon(\omega)}. \tag{29}$$

We find reflection and transmission coefficients for TM plane wave:

$$r_p = \frac{r_p^f + a^2 T}{1 + a^2 T}, \quad t_p = \frac{t_p^f}{1 + a^2 T}, \quad r_{p\to s} = t_{p\to s} = \frac{a T}{1 + a^2 T}, \quad (30)$$

where

$$r_p^f = \frac{\varepsilon(\omega) k_z - k_z^{(2)}}{\varepsilon(\omega) k_z + k_z^{(2)}}, \quad t_p^f = \frac{2\varepsilon(\omega) k_z}{\varepsilon(\omega) k_z + k_z^{(2)}} \quad (31)$$

are TM Fresnel coefficients for diffraction on a flat dielectric semispace and

$$T = \frac{4 k_z k_z^{(2)}}{(k_z + k_z^{(2)})(\varepsilon(\omega) k_z + k_z^{(2)})} \quad (32)$$

Special case: Chern-Simons layer in vacuum

In vacuum the reflection coefficients for TE mode from a Chern-Simons layer have the form:

$$r_s = -\frac{a^2}{1+a^2}, \qquad t_s = \frac{1}{1+a^2},$$
$$r_{s\to p} = \frac{a}{1+a^2}, \qquad t_{s\to p} = -\frac{a}{1+a^2}, \tag{33}$$

for TM mode:

$$r_p = \frac{a^2}{1+a^2}, \qquad t_p = \frac{1}{1+a^2},$$
$$r_{p\to s} = \frac{a}{1+a^2}, \qquad t_{p\to s} = \frac{a}{1+a^2}. \tag{34}$$

(V.N.Marachevsky, Theor.Math.Phys., 2017)

Casimir energy of two Chern-Simons layers in vacuum

The Casimir energy of two Chern-Simons layers in vacuum is (V.N.Marachevsky, Theor.Math.Phys., 2017)

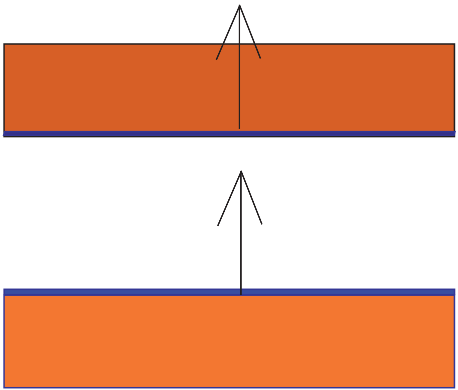
$$E(a_1, -a_2, l) = \frac{1}{2} \iiint \frac{d\omega dk_x dk_y}{(2\pi)^3} \ln\det(I - R_{up} R_{down}) = \frac{1}{4\pi^2} \int_0^{+\infty} dr r^2 \ln\det(I - e^{-2Lr} R(a_2) R(a_1)) = \frac{1}{4\pi^2} \int_0^{+\infty} dr r^2 \ln\det(I - e^{-2Lr} Q), \tag{35}$$

where

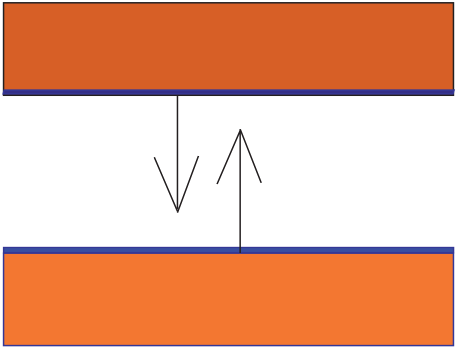
$$Q = a_1 a_2 \begin{pmatrix} \frac{1}{(a_1 - i)(a_2 + i)} & 0 \\ 0 & \frac{1}{(a_1 + i)(a_2 - i)} \end{pmatrix}. \tag{36}$$

$$E(a_1,-a_2,L) = -\frac{1}{16\pi^2 L^3}\left(\mathrm{Li}_4\left(\frac{a_1 a_2}{(a_1-i)(a_2+i)}\right) + \mathrm{Li}_4\left(\frac{a_1 a_2}{(a_1+i)(a_2-i)}\right)\right), \qquad (37)$$

where $\mathrm{Li}_4(x) = \sum_{k=1}^{+\infty} x^k/k^4 = -\frac{1}{2}\int_0^{+\infty} drr^2 \ln(1-xe^{-r})$.
Note that for $a_1 = -a_2$ the force is attractive for every a_1 (due to a theorem that the Casimir force between mirror objects is attractive).
For $a_1 = a_2$ (V. N. Markov and Yu. M. Pis'mak, J. Phys. A: Math. Gen. , 2006) one gets the Casimir energy of two Chern-Simons layers with identically selected directions of the layers in space. In this case the force is repulsive at all distances L for $a_1 \in [0, a_0]$, where $a_0 \approx 1.032502$, and attractive at all distances L for $a_1 > a_0$.



$a_1 = a_2$ case is shown, leads to repulsion for two layers in vacuum for $a_1 \in [0, a_0]$, where $a_0 \approx 1.032502$, and to attraction for $a_1 > a_0$.



$a_1 = -a_2$ is shown, leads to attraction in vacuum and for coinciding dielectrics.

Casimir effect results for Chern-Simons layers at the surfaces of dielectrics and metals

Consider two dielectric semispaces with Chern-Simons terms characterized by constants a_1, a_2 on their surfaces respectively. Assume there is a vacuum slit L between semispaces.
The reflection matrix $R_{down} = R(a_1)$ from the $z \leq 0$ semispace is defined by:

$$R(a_1) = \begin{pmatrix} r_s & r_{p\to s} \\ r_{s\to p} & r_p \end{pmatrix} = \frac{1}{1+a_1^2 T} \begin{pmatrix} r_s^f - a_1^2 T & a_1 T \\ a_1 T & r_p^f + a_1^2 T \end{pmatrix}. \tag{38}$$

The reflection matrix from the $z \geq L$ semispace is defined after euclidean rotation by

$$R_{up} = SR(a_2)S, \tag{39}$$

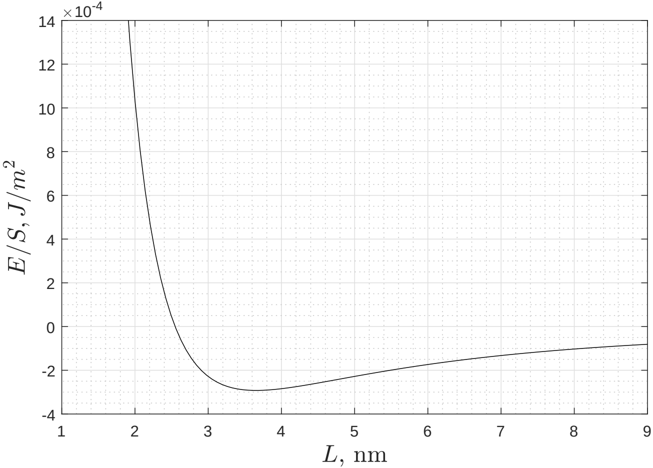
where

$$S = \begin{pmatrix} e^{-L\sqrt{\omega^2+k_x^2+k_y^2}} & 0 \\ 0 & e^{-L\sqrt{\omega^2+k_x^2+k_y^2}} \end{pmatrix} \tag{40}$$

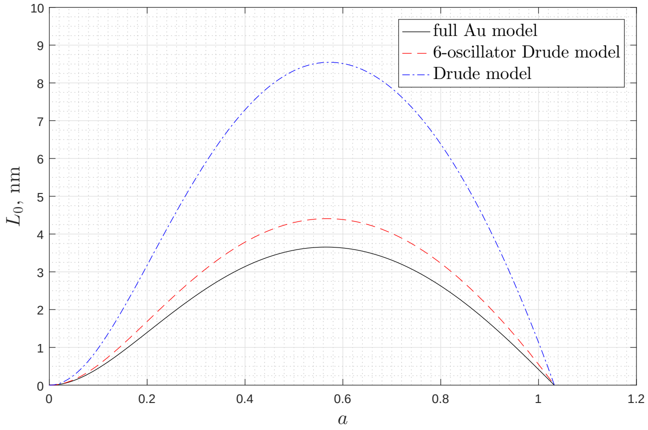
is a matrix due to a change of the coordinate system
$x_1 = x, y_1 = -y, z_1 = -z + L$.

The Casimir energy is equal

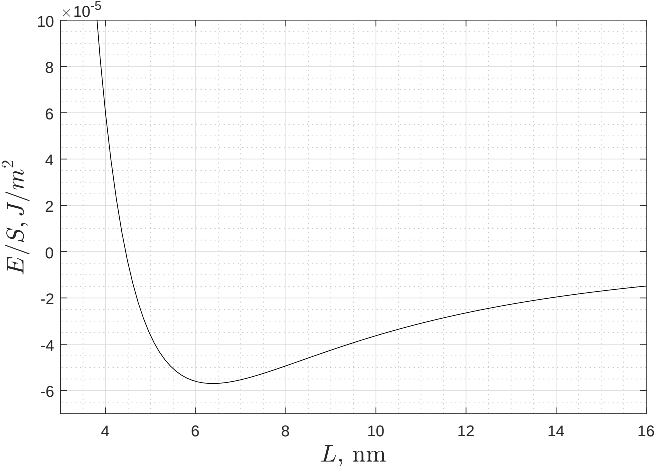
$$E(a_1, -a_2, L) = \frac{1}{2} \iiint \frac{d\omega dk_x dk_y}{(2\pi)^3} \ln\det(I - R_{up} R_{down}) = \frac{1}{4\pi^2} \int_0^{+\infty} drr^2 \ln\det(I - e^{-2Lr} R(a_2) R(a_1)). \quad (41)$$



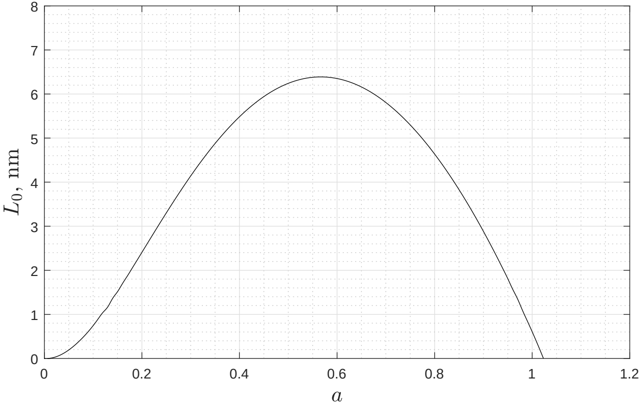
Energy on a unit surface for Chern-Simons layers on Au semispaces obtained from full set of known optical data for Au. Chern-Simons constant is $a_1 = a_2 = 0.565$, which corresponds to the minimum of energy at $L_0 = 3.65\,\mathrm{nm}$.



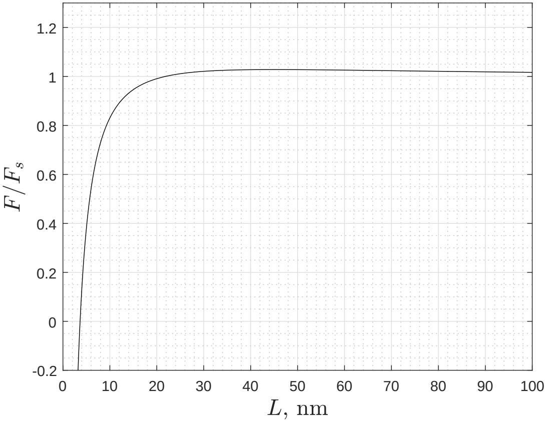
Position of the minimum of the energy L_0 for Chern-Simons layers on Au semispaces, $a \equiv a_1 = a_2$. Results for three models of Au dielectric permittivity are shown.



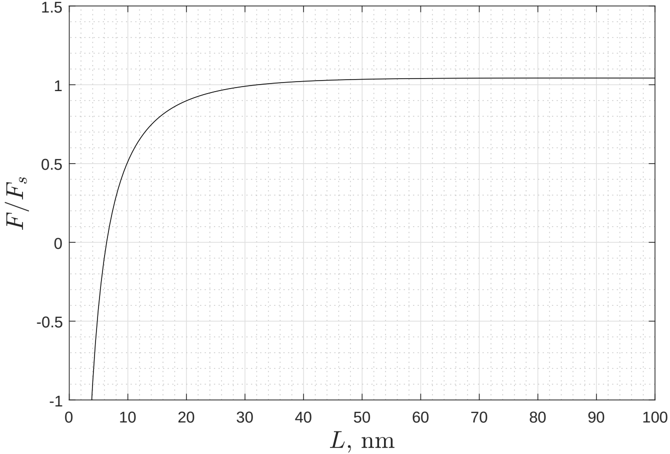
Energy on a unit surface obtained for Chern-Simons layers on intrinsic Si semispaces. Chern-Simons constant is $a_1 = a_2 = 0.567$, which corresponds to the minimum of the energy at $L_0 = 6.39\,\mathrm{nm}$.



Position of the minimum of the energy L_0 for Chern-Simons layers on intrinsic Si semispaces, $a \equiv a_1 = a_2$.



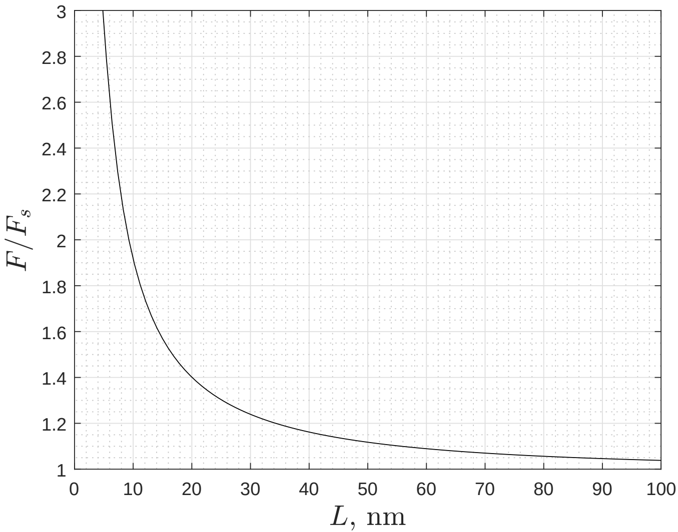
Ratio of the force F with Chern-Simons layers at the boundaries of two Au semispaces to the Lifshitz force F_s between two Au semispaces separated by a distance L, $a_1 = a_2 = 0.565$.



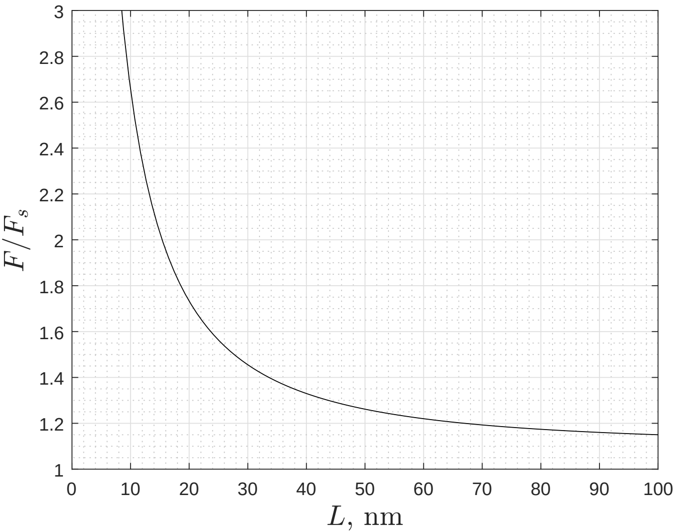
Ratio of the force F with Chern-Simons layers at the boundaries of two intrinsic Si semispaces to the Lifshitz force F_s between two intrinsic Si semispaces separated by a distance L. Chern-Simons constants are $a_1 = a_2 = 0.567$.

Explaining the minimum of the Casimir energy

Lifshitz force power law between two dielectrics/metals effectively changes from retarded L^{-4} to nonretarded L^{-3} behaviour at distances of the order $L \sim 10$ nm.
On the other hand, the force between two Chern-Simons layers in vacuum has L^{-4} behavior at all separations and thus dominates the total force at separations of the order $L \lesssim 10$ nm. For the condition $a \equiv a_1 = a_2$ the Casimir force between two Chern-Simons layers in vacuum is repulsive at all distances L for an interval $a \in [0, a_0]$, where $a_0 \approx 1.032502$.
As a result, the sum of the Lifshitz force and the force between two Chern-Simons layers in vacuum effectively leads to a repulsive force at short separations and to an attractive force at large separations.



Ratio of the force F with Chern-Simons layers at the boundaries of two Au semispaces to the Lifshitz force F_s between two Au semispaces separated by a distance L, $a_1 = -a_2 = 0.565$.



Ratio of the force F with Chern-Simons layers at the boundaries of two intrinsic Si semispaces to the Lifshitz force F_s between two intrinsic Si semispaces, $a_1 = -a_2 = 0.567$.

The attractive Casimir force in the case $a_1 = -a_2$ is explained by a theorem that the Casimir force is attractive for two objects obtained by mirror images of each other and separated by a vacuum slit .

Conclusions

1. A diffraction problem for reflection of an electromagnetic wave from a dielectric with Chern-Simons layer at its surface is solved.
2. The Casimir energy of two Chern-Simons layers and two Chern-Simons layers on top of dielectrics (metals) separated by a vacuum slit is derived in a scattering approach in terms of reflection coefficients.
3. Existence of a regime with the minimum of the Casimir energy due to presence of Chern-Simons layers at the surfaces of dielectrics/metals at a distance of the order 10 nm, the Casimir force in this case is attractive at large distances and repulsive at short distances between the two dielectrics/metals with Chern-Simons surface layers.