Casimir repulsion and attraction due to presence of Chern-Simons layers at the surfaces of dielectrics and metals

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In this talk

2. The solution of a diffraction problem for Chern-Simons layer in vacuum and at the surface of a dielectric.
3. Casimir energy of two Chern-Simons layers in vacuum.
4. Casimir energy of two dielectrics with Chern-Simons layers at their surfaces.
5. Appearance of the minimum in the Casimir energy due to presence of Chern-Simons layers at the surfaces of dielectrics/metals.
Rayleigh decomposition for 1d gratings.

Rayleigh expansion for an incident electromagnetic wave on a single grating

\[ E_y(x, z) = I_p^{(e)} \exp(i \alpha_p x - i \beta_p^{(1)} z) + \sum_{n=-\infty}^{+\infty} R_{np}^{(e)} \exp(i \alpha_n x + i \beta_n^{(1)} z), \]

\[ H_y(x, z) = I_p^{(h)} \exp(i \alpha_p x - i \beta_p^{(1)} z) + \sum_{n=-\infty}^{+\infty} R_{np}^{(h)} \exp(i \alpha_n x + i \beta_n^{(1)} z). \]

Here \( \alpha_p = k_x + 2\pi p/d \) and \( \beta_p^{(1)2} = \omega^2 - k_y^2 - \alpha_p^2. \)

The reflection matrix is constructed as follows:

\[
R_1(\omega) = \begin{pmatrix}
R_{n1l1}^{(e)}(I_p^{(e)} = \delta_{pl1}, I_p^{(h)} = 0) & R_{n1l2}^{(e)}(I_p^{(e)} = 0, I_p^{(h)} = \delta_{pl2}) \\
R_{n3l3}^{(h)}(I_p^{(e)} = \delta_{pl3}, I_p^{(h)} = 0) & R_{n4l4}^{(h)}(I_p^{(e)} = 0, I_p^{(h)} = \delta_{pl4})
\end{pmatrix}.
\]

Rayleigh expansion is exact outside gratings. The unknown coefficients can be determined from the exact solution of Maxwell equations.
\[ z_1 = -z + L \]
$$R_{\text{down}} R_{\text{up}} \psi_i = \psi_i$$

$$\det(I - R_{\text{down}} R_{\text{up}}) = 0$$

argument principle
Argument principle

\[
\frac{1}{2\pi i} \oint \phi(\omega) \frac{d}{d\omega} \ln f(\omega) d\omega = \sum \phi(\omega_0) - \sum \phi(\omega_\infty)
\] (1)

\[\phi(\omega) = \hbar \omega/2\]

\[f(\omega) = \det(I - R_{\text{down}}(\omega)R_{\text{up}}(\omega))\]
Casimir energy of two gratings

\[ E = \frac{\hbar c}{(2\pi)^3} \int_0^{+\infty} d\omega \int_{-\infty}^{+\infty} dk_y \int_{-\pi/d}^{\pi/d} dk_x \ln \det \left( I - R_{\text{down}}(i\omega)R_{\text{up}}(i\omega) \right) \]  

(2)

\[ R_{\text{up}}(i\omega) = Q^* K(i\omega) R(i\omega) K(i\omega) Q, \]  

(3)

\[ K(i\omega) = \begin{pmatrix} G_1 & 0 \\ 0 & G_1 \end{pmatrix}, \]  

(4)

with matrix elements \( e^{-L\sqrt{\omega^2 + k_y^2 + (k_x + \frac{2\pi p}{d})^2}} \), \( p = -N \ldots N \) on the main diagonal of a matrix \( G_1 \),

\[ Q = \begin{pmatrix} G_2 & 0 \\ 0 & G_2 \end{pmatrix}, \]  

(5)

with matrix elements \( e^{2\pi i m s/d} \), \( p = -N \ldots N \) on the main diagonal of a matrix \( G_2 \). (A.Lambrecht and V.N.Marachevsky, PRL 101, 160403 (2008) ).
Chern-Simons Casimir effect


This talk is based on:

V.N.Marachevsky, Chern-Simons layers in the vacuum, Theor.Math.Phys. 190(2), 315 (2017);
Chern-Simons layer on a dielectric semispace

The action with Chern-Simons layer at $z = 0$ has the form:

$$S = \frac{a}{2} \int \varepsilon^{z\nu\rho\sigma} A_\nu F_{\rho\sigma} dtdxdy.$$  \hfill (6)

Equations of electromagnetic field in the presence of Chern-Simons action (6) can be written as follows:

$$\partial_\mu F_{\mu\nu} + a \varepsilon^{z\nu\rho\sigma} F_{\rho\sigma} \delta(z) = 0.$$  \hfill (7)

Consider a flat Chern-Simons layer put at $z = 0$ on a dielectric semispace $z < 0$ characterized by a frequency dependent dielectric permittivity $\varepsilon(\omega)$, the magnetic permeability $\mu = 1$. Boundary conditions on the components of the electromagnetic field follow:

$$E_z|_{z=0}^+ - \varepsilon(\omega)E_z|_{z=0}^- = -2aH_z|_{z=0},$$  \hfill (8)

$$H_x|_{z=0}^+ - H_x|_{z=0}^- = 2aE_x|_{z=0},$$  \hfill (9)

$$H_y|_{z=0}^+ - H_y|_{z=0}^- = 2aE_y|_{z=0}.$$  \hfill (10)
**Diffraction problem**

Consider TE (s-polarized) electromagnetic plane wave diffracting from a Chern-Simons layer located at $z = 0$ on a dielectric semispace ($z < 0$) defined by a dielectric permittivity $\varepsilon(\omega)$ (the factor $\exp(i\omega t + ik_y y)$ is dropped for simplicity of notations):

\[ E_x = \exp(-ik_z z) + r_s \exp(ik_z z), \quad z > 0 \quad (11) \]
\[ E_x = t_s \exp(-ik_z^{(2)} z), \quad z < 0 \quad (12) \]
\[ H_x = r_s \exp(i k_z z), \quad z > 0 \quad (13) \]
\[ H_x = t_s \exp(-i k_z^{(2)} z), \quad z < 0. \quad (14) \]

Here $k_z = \sqrt{\omega^2 - k_y^2}$, $k_z^{(2)} = \sqrt{\varepsilon(\omega)\omega^2 - k_y^2}$. 
From the condition $H_x|_{z=0^+} - H_x|_{z=0^-} = 2aE_x|_{z=0}$ it follows

$$r_s \to p - t_s \to p = 2at_s. \quad (15)$$

From $E_x|_{z=0^+} = E_x|_{z=0^-}$ we obtain

$$1 + r_s = t_s. \quad (16)$$

From the condition $E_y|_{z=0^+} = E_y|_{z=0^-}$ and Maxwell equation $E_y = -\frac{1}{i\omega\varepsilon(\omega)} \partial_z H_x$ it follows that

$$r_s \to p k_z = -\frac{k_z^{(2)}}{\varepsilon(\omega)} t_s \to p. \quad (17)$$

From the condition (10) and Maxwell equation $H_y = \frac{1}{i\omega} \partial_z E_x$ we get

$$k_z(-1 + r_s) + k_z^{(2)} t_s = 2a \frac{k_z^{(2)}}{\varepsilon(\omega)} t_s \to p. \quad (18)$$
Solving equations (15)-(18) we find reflection and transmission coefficients for TE plane wave:

\[ r_s = \frac{r_s^f - a^2 T}{1 + a^2 T}, \quad t_s = \frac{t_s^f}{1 + a^2 T}, \]

\[ r_{s \rightarrow p} = \frac{a T}{1 + a^2 T}, \quad t_{s \rightarrow p} = -\frac{a T}{1 + a^2 T} \frac{\varepsilon(\omega) k_z}{k_z^{(2)}}, \quad (19) \]

where

\[ T = \frac{4 k_z k_z^{(2)}}{(k_z + k_z^{(2)})(\varepsilon(\omega) k_z + k_z^{(2)})}, \quad (20) \]

and

\[ r_s^f = \frac{k_z - k_z^{(2)}}{k_z + k_z^{(2)}}, \quad t_s^f = \frac{2 k_z}{k_z + k_z^{(2)}} \quad (21) \]

are TE Fresnel coefficients for diffraction on a flat dielectric semispace.
Consider TM ($p$-polarized) electromagnetic plane wave diffracting from a Chern-Simons layer located at $z = 0$ on a dielectric semispace ($z < 0$) defined by a frequency dependent dielectric permittivity $\varepsilon(\omega)$:

\[
H_x = \exp(-ik_z z) + r_p \exp(ik_z z), \quad z > 0 \tag{22}
\]

\[
H_x = t_p \exp(-ik_z^{(2)} z), \quad z < 0 \tag{23}
\]

\[
E_x = r_{p \to s} \exp(ik_z z), \quad z > 0 \tag{24}
\]

\[
E_x = t_{p \to s} \exp(-ik_z^{(2)} z), \quad z < 0. \tag{25}
\]
From the condition $E_x|_{z=0^+} = E_x|_{z=0^-}$ it follows

$$ r_{p\rightarrow s} = t_{p\rightarrow s}. \quad (26) $$

From the condition $E_y|_{z=0^+} = E_y|_{z=0^-}$ and equation $E_y = -\frac{1}{i\omega\varepsilon(\omega)} \partial_z H_x$ we get

$$ k_z(1 - r_p) = \frac{k_z^{(2)}}{\varepsilon(\omega)} t_p. \quad (27) $$

From (9)

$$ 1 + r_p - t_p = 2a r_{p\rightarrow s}. \quad (28) $$

From the condition (10) and Maxwell equations $H_y = \frac{1}{i\omega} \partial_z E_x$, $E_y = -\frac{1}{i\omega\varepsilon(\omega)} \partial_z H_x$ we obtain

$$ k_z r_{p\rightarrow s} + k_z^{(2)} t_{p\rightarrow s} = 2a t_p \frac{k_z^{(2)}}{\varepsilon(\omega)}. \quad (29) $$
We find reflection and transmission coefficients for TM plane wave:

\[ r_p = \frac{r_p^f + a^2 T}{1 + a^2 T}, \quad t_p = \frac{t_p^f}{1 + a^2 T}, \quad r_{p\rightarrow s} = t_{p\rightarrow s} = \frac{aT}{1 + a^2 T}, \quad (30) \]

where

\[ r_p^f = \frac{\varepsilon(\omega)k_z - k_z^{(2)}}{\varepsilon(\omega)k_z + k_z^{(2)}}, \quad t_p^f = \frac{2\varepsilon(\omega)k_z}{\varepsilon(\omega)k_z + k_z^{(2)}} \quad (31) \]

are TM Fresnel coefficients for diffraction on a flat dielectric semispace and

\[ T = \frac{4k_z k_z^{(2)}}{(k_z + k_z^{(2)})(\varepsilon(\omega)k_z + k_z^{(2)})} \quad (32) \]
Special case: Chern-Simons layer in vacuum

In vacuum the reflection coefficients for TE mode from a
Chern-Simons layer have the form:

\[ r_s = -\frac{a^2}{1 + a^2}, \quad t_s = \frac{1}{1 + a^2}, \]
\[ r_{s\rightarrow p} = \frac{a}{1 + a^2}, \quad t_{s\rightarrow p} = -\frac{a}{1 + a^2}, \]  
(33)

for TM mode:

\[ r_p = \frac{a^2}{1 + a^2}, \quad t_p = \frac{1}{1 + a^2}, \]
\[ r_{p\rightarrow s} = \frac{a}{1 + a^2}, \quad t_{p\rightarrow s} = \frac{a}{1 + a^2}. \]  
(34)


\[ E(a_1, -a_2, l) = \frac{1}{2} \int\int\int \frac{d\omega dk_x dk_y}{(2\pi)^3} \ln \det(I - R_{up} R_{down}) = \]

\[ \frac{1}{4\pi^2} \int_0^{+\infty} dr r^2 \ln \det(I - e^{-2Lr} R(a_2)R(a_1)) = \quad (35) \]

\[ \frac{1}{4\pi^2} \int_0^{+\infty} dr r^2 \ln \det(I - e^{-2Lr} Q), \]

where

\[ Q = a_1 a_2 \begin{pmatrix} \frac{1}{(a_1-i)(a_2+i)} & 0 \\ 0 & \frac{1}{(a_1+i)(a_2-i)} \end{pmatrix}. \quad (36) \]
\[ E(a_1, -a_2, L) = -\frac{1}{16\pi^2 L^3} \left( \text{Li}_4 \left( \frac{a_1 a_2}{(a_1 - i)(a_2 + i)} \right) \right) \\
+ \text{Li}_4 \left( \frac{a_1 a_2}{(a_1 + i)(a_2 - i)} \right) \right) , \quad (37) \]

where \( \text{Li}_4(x) = \sum_{k=1}^{+\infty} x^k / k^4 = -\frac{1}{2} \int_0^{+\infty} drr^2 \ln(1 - xe^{-r}) \). Note that for \( a_1 = -a_2 \) the force is attractive for every \( a_1 \) (due to a theorem that the Casimir force between mirror objects is attractive). For \( a_1 = a_2 \) (V. N. Markov and Yu. M. Pis’mak, J. Phys. A: Math. Gen., 2006) one gets the Casimir energy of two Chern-Simons layers with identically selected directions of the layers in space. In this case the force is repulsive at all distances \( L \) for \( a_1 \in [0, a_0] \), where \( a_0 \approx 1.032502 \), and attractive at all distances \( L \) for \( a_1 > a_0 \).
The $a_1 = a_2$ case is shown, leading to repulsion for two layers in vacuum for $a_1 \in [0, a_0]$, where $a_0 \approx 1.032502$, and to attraction for $a_1 > a_0$. 
\( a_1 = -a_2 \) is shown, leads to attraction in vacuum and for coinciding dielectrics.
Casimir effect results for Chern-Simons layers at the surfaces of dielectrics and metals
Casimir energy

Consider two dielectric semispaces with Chern-Simons terms characterized by constants $a_1$, $a_2$ on their surfaces respectively. Assume there is a vacuum slit $L$ between semispaces. The reflection matrix $R_{\text{down}} = R(a_1)$ from the $z \leq 0$ semispace is defined by:

$$R(a_1) = \begin{pmatrix} r_s & r_{p \rightarrow s} \\ r_{s \rightarrow p} & r_p \end{pmatrix} = \frac{1}{1 + a_1^2 T} \begin{pmatrix} r^f_s - a_1^2 T & a_1 T \\ a_1 T & r^f_p + a_1^2 T \end{pmatrix}.$$  

(38)

The reflection matrix from the $z \geq L$ semispace is defined after euclidean rotation by

$$R_{\text{up}} = SR(a_2)S,$$  

(39)

where

$$S = \begin{pmatrix} e^{-L \sqrt{\omega^2 + k_x^2 + k_y^2}} & 0 \\ 0 & e^{-L \sqrt{\omega^2 + k_x^2 + k_y^2}} \end{pmatrix}$$

(40)

is a matrix due to a change of the coordinate system $x_1 = x, y_1 = -y, z_1 = -z + L$. 
The Casimir energy is equal

\[ E(a_1, -a_2, L) = \frac{1}{2} \int \int \int d\omega dk_x dk_y \ln \det(I - R_{up}R_{down}) = \]

\[ \frac{1}{4\pi^2} \int_0^{+\infty} dr r^2 \ln \det(I - e^{-2Lr} R(a_2)R(a_1)). \]
Energy on a unit surface for Chern-Simons layers on Au semispaces obtained from full set of known optical data for Au. Chern-Simons constant is $a_1 = a_2 = 0.565$, which corresponds to the minimum of energy at $L_0 = 3.65 \text{ nm}$. 
Position of the minimum of the energy $L_0$ for Chern-Simons layers on Au semispaces, $a \equiv a_1 = a_2$. Results for three models of Au dielectric permittivity are shown.
Energy on a unit surface obtained for Chern-Simons layers on intrinsic Si semispaces. Chern-Simons constant is $a_1 = a_2 = 0.567$, which corresponds to the minimum of the energy at $L_0 = 6.39 \text{ nm}$.
Position of the minimum of the energy $L_0$ for Chern-Simons layers on intrinsic Si semispaces, $a \equiv a_1 = a_2$. 
Ratio of the force $F$ with Chern-Simons layers at the boundaries of two Au semispaces to the Lifshitz force $F_s$ between two Au semispaces separated by a distance $L$, $a_1 = a_2 = 0.565$. 
Ratio of the force $F$ with Chern-Simons layers at the boundaries of two intrinsic Si semispaces to the Lifshitz force $F_s$ between two intrinsic Si semispaces separated by a distance $L$. Chern-Simons constants are $a_1 = a_2 = 0.567$. 
Explaining the minimum of the Casimir energy

Lifshitz force power law between two dielectrics/metals effectively changes from retarded $L^{-4}$ to nonretarded $L^{-3}$ behaviour at distances of the order $L \sim 10$ nm. On the other hand, the force between two Chern-Simons layers in vacuum has $L^{-4}$ behavior at all separations and thus dominates the total force at separations of the order $L \lesssim 10$ nm. For the condition $a \equiv a_1 = a_2$ the Casimir force between two Chern-Simons layers in vacuum is repulsive at all distances $L$ for an interval $a \in [0, a_0]$, where $a_0 \approx 1.032502$.

As a result, the sum of the Lifshitz force and the force between two Chern-Simons layers in vacuum effectively leads to a repulsive force at short separations and to an attractive force at large separations.
Ratio of the force $F$ with Chern-Simons layers at the boundaries of two Au semispaces to the Lifshitz force $F_s$ between two Au semispaces separated by a distance $L$, $a_1 = -a_2 = 0.565$. 
Ratio of the force $F$ with Chern-Simons layers at the boundaries of two intrinsic Si semispaces to the Lifshitz force $F_s$ between two intrinsic Si semispaces, $a_1 = -a_2 = 0.567$. 

![Graph depicting the ratio $F/F_s$ vs. separation $L$, nm.](image-url)
The attractive Casimir force in the case $a_1 = -a_2$ is explained by a theorem that the Casimir force is attractive for two objects obtained by mirror images of each other and separated by a vacuum slit.
Conclusions

1. A diffraction problem for reflection of an electromagnetic wave from a dielectric with Chern-Simons layer at its surface is solved.

2. The Casimir energy of two Chern-Simons layers and two Chern-Simons layers on top of dielectrics (metals) separated by a vacuum slit is derived in a scattering approach in terms of reflection coefficients.

3. Existence of a regime with the minimum of the Casimir energy due to presence of Chern-Simons layers at the surfaces of dielectrics/metals at a distance of the order 10 nm, the Casimir force in this case is attractive at large distances and repulsive at short distances between the two dielectrics/metals with Chern-Simons surface layers.