Ultra-high energy particle collisions near black holes and singularities and super-Penrose process

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Two kinds of energies as a result of collisions

1) High (unbound) energy in the centre of mass frame $E_{c.m.}$.

Black holes, naked singularities, quasiblack holes, star-like configurations, wormholes

BHs: rotating or electrically charged
Proximity to horizon
Ergoregion (high angular momentum), Extremely rapid rotation

Collisions outside and inside BH
In magnetic field
Sclar field

Particle moving towards horizon (BSW effect), Banados-Silk-Wesr PRL 2009
Head-on collisions
Fine-tuned (critical) and typical (usual) particles

2) Possibility to get high (unbounded) energies $E$ at infinity (debris after collision) – super-Penrose process
Physical explanation and properties of BSW effect

Universal character of BSW effect near BH

Kinematic nature of the BSW effect. Role of critical trajectories

BSW effect and acceleration horizons

Geometric explanation

Kinematic explanation for collisions inside BH

Extremal versus nonextremal BHs

Kinematic censorship

Role of self-force due to gravitational radiation

BSW effect versus Penrose process: what can be seen at infinity?
Part 1

High energy processes near BHs

Key quantity: energy in centre of mass frame

1 particle

\[ m^2 = \left| P_\mu P^{\mu} \right| \]

2 particles colliding in some point

\[ E^2_{cm} = \left| P_\mu P^{\mu} \right| \]

Total momentum

\[ P_\mu = p^{(1)}_\mu + p^{(2)}_\mu \]

\[ P_a = (E_{c.m.}, 0, 0, 0) \quad u^\mu u_\mu = -1 \]

Individual E finite, energy in CM frame unbounded
Two different kinds of energy

Killing energy

\[ E = -p_\mu \xi^\mu \]

\( \xi^\mu \) Killing vector

\( E \)

conserved, integral of motion since metric is static or stationary

Energy in the CM frame

\( E_{c.m.} \)

not conserved. Moreover, it is defined in one point only. point of collision
Head-on collision

1975 - 1977  T. Piran, J. Katz and J. Shanam

Two particles move in opposite directions near BH

Almost infinite relative blue shift

E in CM frame almost diverges

Special scenario. Particle near black (not white) hole moving away from horizon and colliding with another particle
Both particles experience blue shift, centre of mass frame is in free fall.
Acceleration of particles as universal property of rotating black holes

O. Z., PRD 2010

Role of horizon

Universality of black hole physics

Unified approach to nonextremal versus extremal black holes
Energy in CM frame

\[ E_{c.m.}^2 = -(m_1 u^\mu_1 + m_2 u^\mu_2)(m_1 u_{1\mu} + m_2 u_{2\mu}) \]

\[ E_{c.m.}^2 = m_1^2 + m_2^2 + 2m_1 m_2 \gamma \]

\[ \gamma = -(u_1 u_2) \]

equatorial plane  \[ \theta = \frac{\pi}{2} \quad (z = 0) \quad \text{Is a symmetry one} \]

\[ m u_0 = -E \quad m u_\phi = L \quad \text{conserved quantities} \]

Integrals of geodesic equations

\[ g_{\mu \nu} u^\mu u^\nu = -1 \]
\[ m_1 = m u^0 = \frac{E - \omega L}{N^2} = \frac{X}{N^2}. \quad X = E - \omega L \]

\[ 2m_1 m_2 \gamma = \frac{X_1 X_2 - \varepsilon_1 \varepsilon_2 Z_1 Z_2}{N^2} - \frac{L_1 L_2}{g_\phi}, \quad Z = \sqrt{X^2 - N^2 (m^2 + \frac{L^2}{g_\phi})} \]

\[ \varepsilon = -1 \quad \text{for particle moving towards horizon} \]

\[ \varepsilon = +1 \quad \text{away from horizon} \]
\[ \varepsilon_1 \varepsilon_2 = -1 \] head-on collision, Piran et al

\[ 2m_1 m_2 \gamma = \frac{X_1 X_2 + Z_1 Z_2}{N^2} - \frac{L_1 L_2}{g_\phi}, \]

\[ E_{c.m.}^2 \] always unbounded near horizon

For any relationship between energies and angular momenta
\[ \varepsilon_1 = \varepsilon_2 = -1 \quad \text{Energy in CM frame} \]

\[ 2m_1m_2\gamma = \frac{X_1X_2 - Z_1Z_2}{N^2} - \frac{L_1L_2}{g_\phi}, \]

\[ Z = \sqrt{X^2 - N^2(m^2 + \frac{L^2}{g_\phi})} \]

Three kinds of mechanism leading to unbounded energy in CM frame

1) \( N \rightarrow 0 \quad \text{proximity to horizons} \quad \text{BSW} \)

2) \( L_2 \rightarrow -\infty \quad \text{inside ergoregion, NOT near horizon} \quad \text{Grib and Pavlov, Kerr metric} \)

3) \( \omega \rightarrow \infty \quad \text{rapid rotation (wormholes)} \)

Generalization OZ
\[ \varepsilon_1 = \varepsilon_2 = -1 \quad \text{BSW} \]

\[ 2m_1 m_2 \gamma = \frac{X_1 X_2 - Z_1 Z_2}{N^2} - \frac{L_1 L_2}{g_\phi}, \]

\[ Z = \sqrt{X^2 - N^2 \left( m^2 + \frac{L^2}{g_\phi} \right)} \]

In general case, \( E_{c.m.}^2 \) remains bound in horizon limit \( N \to 0 \)

Special conditions for unbounded \( E_{c.m.}^2 \)

Two kinds of particles (trajectories)

Usual \( X_H \equiv E - \omega_H L \neq 0 \)

Critical \( X_H \equiv E - \omega_H L = 0 \)
Different limiting transitions

1) point of collision approaches the horizon, and \( L_1 \rightarrow L_{1(H)} = \frac{E_1}{\omega_H} \)

2) \( L_1 \rightarrow L_{1(H)} \) and \( N \rightarrow 0 \) afterwards

In both cases

\[
\lim_{L_1 \rightarrow L_{1(H)}} \lim_{N \rightarrow 0} E_{cm} = \lim_{N \rightarrow 0} \lim_{L_1 \rightarrow L_{1(H)}} E_{cm} = \infty.
\]

particle 1 is critical, particle 2 is usual
Extremal versus nonextremal

Problems with attaining extremality, $a=0.998$ (Thorne)

\[ Z = \sqrt{X^2 - N^2 \left( m^2 + \frac{L^2}{g_\phi} \right)} \]

Conditions of regularity: \[ X_H = O(N^2) \quad \text{for critical particle} \]

\[ Z^2 < 0 \]

For NBH, critical particle cannot reach horizon

Grib and Pavlov: nonextremal Kerr, O. Z. generalization

\[ E_{c.m.} \approx \frac{m}{\sqrt{\delta}} \sqrt{\frac{2(L_H - L_2)}{1 - \sqrt{1 - a^2}}} \quad L_1 = L_{(H)} - \delta \quad \text{slightly noncritical} \]
Multiple scattering (Grib and Pavlov)

1) Particle 2 comes from infinity or is created in inner region

2) Collides with particle 2 there. Near-critical + usual
Geometric explanation

\[ \sigma_{a\beta} = a_a a_\beta + b_\alpha b_\beta \]

lightlike vectors and \( N^\mu \)

spacelike vectors \( a^\mu, b^\mu \) orthogonal to them
Four-velocity

\[ u_i^\mu = \frac{l^\mu}{2\alpha_i} + \beta_i N^\mu + s_i^\mu, \quad s_i^\mu = A_i a^\mu + B_i b^b \]

\[ -(u_1 u_2) = \frac{1}{2} \left( \frac{\beta_1}{\alpha_2} + \frac{\beta_2}{\alpha_1} \right) - (s_1 s_2). \]

\[ \alpha = 0 \]

\[ E_{c.m.}^2 = m_1^2 + m_2^2 - 2m_1 m_2 (u_1 u_2) \]

\[ E_{c.m.}^2 = m_1^2 + m_2^2 + m_1 m_2 \left[ \frac{\beta_1}{\alpha_2} + \frac{\beta_2}{\alpha_1} - 2(s_1 s_2) \right]. \]
Now, special condition

Kruskal-like coordinates

\[ Cu^X u^Y = 1 \]

\[ u^X = O(\alpha) \to 0 \quad \tau = O(-\ln X) \to \infty \]

Proper time grows unbound (T. Jacobson, Grib and Pavlov, O. Z.)
Kinematic explanation

\[ E_{\text{c.m.}}^2 = -(p_1^\mu + p_2^\mu)(p_{1\mu} + p_{2\mu}) = m_1^2 + m_2^2 - 2m_1m_2u_1^\mu u_2^\mu. \]

\[ \gamma = -u_1^\mu u_2^\mu = \frac{1}{\sqrt{1 - w^2}} \]

BSW effect occurs if \( w \rightarrow 1 \) \( w \) is relative velocity

\[ w^2 = 1 - \frac{(1-v_1^2)(1-v_2^2)}{[1-v_1v_2(\vec{n}_1 \cdot \vec{n}_2)]^2} \]

The most interesting case: \( v_1 < 1 \), \( v_2 \rightarrow 1 \)

Collision of rapid particle with target

Relative velocity close to \( c \)
\[ ds^2 = -N^2 \, dt^2 + g_{\phi\phi} \, (d\phi - \omega dt)^2 + dl^2 + g_{zz} \, dz^2 \]

Attached to observer

\[ h_{(0)\mu} = -N(1,0,0,0), \]
\[ h_{(1)\mu} = (0,1,0,0), \]
\[ h_{(2)\mu} = \sqrt{g_{zz}} \, (0,0,0,1), \]
\[ h_{(3)\mu} = \sqrt{g_{\phi\phi}} \, (-\omega,0,0,1) \]

\[ -u_\mu \, h_{(0)}^\mu = \frac{E - \omega L}{N}, \quad V_\mu = h_{\mu(0)} \]  

If

\[ u_\mu \, h_{(3)}^\mu = \frac{L}{\sqrt{g_{\phi\phi}}}. \]

then

\[ V_\mu \, \xi^{(3)\mu} = 0 \]

ZAMO

\[ h_{(0)\mu} = -N(1,0,0,0), \]
\[ h_{(1)\mu} = (0,1,0,0), \]
\[ h_{(2)\mu} = \sqrt{g_{zz}} \, (0,0,0,1), \]
\[ h_{(3)\mu} = \sqrt{g_{\phi\phi}} \, (-\omega,0,0,1) \]

\[ E - \omega L = \frac{mN}{\sqrt{1-v^2}}, \]

Horizon limit

1) **Usual** particle,

\[ E \neq \omega_+ L \quad \nu \to 1 \]

2) **Critical** particle

\[ E = \omega_+ L \quad \nu \to \nu_0 < 1 \]
Acceleration versus deceleration

Naïve expectation: to achieve large $E_{c.m.}$ we must have large velocities and individual energies.

No! The condition of criticality selects slow particle among all possible ones

$$E - \omega L = \frac{mN}{\sqrt{1-v^2}},$$

“Almost” any particle is rapid (usual one)

Special subset of slow particles is responsible for large energy in CM frame

Strong gravity ensures BSW effect since it almost “halts” this kind of particles.
Role of gravitational radiation

Naively: it bounds the growth of $E$ in CM, restricts BSW effect

More careful inspection: under rather general assumptions (radial acceleration is finite in OZAMO frame, azimuthal force tends to zero not too slowly) the critical trajectories do exist. As a consequence, the BSW effect persists.

Details: I. V. Tanatarov and O. Z., PRD 2013

BSW effect survives!
Acceleration of particles by nonrotating charged black holes

O. Z. JETP Letters 2010

Role of rotation

\[ L_1 = \frac{E_1}{\omega_H} \quad \text{if} \quad \omega_H \to 0 \quad L_1 \to \infty \]

Angular momentum versus charge

Reissner-Nordstrom Pure radial motion

\[ \omega_H = 0 \quad \text{and} \quad L_1 = L_2 = 0 \]

particles charged, nongeodesic motion
\[ ds^2 = -dt^2 f + \frac{dr^2}{f} + r^2 d\omega^2. \]

\[ m^2 \dot{r}^2 = (E - \frac{qQ}{r})^2 - m^2 f. \]

\[ f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \]

\[ mu^0 = mt = \frac{1}{f} \left(E - \frac{qQ}{r}\right), \]

\[ X_i = E_i - \frac{q_iQ}{r}, Z_i = \sqrt{X_i^2 - m^2 f}. \]

\[ \frac{E_{cm}^2}{2m^2} = 1 + \frac{X_1X_2 - Z_1Z_2}{fm^2}. \]

Rotating BH  
1 critical + 1 usual  
Static charged BH  
Q  

\[ E_{c.m.}^2 \approx \frac{\text{const}}{f} \]
Alternative mechanisms of getting unbounded energies in CM frame

Patil, Joshi, Kimura, Nakao

RN metric, naked singularity

\( Q \approx M \)

Black hole

\( Q > M \)

Naked singularity

\( Q < M \)

Small N

Small f in point of collision
Collisions near inner horizon

Two particles (r,t)

\[ \lim_{r \to r_H} E_{c.m.}(r) = \infty \]

Inside:
Two different points with same r (U,V) Kruskal coordinates.
Collisions near inner horizon

Again, one of two particle should be critical. Then, the following cases are possible.

Fig. 2. The weak version of BSW effect. Near-horizon collision between Critical particle 1 and usual one 2.

Fig. 1. Impossibility of strong version of BSW effect. Critical particle 1 passes through bifurcation point whereas usual one 2 hits left horizon

Kinematic censorship preserved

Fig. 2. The weak version of BSW effect. Near-horizon collision between Critical particle 1 and usual one 2.
Fig. 3. Impossibility of strong version. Critical particle 1 passes through bifurcation point, whereas a usual one 2 hits left horizon.

Fig. 4. Impossibility of strong version of PS effect. Two usual particles hit different branches of horizon.

Kinematic censorship
Kinematic censorship as general principle (Yu. Pavlov, O.Z.)

In any act of collision energy remains finite

Extremal black holes: infinite proper time

Nonextremal black hole, outside: interval shrinks to point

Nonextremal black hole, inside: two different branches of horizon
Part 2

High energy collisions near black holes and super-Penrose process
“Standard” Penrose process

Decay of particle

\[ 0 \rightarrow 1 + 2 \]

\[ E_0 = E_1 + E_2 \quad E_2 < 0 \quad E_1 > E_0 \]

Efficiency

\[ \eta = \frac{E_1 - E_0}{E_0} \]

Collisional Penrose process

\[ 1 + 2 \rightarrow 3 + 4 \]
BSW process

Unbounded energy in the centre of mass (CM) frame

\[ E_{\text{c.m.}} \quad \text{versus} \quad \text{Killing energy measured at infinity} \]

Even in spite of unbounded \[ E_{\text{c.m.}} \]

typically quite modest

Equatorial plane

Kerr Excess less than 50 % Mejer et al, 2012
Harada et al 2012

Dirty black holes

Dirty = surrounded by matter, NOT Kerr BH
Particles 1 and 2 fall from infinity, collide

1  Fined-tuned (critical)

2  Not fined-tuned (usual)

Particle 4 falls into a BH,

4  Particle 4 falls into a BH,

Particle 3 escapes to infinity

Particle 3 moves immediately after collision towards BH and bounces or moves to infinity at once

From analysis of conservation laws:
Particle 3 is critical or near-critical, particle 4 is usual
J. Schnittman (2014)

1  Near-critical moves from BH

2  Not fine-tuned (usual)

head-on collision

Amplification, factor about 14  Kerr, numerics

Harada et al 2015 Analytical derivation for Kerr

O.Z. dirty black holes, analytically
Unbounded efficiency (super-Penrose process)

Is it possible? Test particles approximation

E. Berti, R. Brito and V. Cardoso, 2015

Kerr, numerics

O. Z. 2015

Dirty BH, analytically

Head-on collision of usual particles

Near horizon, particle should move towards BH

White holes (Grib and Pavlov 2014)

or special scenario of multiple collisions in case of BH
Particle 1 (moves from BH) is usual

**Unbounded efficiency (super-Penrose process)**

E. Berti, R. Brito and V. Cardoso, 2015

Kerr, **numerics**

O. Z. 2015

Dirty BH, **analytically**

Near horizon, particle should move towards BH

White holes (Grib and Pavlov 2014)

or special scenario n case of BH
We can try to prepare required state for SPP (usual particle moving \textit{from} BH)

Is it possible to obtain it as a result of previous collision?

Full scenario

Step 1. Particles 1 and 2 \textit{ingoing}: fall from infinity and collide near BH

\begin{align*}
\text{1} & \quad \text{2}
\end{align*}

Step 2. They produce usual \textit{outgoing} particle 3

\begin{align*}
\text{3} & \quad \text{4}
\end{align*}

Step 3. Particle 3 collides with particle 4 falling from infinity (head-on collision)

Result: particle 5 with unbounded energy moving to infinity

\begin{align*}
\text{5}
\end{align*}

One of particles (say, 2) falling from infinity has to have mass (N is lapse function)

\begin{align*}
m_2 = O(N^{-2})
\end{align*}

Kerr metric, E. Leiderschneider and T. Piran 2015
General approach (O.Z., 2015)

\[ ds^2 = -N^2 dt^2 + g_\phi (d\phi - \omega dt)^2 + \frac{dr^2}{A} + g_\theta d\theta^2 \]

Equatorial plane, redefine radial coordinate

Effective metric

\[ ds^2 = -N^2 dt^2 + g_\phi (d\phi - \omega dt)^2 + \frac{dr^2}{N^2} \]
Conservation laws

\[ E_{\text{in}} = E_{\text{fin}} \quad L_{\text{in}} = L_{\text{fin}} \quad \text{Consequence:} \quad X_{\text{in}} = X_{\text{fin}} \]

Let \( p \) particles collide and produce \( q \) new particles.

\[ \sum_{i=1}^{p} \sigma_i Z_i = \sum_{k=1}^{q} \sigma_k Z_k. \quad \text{radial momentum} \]

Conservation laws + forward-in-time condition \( X>0 \)

Near-horizon limit, \( N_c \rightarrow 0 \)

**Statement.** If in the initial configuration usual outgoing particles are absent, they cannot appear after collision.
Previous statement applis to case with finite masses, etc.

For finite masses and angular momenta, We cannot obtain a usual outgoing particle as a result of previous collision

If we relax this condition, it is possible to obtain a usual outgoing Particle, provided

$$m_2 = O(N^{-2})$$

Attempt to find loophole

Fractional degrees allow

$$X = O(N^s) \quad 0 < s < 1$$

Inconsistent with conservaiton laws

Generalizes observation of E. Leiderschneider and T. Piran

Collision with a supermassive particle

Collision near past horizons (white holes)

BH is unsuitable for SP P
Super-Penrose process (naked singularity)

Both particles are ingoing and usual, come from infinity. Particle 1 bounces back from potential barrier. Collides with particle 2.
Ingoing $\rightarrow$ outgoing
Head-on collisions

Debris from head-on collision. Significant enhancement

Critical particle moves away from black hole (outgoing)

Usual particle moves towards black hole

Outgoing usual particle
O. Z. (2014) analytically
V. Cardoso et al (2014) numeric findings
Super-Penrose process (naked singularity)

Both particles 1 and 2 are ingoing and usual, come from infinity.

Particle 1 bounces back from potential barrier. Ingoing $\rightarrow$ outgoing

Head-on collision, SPP (OZ 2014)


General approach, Tanatarov and O. Z. 2017
Particle with mass \( \mu \) and Killing energy \( E \) decays into two massless fragments. Fragment’s frequency measured at infinity \( v_\infty \)

its emitted frequency measured in the rest frame of the decaying particle \( v \)

\[
\frac{E}{\mu} - \sqrt{\frac{E^2}{\mu^2} + g_{tt}} \leq v_\infty \leq \frac{E}{\mu} + \sqrt{\frac{E^2}{\mu^2} + g_{tt}}
\]

Wald 1974
Wald inequalities for collisional Penrose process

1+2 = compound particle, decays to massless 3 and 4

\[ \mu = E_{\text{c.m.}} = 2\hbar \nu \]

\[ E - \sqrt{E^2 + \mu^2 g_{tt}} \leq 2\hbar \nu_{\infty} \leq E + \sqrt{E^2 + \mu^2 g_{tt}} \]

Thus \( \hbar \nu_{\infty} \) can be large (diverge) only if \( \mu \) is large (diverging)

\[ \mu \rightarrow \infty \quad \hbar \nu_{\infty} \approx \frac{\mu}{2} \sqrt{g_{tt}} \]
Conclusions

High energy collisions due to horizon

Role of critical trajectories
Force does not spoil effect

Rotating or charged BH
Universality

Energy of debris at infinity

Modest extraction in standard scenarios
Enhancement in head-on collision
SPP near BH is impossible
Near naked sing. possible

Alternative scenarios (far from horizon – large L or rapid rotation)
Thank you!