Cosmological bounce in Horndeski theory and beyond.

based on arXiv:1705.06626

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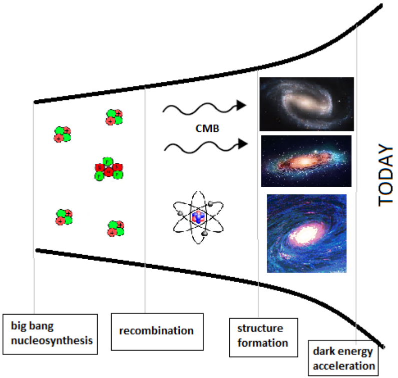
Quarks-2018

We know quite a lot about the Universe and it's evolution from at least hundreds KeV (BBN) to 2.4K (today).



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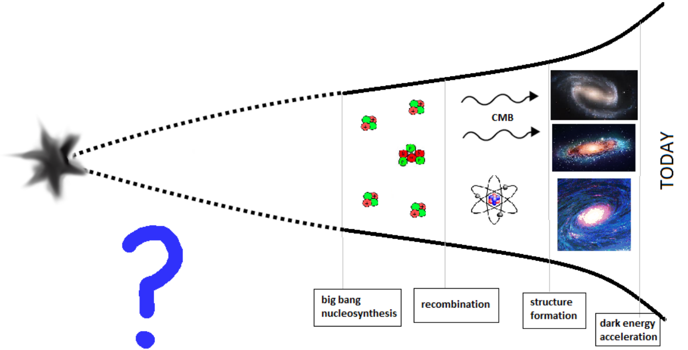
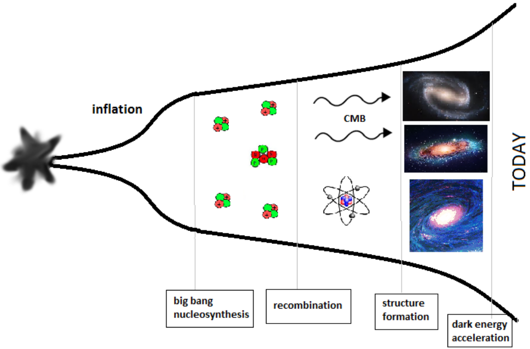


However, we believe there are preceding stages...

One of the most attractive and conventional options is *inflation*



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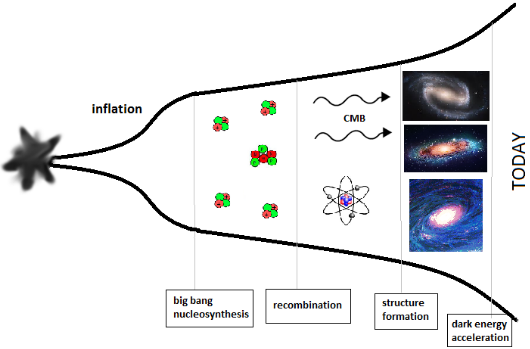
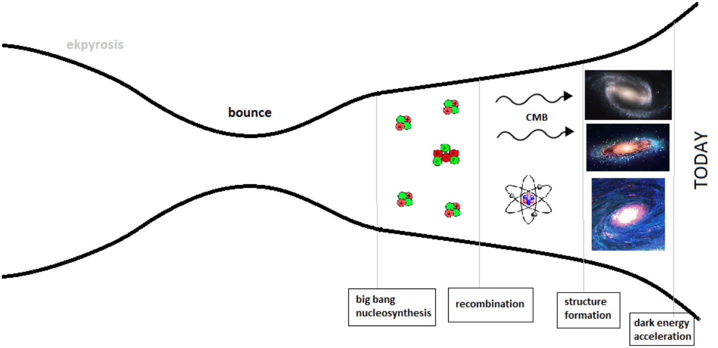


Yet not fully justified...

Bouncing solution

An alternative is *bouncing solution*



Bouncing solution

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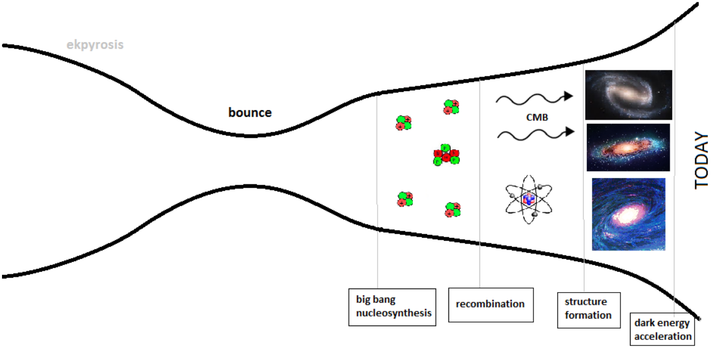

But it requires Null Energy Condition violation...

Null Energy condition

$T_{\mu\nu} k^\mu k^\nu > 0 \quad \longleftrightarrow \quad p + \rho > 0 \quad \longrightarrow \quad$ NEC–violation: $\; p + \rho \leq 0$

Friedmann equations

$\dot{H} = -4\pi G (p + \rho) + \frac{\kappa}{a^2}$

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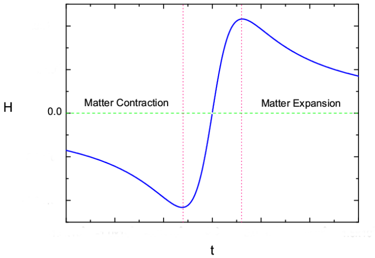

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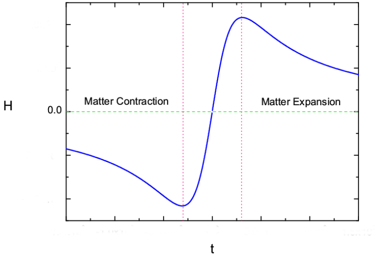
$\dot{H} = -4\pi G(p+\rho) \quad \leq 0$



It is impossible to violate NEC with a conventional matter in a healthy way.

Horndeski theory

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right),$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}{}^{\nu}\right]$$

where π is the Galileon field, $X = g^{\mu\nu}\pi_{,\mu}\pi_{,\nu}$, $\pi_{,\mu} = \partial_\mu\pi$, $\pi_{;\mu\nu} = \triangledown_\nu\triangledown_\mu\pi$, $\Box\pi = g^{\mu\nu}\triangledown_\nu\triangledown_\mu\pi$, $G_{4X} = \partial G_4/\partial X$

NEC-violation without pathologies (ghost or gradient instabilities)

T. Qiu, J. Evslin, Y. F. Cai, M. Li and X. Zhang, 1108.0593
D. A. Easson, I. Sawicki and A. Vikman, 1109.1047
M. Osipov and V. Rubakov, 1303.1221
T. Qiu, X. Gao and E. N. Saridakis, 1303.2372

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But situation is not so bright for complete cosmological models

If there are no pathologies during or near the NEC-violation phase, they will appear somewhere else:

Y. F. Cai, D. A. Easson and R. Brandenberger, 1206.2382
M. Koehn, J. L. Lehners and B. A. Ovrut, 1310.7577
L. Battarra, M. Koehn, J. L. Lehners and B. A. Ovrut, 1404.5067
T. Qiu and Y. T. Wang, 1501.03568
T. Kobayashi, M. Yamaguchi and J. Yokoyama, 1504.05710
Y. Wan, T. Qiu, F. P. Huang, Y. F. Cai, H. Li and X. Zhang, 1509.08772
A. Ijjas and P. J. Steinhardt, 1606.08880

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Theorem: there is no healthy bounce in Horndeski theory

M. Libanov, S. M and V. Rubakov, 1605.05992
R. Kolevatov and S. M, 1607.04099
T. Kobayashi, 1606.05831
S. Akama and T. Kobayashi, 1701.02926

$$S = \int \mathrm{d}t\mathrm{d}^3xa^3\left[\frac{\mathcal{G_T}}{8}\left(\dot{h}^T_{ik}\right)^2 - \frac{\mathcal{F_T}}{8a^2}\left(\partial_i h^T_{kl}\right)^2 + \mathcal{G_S}\dot{\zeta}^2 - \mathcal{F_S}\frac{(\nabla\zeta)^2}{a^2}\right]$$

where the coefficients are related:

$$\mathcal{G_S} = \frac{\Sigma\mathcal{G_T}^2}{\Theta^2} + 3\mathcal{G_T},$$

$$\mathcal{F_S} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F_T},$$

$$\xi = \frac{a\mathcal{G_T}^2}{\Theta}.$$

The speeds of sound for tensor and scalar perturbations are, respectively,

$$c_\mathcal{T}^2 = \frac{\mathcal{F_T}}{\mathcal{G_T}}, \qquad c_\mathcal{S}^2 = \frac{\mathcal{F_S}}{\mathcal{G_S}}$$

A healthy and stable solution requires correct signs for kinetic and gradient terms as well as subluminal propagation:

$$\mathcal{G_T} > \mathcal{F_T} > 0, \quad \mathcal{G_S} > \mathcal{F_S} > 0$$

No-go theorem in Horndeski theory

$$\mathcal{F}_S = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_T \qquad \longrightarrow \qquad \xi(t_2) - \xi(t_1) = \int_{t_1}^{t_2} a(t)\left(\mathcal{F}_T + \mathcal{F}_S\right)\mathrm{d}t$$

Suppose that $\xi(t_2) > 0$. As we have

$$\xi(t_1) = \xi(t_2) - \int_{t_1}^{t_2} a(t)\left(\mathcal{F}_T + \mathcal{F}_S\right)\mathrm{d}t,$$

taking a long enough period of time (for instance, $t_1 \to -\infty$) results in $\xi(t_1) < 0$.
Another possibility is that $\xi(t_1) < 0$:

$$\xi(t_2) = -|\xi(t_1)| + \int_{t_1}^{t_2} a(t)\left(\mathcal{F}_T + \mathcal{F}_S\right)\mathrm{d}t,$$

Then taking $t_2 \to \infty$ gives $\xi(t_2) > 0$.

Hence, there must be a moment of time when $\xi(t)$ changes sign, i.e, it crosses zero, $\xi(t_0) = 0$.

No-go theorem in Horndeski theory

The definition of ξ:

$$\xi = \frac{a{\mathcal{G}_{\mathcal{T}}}^2}{\Theta}.$$

Hence to make ξ cross zero it requires $\Theta \to \infty$ or $\mathcal{G}_{\mathcal{T}} \to 0$.

Neither of these requirements can be met:

- $\mathcal{G}_{\mathcal{T}} = 0$ corresponds to a strong coupling regime
- infinite Θ means a singularity in the Lagrangian

Beyond Horndeski

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_{\mathcal{BH}}\right),$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}{}^{\nu}\right],$$

$$\mathcal{L}_{\mathcal{BH}} = F_4(\pi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'} +$$
$$+F_5(\pi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'}\pi_{;\sigma\sigma'}$$

$$S = \int \mathrm{d}t\mathrm{d}^3x a^3 \left[\frac{\hat{\mathcal{G}}_\mathcal{T}}{8} \left(\dot{h}_{ik}^T \right)^2 - \frac{\mathcal{F}_\mathcal{T}}{8a^2} \left(\partial_i h_{kl}^T \right)^2 + \mathcal{G}_\mathcal{S} \dot{\zeta}^2 - \mathcal{F}_\mathcal{S} \frac{(\nabla\zeta)^2}{a^2} \right]$$

where the modified coefficients are

$$\mathcal{G}_\mathcal{S} = \frac{\Sigma {\mathcal{G}_\mathcal{T}}^2}{\Theta^2} + 3\mathcal{G}_\mathcal{T}, \qquad \mathcal{G}_\mathcal{S} = \frac{\Sigma {\hat{\mathcal{G}}_\mathcal{T}}^2}{\Theta^2} + 3\hat{\mathcal{G}}_\mathcal{T},$$

$$\mathcal{F}_\mathcal{S} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_\mathcal{T}, \qquad \longrightarrow \qquad \mathcal{F}_\mathcal{S} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_\mathcal{T},$$

$$\xi = \frac{a{\mathcal{G}_\mathcal{T}}^2}{\Theta}, \qquad \xi = \frac{a\mathcal{G}_\mathcal{T}\hat{\mathcal{G}}_\mathcal{T}}{\Theta} = \frac{a\mathcal{G}_\mathcal{T}\left(\mathcal{G}_\mathcal{T} + \mathcal{D}\dot{\pi}\right)}{\Theta}.$$

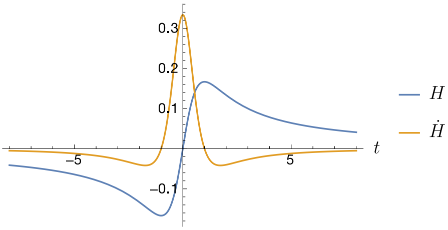
The speeds of sound for tensor and scalar perturbations are, again, respectively,

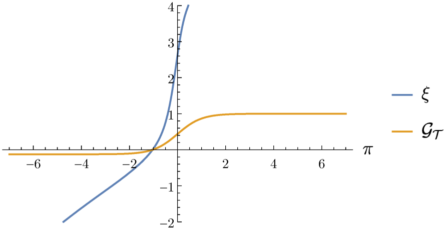
$$c_\mathcal{T}^2 = \frac{\mathcal{F}_\mathcal{T}}{\hat{\mathcal{G}}_\mathcal{T}}, \qquad c_\mathcal{S}^2 = \frac{\mathcal{F}_\mathcal{S}}{\mathcal{G}_\mathcal{S}}$$

Again, we require correct signs for kinetic and gradient terms as well as subluminal propagation:

$$\hat{\mathcal{G}}_\mathcal{T} > \mathcal{F}_\mathcal{T} > 0, \quad \mathcal{G}_\mathcal{S} > \mathcal{F}_\mathcal{S} > 0$$

Bouncing solution: an example





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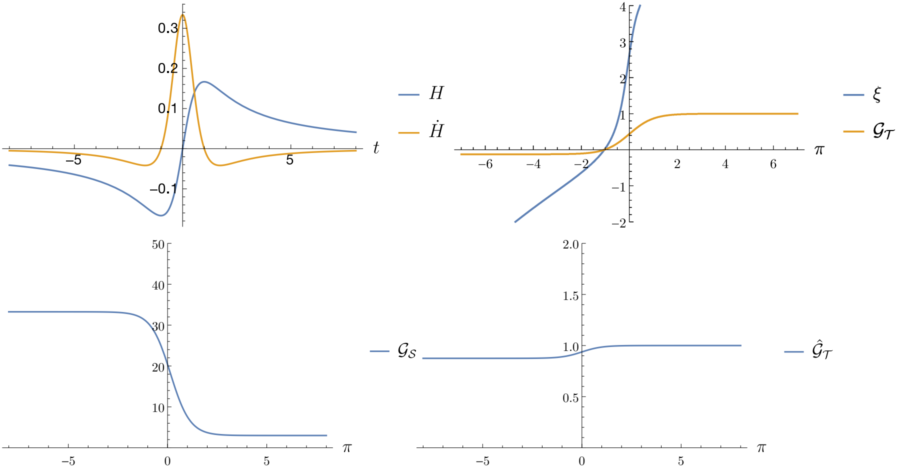

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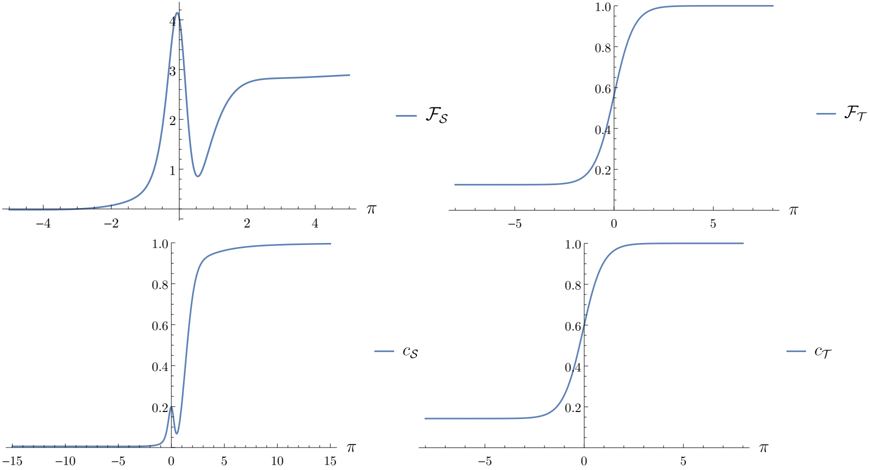


Θ issue

Is it possible for Θ to safely have zero value?

$$S = \int \mathrm{d}t\mathrm{d}^3x a^3 \Big[\frac{\hat{\mathcal{G}}_\mathcal{T}}{8}\left(\dot{h}_{ik}^T\right)^2 - \frac{\mathcal{F}_\mathcal{T}}{8a^2}\left(\partial_i h_{kl}^T\right)^2 + \mathcal{G}_\mathcal{S}\dot{\zeta}^2 - \mathcal{F}_\mathcal{S}\frac{(\nabla\zeta)^2}{a^2}\Big]$$

$$\mathcal{G}_\mathcal{S} = \frac{\Sigma\hat{\mathcal{G}}_\mathcal{T}^2}{\Theta^2} + 3\hat{\mathcal{G}}_\mathcal{T},$$

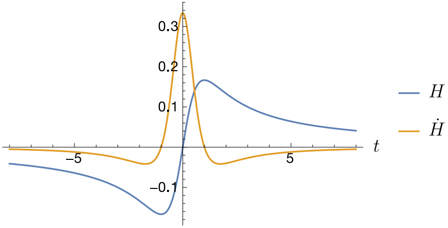
$$\mathcal{F}_\mathcal{S} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_\mathcal{T},$$

$$\xi = \frac{a\mathcal{G}_\mathcal{T}\hat{\mathcal{G}}_\mathcal{T}}{\Theta} = \frac{a\left(\hat{\mathcal{G}}_\mathcal{T} - \mathcal{D}\dot{\pi}\right)\hat{\mathcal{G}}_\mathcal{T}}{\Theta}.$$

At least naively, it seems that $\Theta = 0$ forces one to fine tune $\hat{\mathcal{G}}_\mathcal{T}$ and $\mathcal{G}_\mathcal{T}$ to have regular $\mathcal{G}_\mathcal{S}$ and $\mathcal{F}_\mathcal{S}$.

If $\Theta \neq 0$ at all times, it is impossible to have Einstein gravity + massless scalar field *both* in distant past and future.

$$\Theta = -K_X X\dot{\pi} + 2G_4 H - 8HG_{4X}X - 8HG_{4XX}X^2 + G_{4\pi}\dot{\pi} + 2G_{4\pi X}X\dot{\pi} -$$
$$- 5H^2 G_{5X}X\dot{\pi} - 2H^2 G_{5XX}X^2\dot{\pi} + 3HG_{5\pi}X + 2HG_{5\pi X}X^2 +$$
$$+ 10HF_4 X^2 + 4HF_{4X}X^3 + 21H^2 F_5 X^2\dot{\pi} + 6H^2 F_{5X}X^3\dot{\pi}$$



Let us consider $\Theta = 0$:

$$S = \int \mathrm{d}t\mathrm{d}^3x a^3 \Big[\mathcal{G_S}\dot{\zeta}^2 - \mathcal{F_S}\frac{(\nabla\zeta)^2}{a^2}\Big]$$

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Despite the seeming singularities in the action and linearized equation, the solution $\zeta(t)$ is regular at any moment of time.

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Despite the seeming singularities in the action and linearized equation, the solution $\zeta(t)$ is regular at any moment of time.

This fact agrees with the recent discussion raised by Anna Ijjas (arXiv:1710.05990).

Conclusion

1. In Horndeski there are no bouncing solution stable at all times.
2. There is a completely healthy bounce in beyond Horndeski theory.
3. It is possible to construct a bouncing solution with conventional Einstein gravity in *both* distant past and future.

Thank you for your attention!