Holography, quantum complexity and quantum chaos in different models

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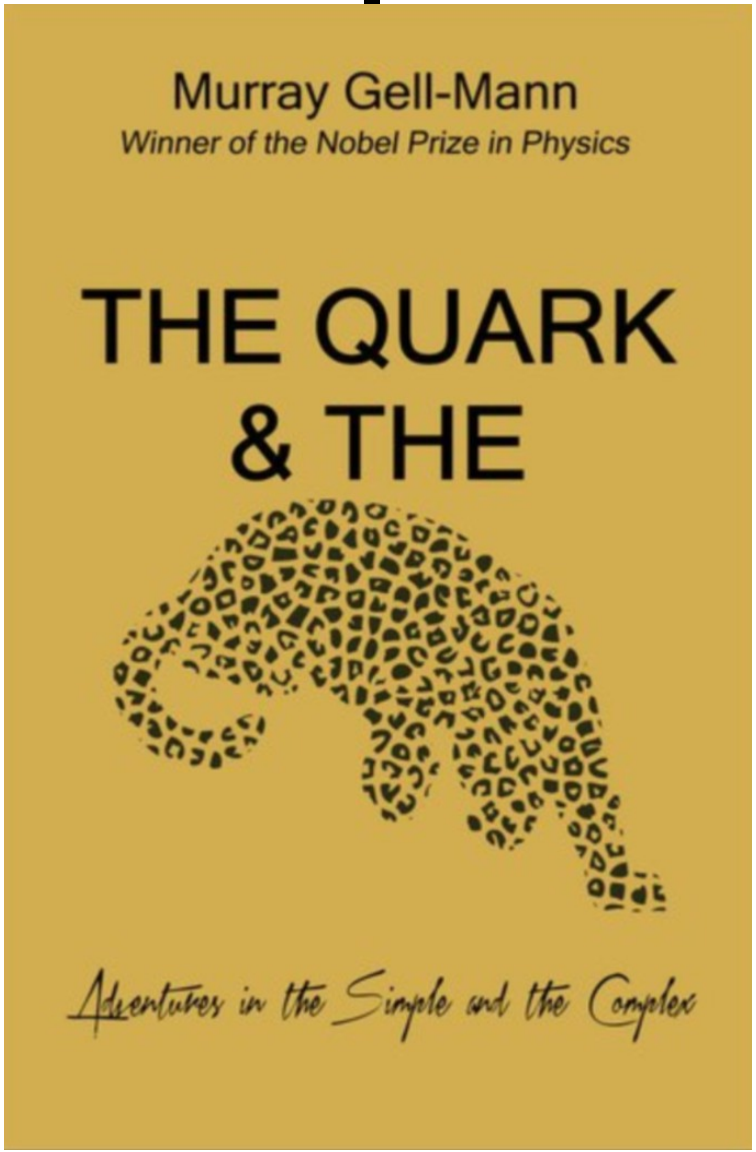
Based on
arXiv 1803.11162, D.A., I. Aref'eva, A. Bagrov, M. Katsnelson
arXiv 1805.XXXX, D.A., I. Aref'eva

The **Quark** and the Jaguar: Adventures in the Simple and **Complex**



The plan of my talk

- The <u>chaos</u> in quantum field theory and gravity:short review
- Where the <u>quantum complexity</u> is necessary in quantum field theory and what is it?
- <u>Holographic local quench</u> as a toy model of quantum system out of equilibrium and complexity evolution.
- Qualitative check of correspondence between gravity and chaos: <u>chaos supression and holography</u>

AdS/CFT correspondence

- Relates gravity in d+1 dimension and strongly coupled quantum system in d dimensions

Gravity(Holography) and quantum information

- The physics of quantum information has played a growing role in our understanding of the emergence of spacetime and gravity in the context of the AdS/CFT correspondence
- Examples:

1. AdS (in dim=2) space naturally emerges from the special «variational ansatz» for the state in conformal quantum system - tensor network

2. Einstein equations implies equations for the entanglement entropy in CFT and vice versa

Hayden et.al., 1601.01694

Faulkner et.al., 1601.01694

Recent ideas with strong connection to the gravity and AdS/CFT

-*Scrambling*

-*Quantum chaos*

-*Quantum complexity*

-Black holes are the fastest scramblers

-Black holes are the fastest quantum computers

-Black holes are-??????

Chaos reigns

- New quantitatives measure of the chaos

1. Commutator square correlator

$$C(t) = -\langle [W(t), V(0)]^2 \rangle$$

$$\langle \cdot \rangle \ = \ Z^{-1} \mathrm{tr}[e^{-\beta H} \cdot]$$

2.Spectral form factor

$$\left| \frac{Z(\beta,t)}{Z(\beta)} \right|^2 = \frac{1}{Z(\beta)^2} \sum_{m,n} e^{-\beta(E_m+E_n)} e^{i(E_m-E_n)t}$$

<u>3.Operator size</u>

4.General idea of out-of-time ordered correlators

Chaos reigns in holographic systems

Bound on chaos!

Maldacena, Stanford, Shenker, 1503.01409

$$F_d - F(t) = \epsilon \exp \lambda_L t$$

$$\lambda_L \leq \frac{2\pi}{\beta} = 2\pi T$$

- **Examples:**

  1. Two dimensional CFT at large central charge (holographic dual of 3-dimensional gravity)

  2. Sachdev-Ye-Kitaev model and other melonic models (probable dual of 2-dimensional dilaton gravity)

New time scales in strongly coupled quantum systems

- **Local thermalization**: (also called diffusion time or collision time)
- **Scrambling:**

  - Time of the chaos onset

  - Time when all information is governed by higher- and-higher-point correlators and non-local measures. Estimation by vanishing of n-point mutual information. *D.A., I. Aref'eva, 1701.07280*
- **Global thermalization**

Eternal black holes: the paradox

- Eternal black holes are dual to the thermofield double of QFT.
- The dual boundary theories very quickly comes to the thermal equilibrium.
- All evolution seems to stop at the scrambling time
- *But the one thing does not stop evolving - the Einstein-Rosen bridge: linearly and eternal growing.*

The complexity conjecture

D Stanford, L Susskind, 1406.2678

The quantum state does not stop evolving.

- Subtle quantum properties continue to equilibrate long after a system is scrambled.
- These properties can be summarized in a quantity called quantum complexity, or just complexity.
- Complexity characterizes «how much elementary operations we have to do to make the target state».

Proposal of definition of QFT complexity

- ***«Entropy is only the tip of the gigantic complexity iceberg»***

D Stanford, L Susskind, *1406.2678*

Proposal of definition of QFT complexity:discrete version

Minimize over all possible operations sets

$$\psi_{\mathrm{T}} = U\psi_{\mathrm{R}} \equiv Q_{22}^{\alpha_3}\, Q_{21}^{\alpha_2}\, Q_{11}^{\alpha_1}\, \psi_{\mathrm{R}}$$

$$Q_{21}\,\psi(x_1,x_2) = \psi(x_1+\epsilon x_2, x_2)$$

$$Q_{11}\,\psi(x_1,x_2) = e^{\epsilon/2}\psi\left(e^{\epsilon}x_1, x_2\right)$$

$$\psi_{\mathrm{R}}(x_1,x_2) = \sqrt{\frac{\omega_0}{\pi}}\exp\left[-\frac{\omega_0}{2}\left(x_1^2+x_2^2\right)\right]$$

$$\psi_{\mathrm{T}}(x_1,x_2) = \frac{\left(\omega_1\omega_2-\beta^2\right)^{1/4}}{\sqrt{\pi}}\exp\left[-\frac{\omega_1}{2}x_1^2-\frac{\omega_2}{2}x_2^2-\beta x_1 x_2\right]$$

$$\mathcal{D}(U) = |\alpha_1|+|\alpha_2|+|\alpha_3|$$

$$= \frac{1}{\epsilon}\left[\frac{1}{2}\log\left(\frac{\omega_1\omega_2-\beta^2}{\omega_0^2}\right)+\sqrt{\frac{\omega_0}{\omega_1}}\frac{|\beta|}{\sqrt{\omega_1\omega_2-\beta^2}}\right]$$

Jefferson, Myers, 1707.08570

Continous complexity=geometric complexity

Geodesics in the parameter space:

set all possible operators O and minimize over Y

Jefferson, Myers, 1707.08570

$$U = \overleftarrow{\mathcal{P}} \exp \int_0^1 \mathrm{d}s \, Y^I(s) \, \mathcal{O}_I$$

$$\psi_\mathrm{T}\left(x_1, x_2\right) = U \psi_\mathrm{R}\left(x_1, x_2\right)$$

$$\mathcal{D}(U) = \int_0^1 \mathrm{d}s \sqrt{G_{IJ} \, Y^I(s) \, Y^J(s)}$$

Free fields

Examples:

Chapman, et.al. 1707.08582,
Jefferson, Myers, 1707.08570

$$b_{\vec{k}} = \beta_k^+ a_{\vec{k}} + \beta_k^- a^\dagger_{-\vec{k}}; \quad b_{\vec{k}} |R(M)\rangle = 0;$$

$$\beta_k^+ = \cosh 2r_k; \quad \beta_k^- = \sinh 2r_k; \quad r_k \equiv \log \sqrt[4]{\frac{M}{\omega_k}}$$

$$\mathcal{C}^{(n)} = 2 \sqrt[n]{\frac{\text{Vol}}{2} \int_{k \leq \Lambda} d^d k |r_k|^n}$$

Holographic complexity

- There are two main proposals:
- Complexity=Volume
- Complexity=Action
- In fact the conventional covariant version of the holographic complexity for arbitrary state (for example interval) is unknown.

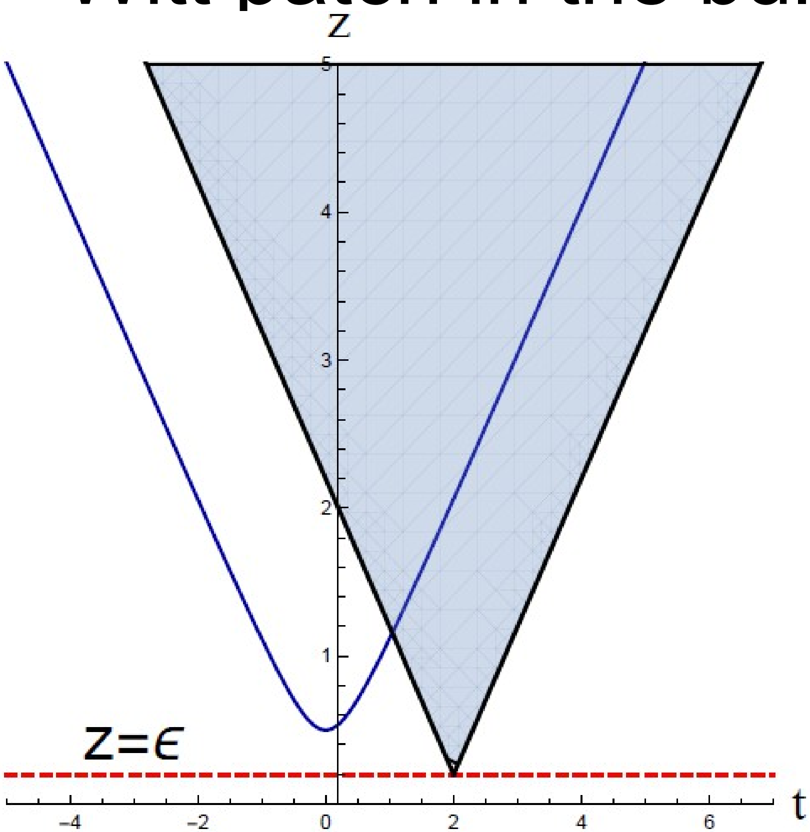
Holographic proposals:CV

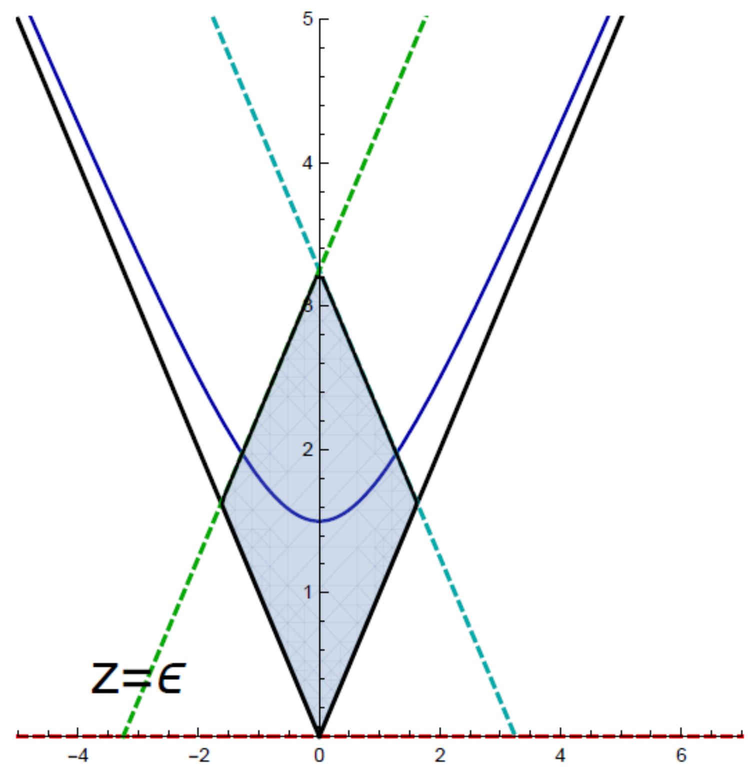
- «CV» is Complexity=Volume conjecture
- The CV complexity of the state on in quantum field theory is (modulo technical details) the volume under the minimal surface spanned on the region corresponding to the state

Alishahiha, 1509.06614;
Carmi, Myers, Rath 1612.00433

Holographic proposals:CA

- <u>Complexity action:</u> *Brown, et.al. 1512.04993*
- Action of the gravitational theory(plus matter fields!!) in the special region called Wheeler-de-Witt patch in the bulk of the AdS space





CA versus CV

- CV conjecture is strongly defined by the entanglement properties of the state
- CV conjecture has strong numerical support in its favor(with tensor networks numerics)
- The simplest formulation of CV incudes additional parameter while the original CA does not
- CV and CA are supported by tensor networks arguments
- In fact in the simplest situations they lead to the very similar results

Local quench

- We need the example that is intuitively easy to understand and non-trivial enough to compare these conjectures
- Local quench — is the local perturbation(for example by the energy injection at one point) of the quantum system.
- Examples:joining two semi-infinite spin chains. Insertion of operator at a point in CFT.
- Exactly solvable at CFT different description
- This process have good holographic description

Holographic local quench

Nozaki, Numasawa, Takayanagi, 1302.5703

$$\mathcal{R}^{\mu\nu} - \frac{1}{2} g^{\mu\nu}\mathcal{R} + \Lambda g^{\mu\nu} = \mathcal{T}^{\mu\nu}$$

$$\mathcal{T}^{\mu\nu} = \frac{8\pi m G_N}{\sqrt{-g}} \cdot \frac{\partial_t X^\mu \partial_t X^\nu}{\sqrt{-g_{\mu\nu} \cdot \partial_t X^\mu(t) \cdot \partial_t X^\nu(t)}} \cdot \delta(z - z(t)) \cdot \delta^{d-1}(x_i)$$

Static (in the space coordinates) point particle deforming the Poincare patch of the AdS space.

Holographic local quench: dual metric

$$
ds^2 = \frac{1}{z^2}\frac{\left(\alpha^2 dx - 2txdt + dx\left(u - z^2\right) + 2xzdz\right)^2}{\alpha^4 + 2\alpha^2\left(u - z^2\right) + \left(z^2 - v\right)^2} - \tag{A.5}
$$

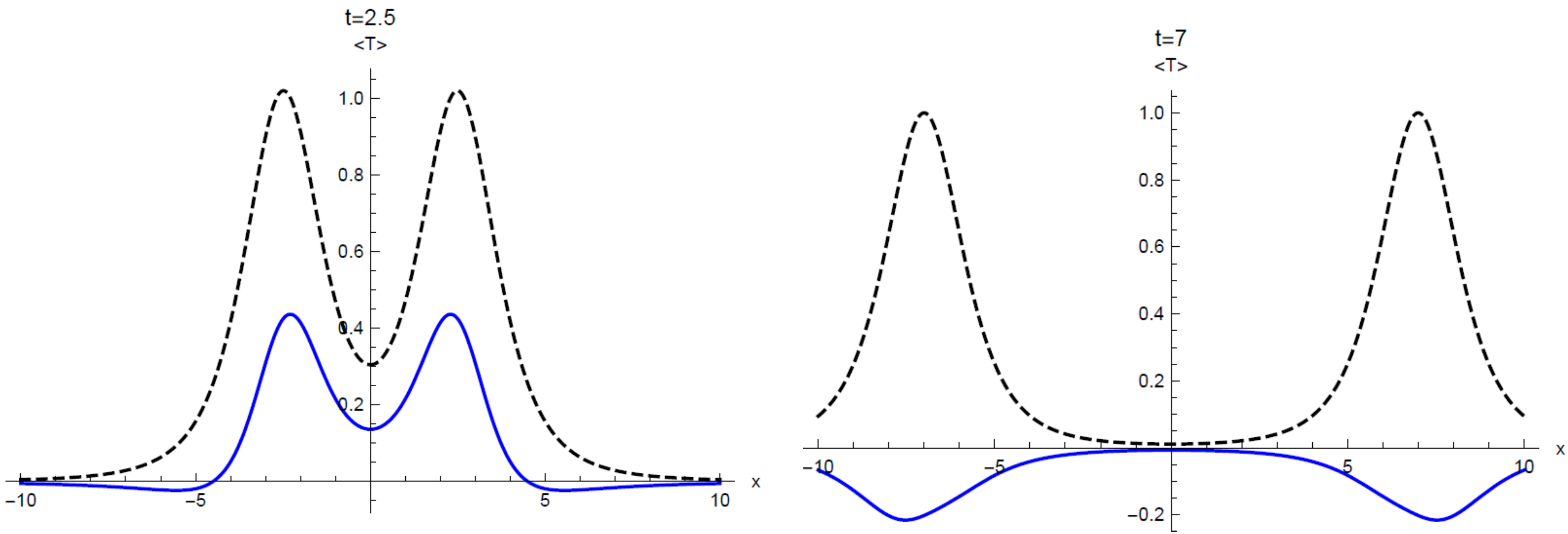
$$
-\frac{1}{z^2}\frac{\left(\alpha^4 + 2\alpha^2\left(u + z^2(1-2M)\right) + \left(z^2 - v\right)^2\right)\left(\alpha^2 dt + \left(u + z^2\right)dt - 2t(xdx + zdz)\right)^2}{\left(\alpha^4 + 2\alpha^2\left(u + z^2\right) + \left(z^2 - v\right)^2\right)^2}
$$

$$
\frac{1}{z^2}\frac{\left(\alpha^4 dz + 2\alpha^2(udz - z(tdt + xdx)) + \left(v - z^2\right)\left(-2tzdt + 2xzdx + \left(v + z^2\right)dz\right)\right)^2}{\left(\alpha^4 + 2\alpha^2\left(u - z^2\right) + \left(z^2 - v\right)^2\right)\left(\alpha^4 + 2\alpha^2\left(-2Mz^2 + u + z^2\right) + \left(z^2 - v\right)^2\right)},
$$

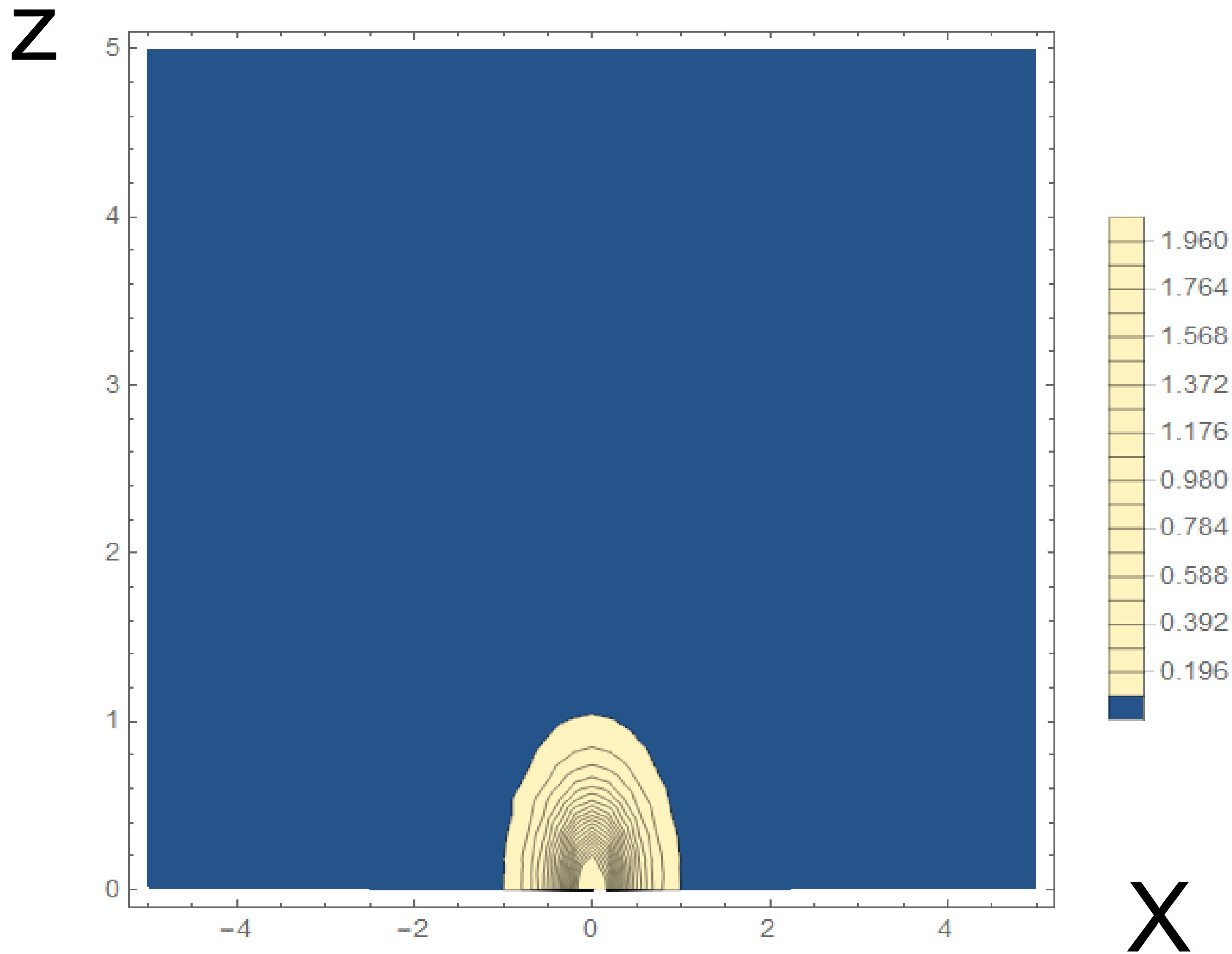
$$
u = t^2 - x^2
$$

$$
v = t^2 + x^2
$$

Two quasiparticles from quench: vev of stress-energy tensor

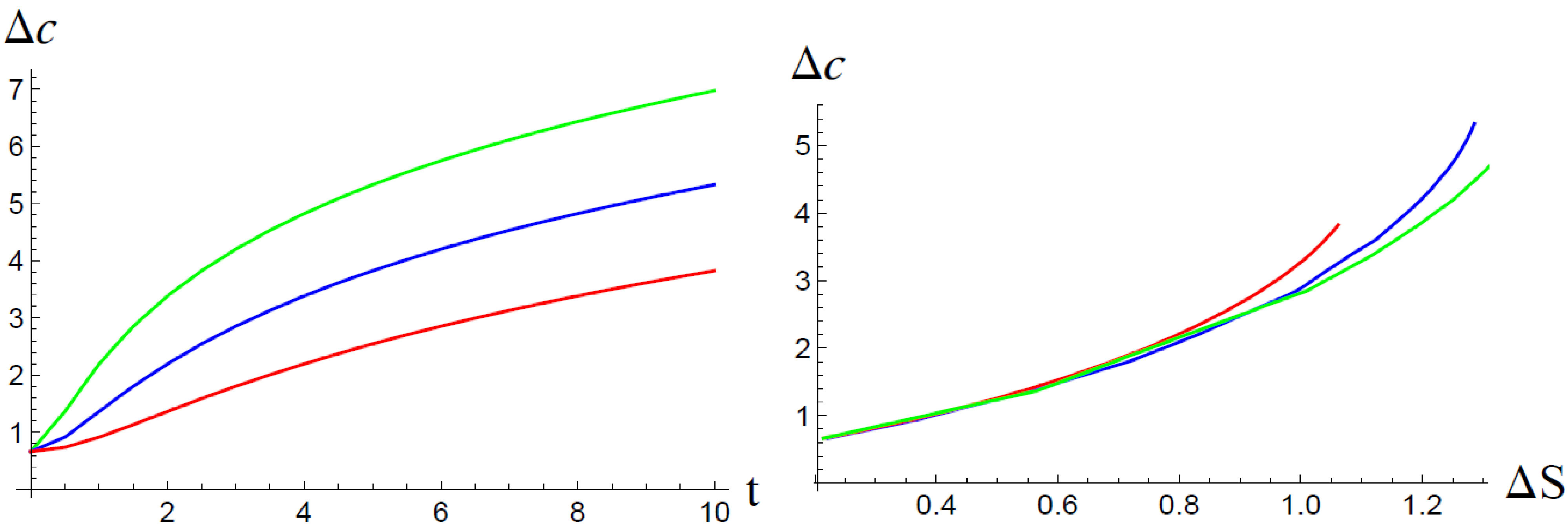


Constant time slice volume density evolution

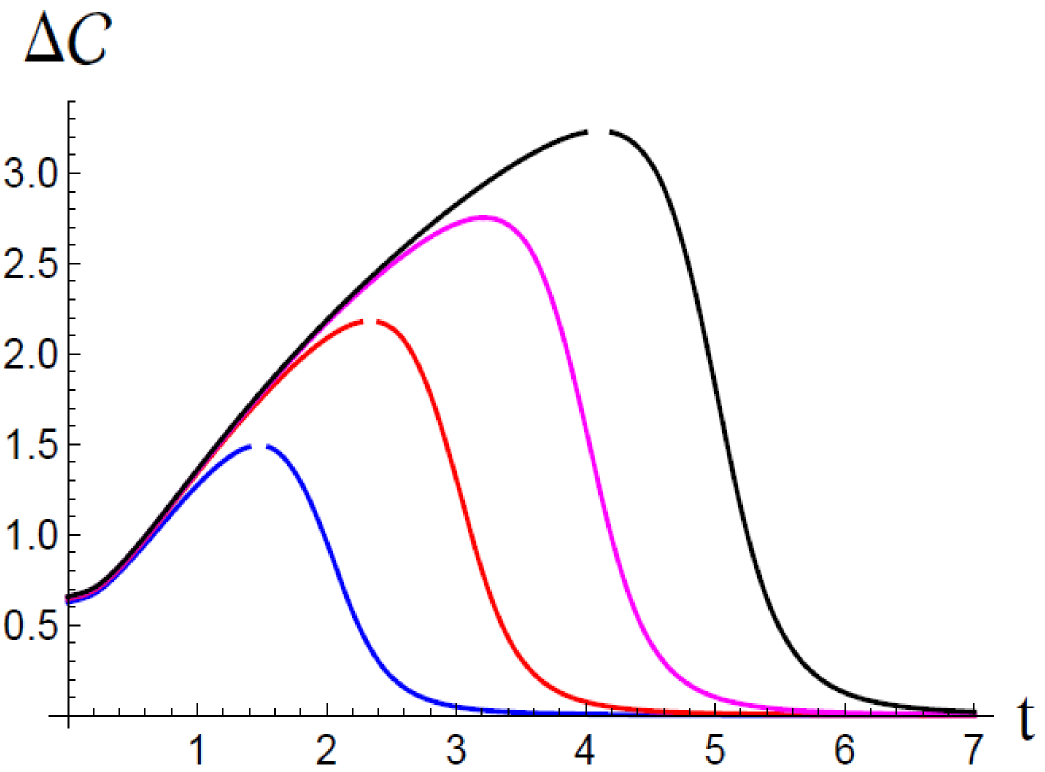


Quench starts at x=0

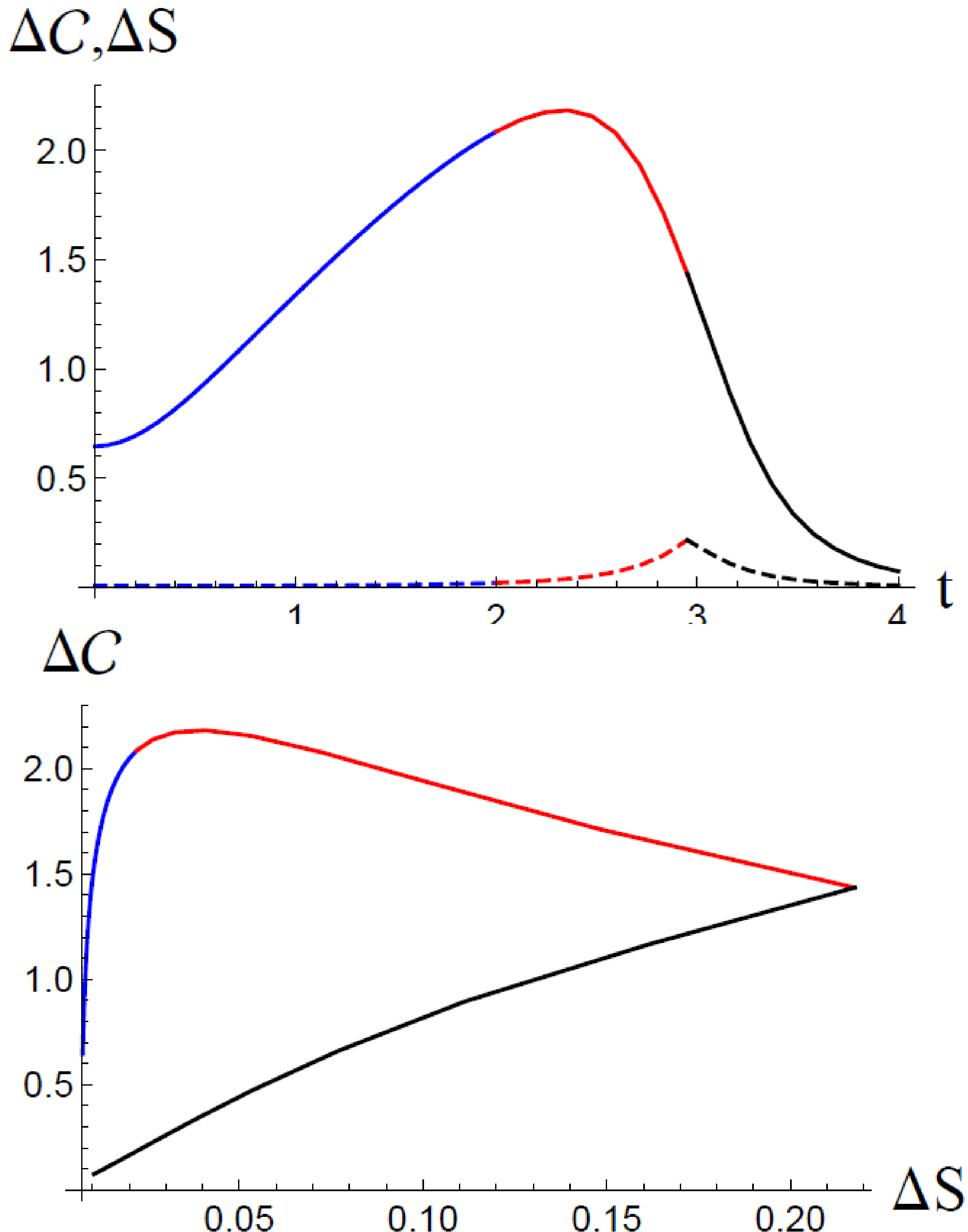
CV for total system grows monotonicaly



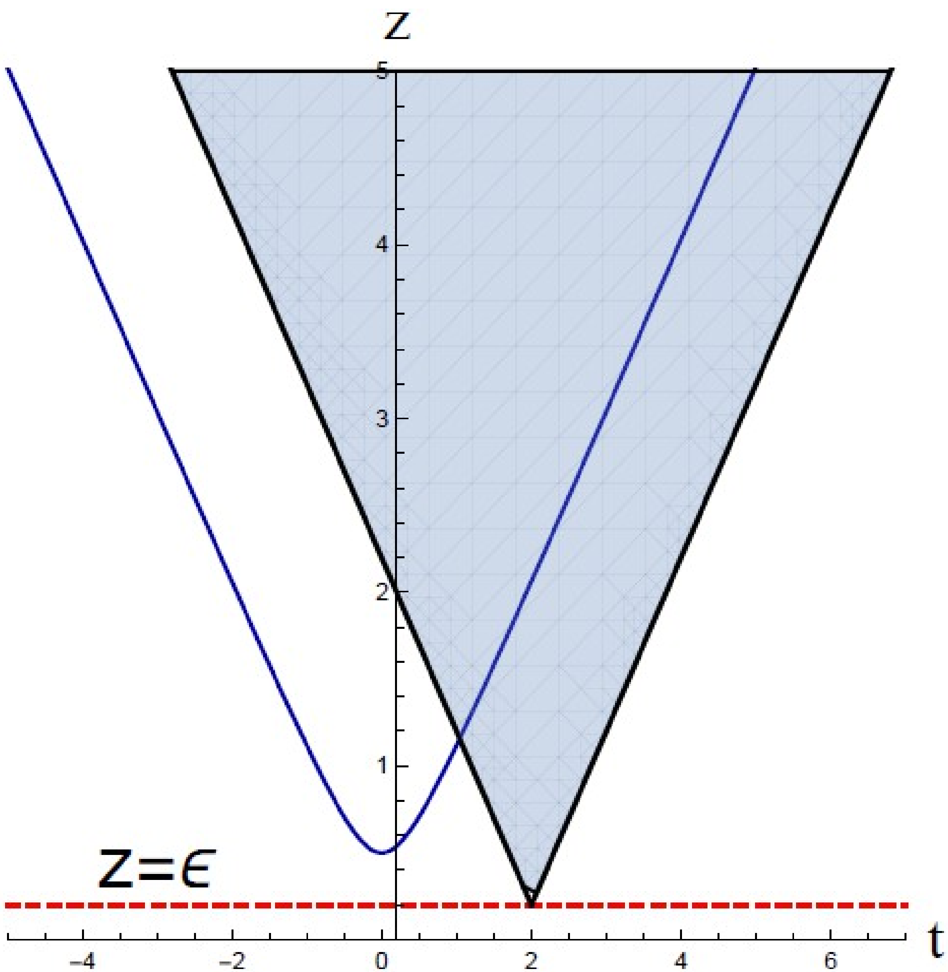
CV (for interval) complexity following quench

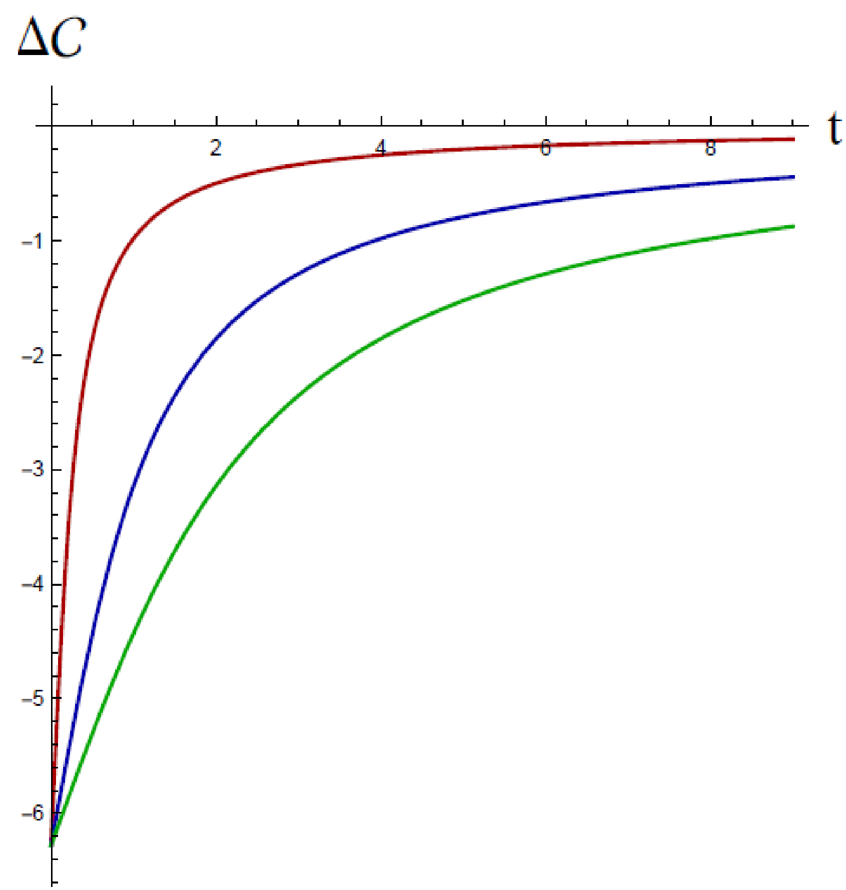


Complexity versus entanglement

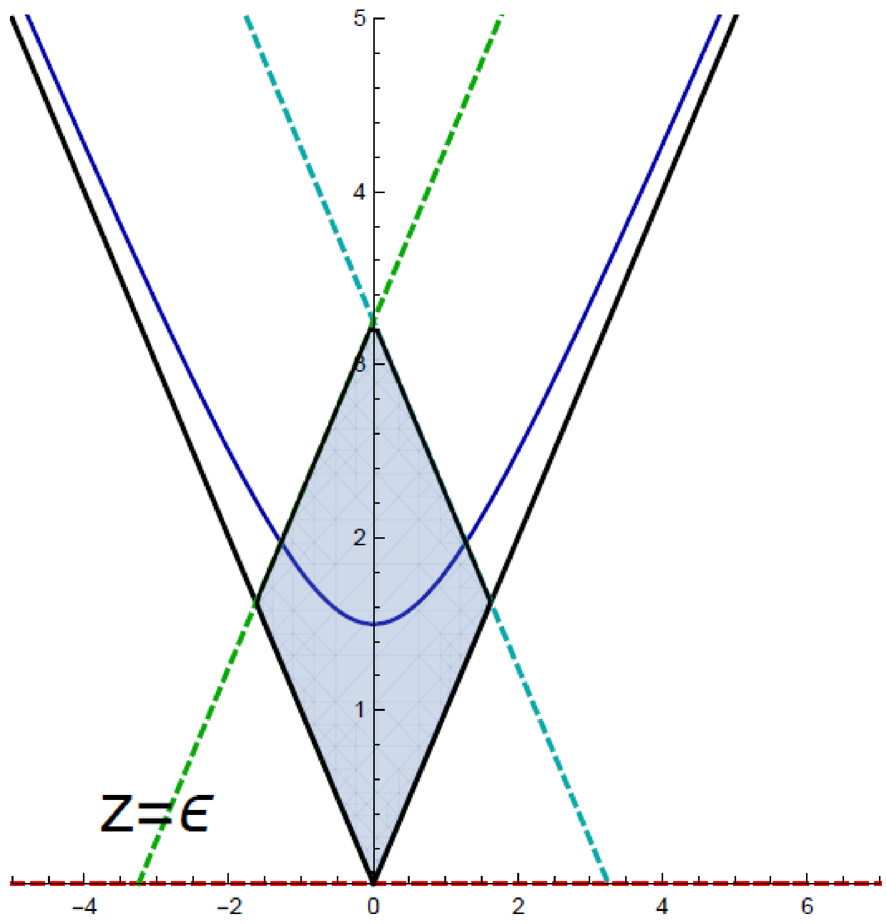


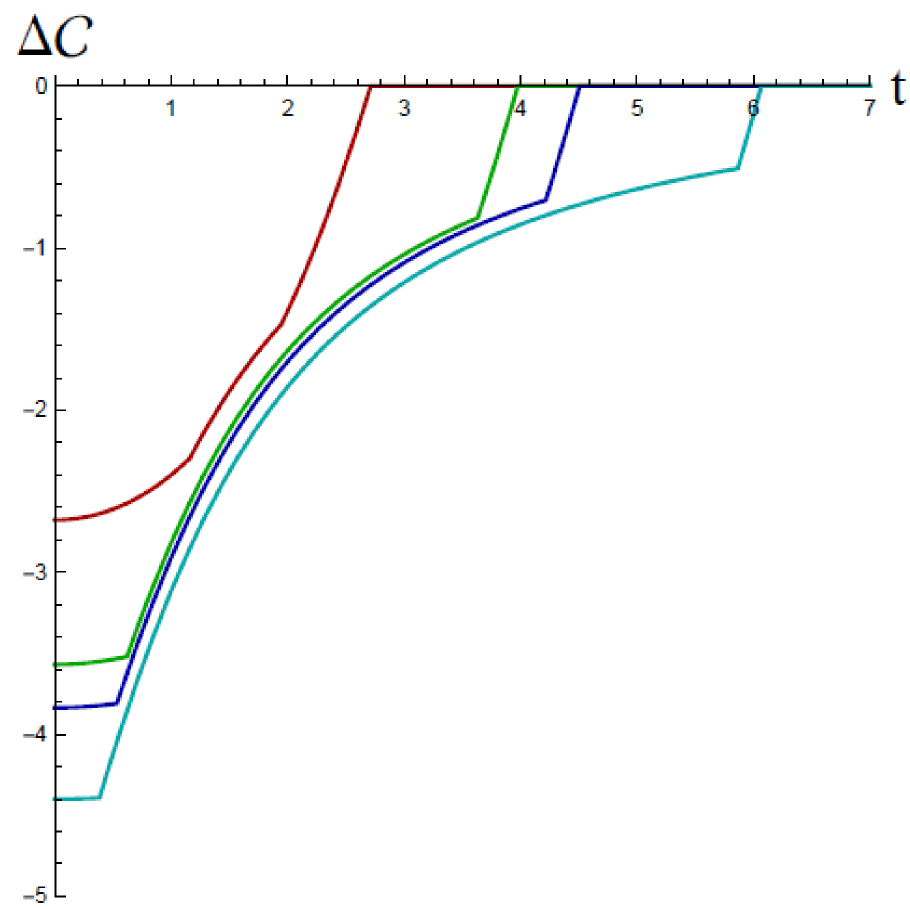
Complexity-Action for total system decrease monotonicaly





Complexity-Action for subsystem





Lloyd bound

$$\mathcal{R} < \frac{2E}{\pi},$$

$$\mathcal{R}(t) = \frac{d\Delta \mathcal{C}}{dt}$$

Conjectural bound of the speed of «computational machine» to procced with the physical process.

For Complexity=Action conjecture this bound is precisely saturated at time t=0 in local quench

For 2D conformal field theory to insert the heavy operator in radial quantization (to quench the system) is the operation of maximal computation complexity?

Can we say something about chaotic evolution **of** perturbing operator?

- yes

Particle=Operator in AdS/CFT

- We model the local quench and study the evolution of the system
- Can we say something about operator characteristics during the evolution?
- First — let us consinder the operator in the probe limit(neglecting backreaction) to simplify the problem

Operator size

Roberts,Stanford, Streicher, 1802.02633

$$W(t) = \sum_{s, a_1 < \ldots < a_s} c_{a_1 \ldots a_s}(t) \psi_{a_1} \ldots \psi_{a_s}$$

- S-grows while the system evolves $\approx e^{\frac{2\pi}{\beta} t}$
- Characterizes how «complex» becomes the operator during the evolution of the system
- Important quantitative chaotic measure in holographic systems

«Why things do fall?»- Susskind,1802.01198

- There is conjectural correspondence (by L.Susskind) between the particle radial momentum falling in the black hole (i.e. operator evolving at finite temperature) and the operator size.
- It occurs that holographic theories precisely saturate some bound of this growth
- Gravity makes things more and more complex

$$\mathbf{Operator\ size} \longleftrightarrow \mathbf{p_z(t)}$$

$$p_z(t) \approx e^{\frac{2\pi}{\beta}t}$$

Why things stop falling?

- We make a quantitative check of this correspondence. We show that finite chemical potential suppreses the chaos both in the holographic model and in the model dual theories.
- Charged operator = charged particle

$$S = -m\int \sqrt{-g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}}d\tau + qA_{\mu}\dot{x}^{\mu}d\tau$$

D.A., I.Aref'eva, 1805.xxxx

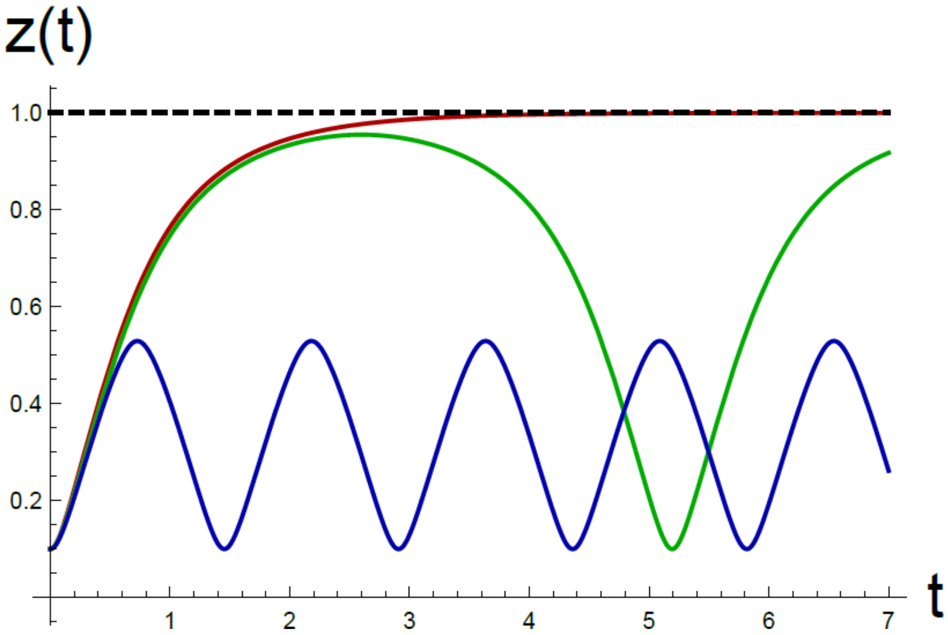
Reissner-Nordstrom black hole and finite chemical potential

$$ds^2 = \frac{1}{z^2}\left(-f(z)dt^2 + \frac{dz^2}{f(z)} + d\bar{x}^2\right)$$

$$f(z) = 1 - M\left(\frac{z}{z_h}\right)^d + Q\left(\frac{z}{z_h}\right)^{2d-2},$$

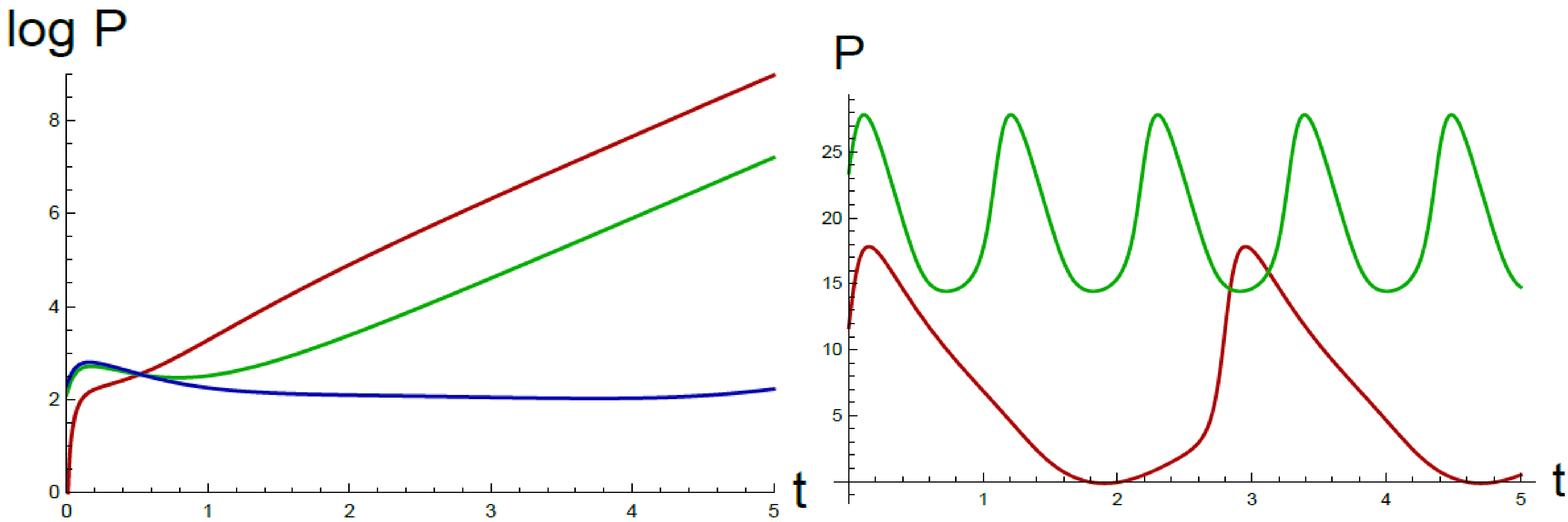
$$A = \mu\left(1 - \left(\frac{z}{z_h}\right)^{d-2}\right)dt$$

Critical charge



$$q_{crit} = \frac{\sqrt{f(z_*)}}{z_* A(z_*)}$$

Momentum stops growing after critical charge



$$\mathbf{Operator\ size} \longleftrightarrow \mathbf{p_z(t)}$$

Quantum models that support these results

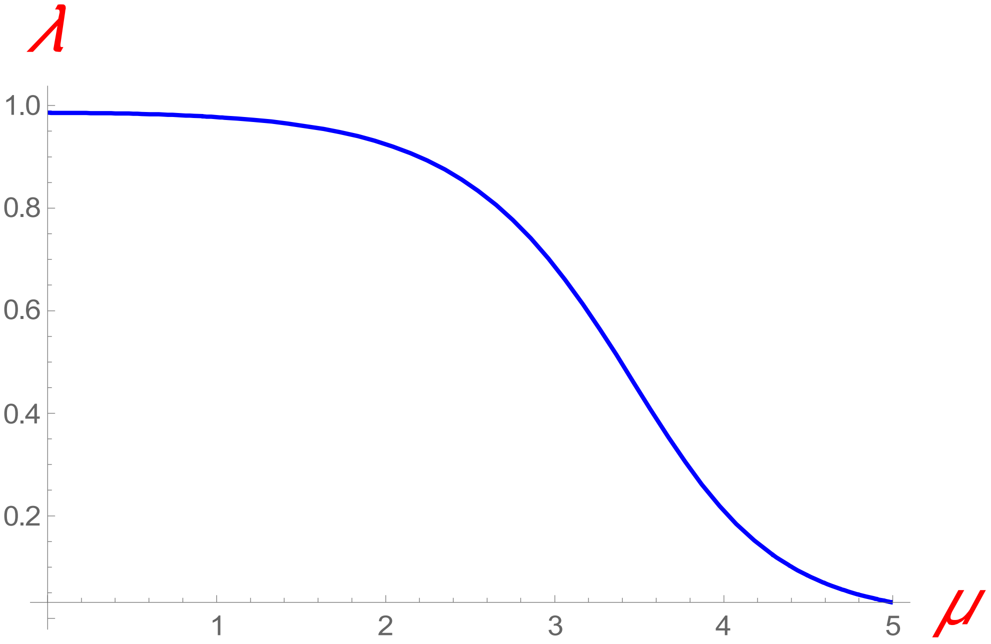
- Sachdev-Ye-Kitaev — randomly all-to-all interacting complex fermions at finite chemical potential
- Matrix quantum mechanics with the mass term in the special limit

Quantum models that support these results:1

- Sachdev-Ye-Kitaev — randomly all-to-all interacting complex fermions

$$H = \sum_s J_{i_1..i_q} \psi^\dagger_{i_1} \ldots \psi^\dagger_{i_{q/2}} \psi_{i_{q/2+1}} \ldots \psi_{i_q}$$

$$G_0 = \frac{1}{i\omega + \mu}$$



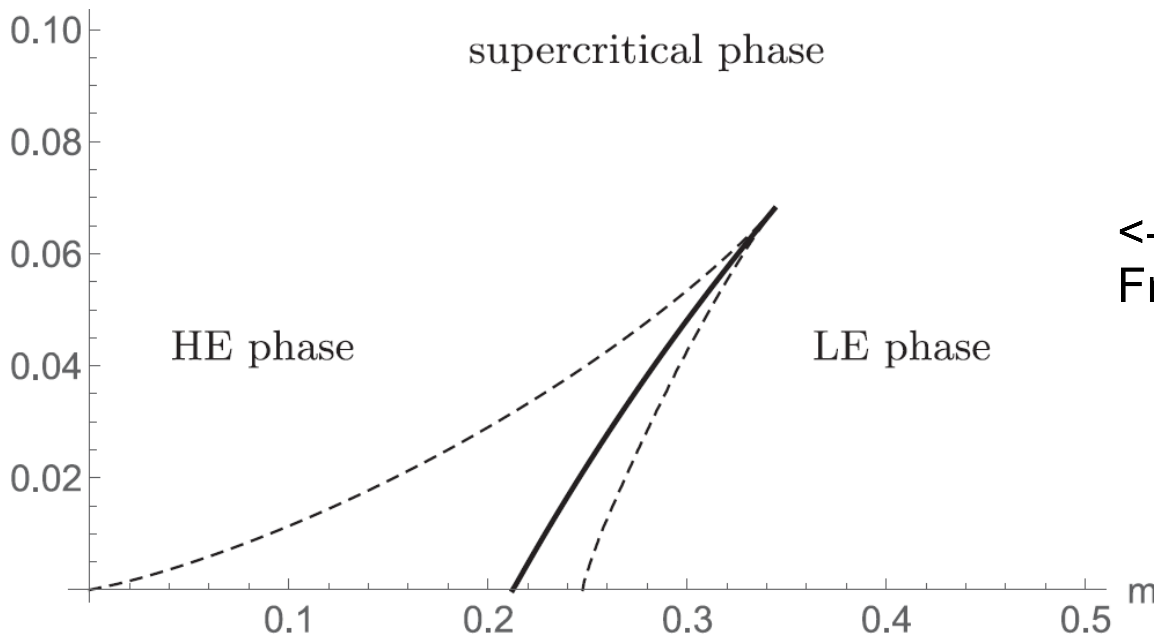
Bhattacharya et.al.,1709.07613

Quantum models that support these results:2

- Matrix quantum mechanics with the mass term in the special limit

Tatsuo Azeyanagi, Frank Ferrari, and Fidel I. Schaposnik Massolo
Phys. Rev. Lett. 120, 061602

$$H = ND\mathrm{tr}\left(m\psi_\mu^\dagger\psi_\mu + \frac{1}{2}\lambda\sqrt{D}\psi_\mu\psi_\nu^\dagger\psi_\mu\psi_\nu^\dagger\right)$$



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From Phys. Rev. Lett. 120, 061602

Conclusion

- We calculated the quantum complexity in the model of the locally excited system. In some sense our results are consistent with the intuitive definition of complexity. CV looks more «physical».
- Our CA results states that 2d CFT local excitation saturates the bound on complexity
- At the finite chemical potential the chaos (corresponding to the local charged excitations) is in accordance with the QFT models (chaos is supressed)