A simple UV completion for Higgs and Higgs-dilaton inflation.

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1. Higgs inflation

2. UV completion with $R^2$-term

3. UV completing Higgs-dilaton inflation
Standard Model Higgs inflation

Action for the Higgs boson in a unitary gauge ($\mathcal{H} = h / \sqrt{2}$)

$$S = \int d^4x \sqrt{-g} \left( - \frac{M_p^2 + \xi h^2}{2} R + \frac{(\partial_\mu h)^2}{2} - \frac{\lambda}{4} (h^2 - v^2)^2 \right)$$

Einstein frame action

$$S = \int d^4x \sqrt{-g} \left( - \frac{M_p^2}{2} R + \frac{(\partial_\mu \chi)^2}{2} - \frac{\lambda}{4} \frac{(h(\chi))^4}{(1 + \xi h(\chi)^2/M_p^2)^2} \right)$$

$$\frac{d\chi}{dh} = \frac{\sqrt{1 + \xi (1 + 6\xi) h^2 / M_p^2}}{1 + \xi h^2 / M_p^2}$$

$$h > M_p / \sqrt{\xi}, \quad V(\chi) \simeq \frac{\lambda M_p^4}{4\xi^2} \left( 1 - e^{-2\chi/(\sqrt{6} M_p)} \right)^2$$

In order to produce CMB normalization one needs $\lambda / \xi^2 \simeq 4 \times 10^{-10}$, $\xi \sim 10^4$. 
Field-dependent cutoff scale

Until which scale it is a valid description?

In the small field domain \( (g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_p) \) the minimal suppression scale of non-renormalizable operator

\[
L_{int} = \frac{\xi h^2 \partial^2 h_{\mu}}{M_p} \rightarrow \Lambda = \frac{M_p}{\xi}
\]

\( \Lambda/M_p^{\text{eff}} \)

\( 1 \)

\( 1/\xi \)

\( M_p/\xi \)

\( M_p/\sqrt{\xi} \)

\( M_p \)
Why UV completion is needed?

- No way to make the cutoff scale higher than the Planck mass is expected ⇒ having $\Lambda \sim M_p$ is already a good improvement
- Connection between the inflationary parameters and low energy physics
- Description of the preheating and reheating process

Y. Ema, R. Jinno et al., arXiv:1609.05209

\[ T_{\text{reh}} \sim 10^{-3} M_p \]
UV completion with $R^2$-term

\[ S_0 = \int d^4x \sqrt{-g} \left( -\frac{M_p^2 + \xi h^2}{2} R + \frac{\beta}{4} R^2 + \frac{(\partial_{\mu} h)^2}{2} - \frac{\lambda}{4} (h^2 - v^2)^2 \right) \]

How does it work?

- Introduce a Lagrange multiplier \( L \) and an auxiliary scalar \( \mathcal{R} \),

\[ S = \int d^4x \sqrt{-g} \left( L_h - \frac{M_p^2 + \xi h^2}{2} \mathcal{R} + \frac{\beta}{4} \mathcal{R}^2 - L \mathcal{R} + L \mathcal{R} \right) \]

- Integrate out the field \( \mathcal{R} \): the problematic \( \xi \) appear only in the potential

\[ S = \int d^4x \sqrt{-g} \left( L_h + L \mathcal{R} - \frac{1}{4\beta} \left( L + \frac{1}{2} \xi h^2 + \frac{1}{2} M_p^2 \right) \right) \]

- \( L \) is a dynamical field connected to the scalar graviton (scalaron)
Einstein frame action

Hereafter we use $M_p = 1/\sqrt{6}$

$$S = \int d^4x \sqrt{-g} \left( -\frac{R}{12} + \frac{1}{2} e^{-2\phi} (\partial h)^2 + \frac{1}{2} (\partial \phi)^2 - \frac{1}{4} e^{-4\phi} \left( \lambda h^4 + \frac{1}{36\beta} (e^{2\phi} - 1 - 6\xi h^2)^2 \right) \right)$$

Y. Ema, arXiv:1701.07665

Bounds on $\beta$:

- No strong coupling $\rightarrow \xi^2/\beta \lesssim 1 < 4\pi$
- CMB normalization can be satisfied if $\lambda < \xi^2/\beta$
Unitarity in the gauge sector

Scattering of the gauge bosons (longitudinal modes)
In the Standard model the growing part of the amplitude is cancelled

\[ W \quad h \quad W \quad W \quad W \quad h \quad W \quad W \]
\[ + \quad + \quad - \quad = 0 \]
\[ (\sim p^2/m_W^2) \]

If the Higgs interactions are modified there is no cancellation anymore

\[ W \quad h \quad W \quad W \quad h \quad W \quad W \]
\[ + \quad + \quad - \quad \sim \Delta p^2/m_W^2 \]
Gauge bosons unitarity cutoff in Higgs inflation
Which field interacts with gauge bosons and fermions?

\[ L_{\text{kin}} = \frac{1}{2} e^{-2\phi} (\partial h)^2 + \frac{1}{2} (\partial \phi)^2 \]

It is a mixture of \( \phi \) and \( h \).

**New variables:** \( h = e^\chi \tanh H, \phi = e^\chi / \cosh H \rightarrow \) Higgs is canonical

\[ L_{\text{kin}} = \frac{1}{2} \cosh^2 H (\partial \chi)^2 + \frac{1}{2} (\partial H)^2 \]

\[ V = \frac{1}{4} \left( \lambda \sinh^4 H + \frac{1}{36\beta} (1 - e^{-2\chi} \cosh^2 H - 6\xi \sinh^2 H)^2 \right) \]

\[ L_{\text{gauge}} = \frac{g^2 h^2}{4} e^{-2\phi} W^+_\mu W^-_\mu = \frac{g^2}{4} \sinh^2 H \ W^+_\mu W^-_\mu \]
Unitarity breaking scale

Standard model:

\[ m_W = \frac{1}{2} g \Lambda \]

\[ R^2 \text{- Higgs:} \]

\[ m_W = \frac{g}{2} \sinh H \]

The growing part of amplitude

\[ A \sim \frac{g^2 p^2}{m_W^2} \left( \frac{4}{g^2} \left( \frac{d m_W (H)}{d H} \right)^2 - 1 \right) \sim \frac{p^2}{6 M_p^2} \]

\[ \Lambda_U = \sqrt{6} M_p \]
Potential: who drives inflation?

$$V = \frac{1}{4} \left( \lambda \sinh^4 H + \frac{1}{36\beta} \left( 1 - e^{-2\chi} \cosh^2 H - 6\xi \sinh^2 H \right)^2 \right)$$
Recovering Higgs inflation

\[ V = \frac{1}{4} \left( \lambda \sinh^4 H + \frac{1}{36\beta} (1 - e^{-2\chi} \cosh^2 H - 6\xi \sinh^2 H)^2 \right) \]

Valley:

\[ 1 - e^{-2\chi} \cosh^2 H - 6\xi \sinh^2 H = 0 \]

After integrating out heavy field and canonical normalization

\[ V = \frac{1}{4} \frac{\lambda}{36\xi^2} (1 - e^{-2\bar{\chi}})^2, \quad \chi \gtrsim M_p \]
Validity of the single-field approximation

Effective mass of the orthogonal perturbation

\[ m(0,0) = \frac{M_p}{\sqrt{3\beta}}, \quad \beta \sim \xi^2 \rightarrow m \sim \Lambda_c = \frac{M_p}{\xi} \]
A simple UV completion for Higgs and Higgs-dilaton inflation.
Higgs inflation

UV completion with $R^2$-term

UV completing Higgs-dilaton inflation

Predictions

$$n_s = 1 - \frac{2}{N} = 0.966, \quad r = \frac{12}{N^2} = 0.0033$$

M. He, A. A. Starobinsky, J. Yokoyama, 1804.00409
Improving the stability of the Higgs potential

\[ \delta \beta \lambda = \frac{1}{16 \pi^2} \frac{2 \xi^2 (1 + 6 \xi)^2}{9 \beta^2} \]
A scale invariant extension of the Standard model

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \left( (\partial_\mu X)^2 - \xi X^2 R - \xi' h^2 R + (\partial_\mu h)^2 \right) - \frac{\lambda}{4} (h^2 - \alpha^2 X^2) \right] \]

- Planck mass and Higgs vev are provided by the dilatov vev $\langle X \rangle$
- Scale invariance is broken spontaneously
- This symmetry can be preserved at the quantum level if the renormalization scale is given by the dilaton vev
- The model can describe inflation with $\xi' \sim 10^3$, $\xi \lesssim 10^{-3}$
Higgs-dilaton inflation

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} [(\partial_\mu X)^2 - \xi X^2 R - \xi' h^2 R + (\partial_\mu h)^2] - \frac{\lambda}{4} (h^2 - \alpha^2 X^2) \right] \]

In the Einstein frame ($\xi \ll \xi'$, inflation)

\[ S = \int d^4x \sqrt{-g} \left( -\frac{M_p}{2} R + \frac{1}{2} \cosh^2 \left( \frac{\phi}{\sqrt{6} M_p} \right) (\partial_\mu r)^2 + \frac{1}{2} (\partial_\mu \phi^2) \right) - V \]

\[ V \approx \frac{\lambda M_p^4}{\xi_i^2} \left( 1 - 6\xi \sinh^2 \left( \frac{\phi}{\sqrt{6} M_p} \right) \right)^2 \]
A simple UV completion for Higgs and Higgs-dilaton inflation.
Cutoff scales

F. Bezrukov, G. Karananas, et. al, arXiv:1212.4148
UV completion with $R^2$ term

$R^2$ term can provide a minimal UV completion up to Planck scale

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \left[ \beta R^2 + (\partial_\mu X)^2 - \xi X^2 R - \xi' h^2 R + (\partial_\mu h)^2 \right] - \frac{\lambda}{4} h^4 \right]$$

In the Einstein frame

$$L = \frac{1}{2} \left( (\partial H)^2 + \cosh^2 H (\partial \phi)^2 + \cosh^2 H \cosh^2 \phi (\partial \rho)^2 \right) -$$

$$- \frac{1}{4} \left( \lambda \sinh^4 H + \frac{1}{36 \beta} \left( 1 - 6\xi \sinh^2 \phi \cosh^2 H - 6\xi' \sinh^2 H \right)^2 \right)$$

Here $H$ is Higgs, $\phi$ is scalaron, $\rho$ is dilaton (Goldstone boson)

$$L_{\text{gauge}} = \frac{g^2}{4} \sinh^2 H \ W^+ \ W^-$$

The cutoff scale is again $\sqrt{6} M_P$
Potential and trajectories

\[ n_s = 1 - 8\xi \coth(4\xi N) \sim 1 - \frac{1}{2N}, \quad \xi \lesssim 0.004 \]
Both Higgs and Higgs-dilaton inflation can be UV completed up to the Planck scale.

The completion can be achieved by means of introducing only one extra term in the lagrangian ($R^2$-term).

At the beginning, inflation is driven by the $R^2$ degree of freedom, then the trajectory turns to a Higgs direction.

$R^2$ term can also improve the stability of Higgs potential for a certain range of top quark masses.
Thanks for your attention!