Laser effect for cosmic axions

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**Bose star formation**

**QCD axion** ($m \sim 10^{-5}$ eV)

Bose condensation by gravitational interaction in miniclusters.

[D. Levkov et al, 2018]

$$M_{bs} \sim 10^{-11} M_{\odot} ; \quad R_{bs} \sim 50 \text{ km}$$

**Fuzzy dark matter** ($m \sim 10^{-22}$ eV)

Bose star appear during structure formation in the center of each galaxy.

[H.-Y. Schive et al, 2014]

$$M_{bs} \sim 10^{8} M_{\odot} ; \quad R_{bs} \sim 100 \text{ pc}$$
Bose star properties

Nonrelativistic approximation:

\[ \frac{a}{f_a} = (\psi e^{-imt} + \text{h.c.})/2 \quad \leftrightarrow \quad \partial_t, \partial_x \ll m; \quad \Phi, \psi \ll 1 \]

\[ \begin{align*}
    i\partial_t \psi &= -\Delta \psi/2m + m(\Phi - g_4^2|\psi|^2/8)\psi \\
    \Delta \Phi &= 4\pi G \times m^2 f_a^2 |\psi|^2 
\end{align*} \]

Gross-Pitaevskii-Puasson system

Using coordinate and field rescaling:

\[ \begin{align*}
    \tilde{t} &= mv_0^2 t; \quad \tilde{x} = mv_0 x \\
    \tilde{\psi} &= g_4 \psi/v_0; \quad \tilde{\Phi} = \Phi/v_0^2 \\
    v_0 &\equiv f_a/g_4 M_{\text{Pl}} 
\end{align*} \]

All physical parameters disappear!

Symmetry: \( \psi \rightarrow e^{i\alpha} \psi \)

The total mass conservation

\[ \tilde{M} = \int d^3 \tilde{x} \tilde{\rho} = \int d^3 \tilde{x} |\tilde{\psi}|^2 \]
**Axion-photon coupling**

Axion field of Bose star oscillates coherently with time:

\[ a / f_a = (\psi e^{-i mt} + h.c.) / 2 \]

May cause parametric resonance of photons!

\[ \mathcal{L}_{em} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \frac{g_{a\gamma}}{4} a F_{\mu\nu} \tilde{F}_{\mu\nu} \]

Axion-photons coupling

\[ \partial_{\mu} F_{\mu\nu} + g_{a\gamma} \partial_{\mu} a \tilde{F}_{\mu\nu} = 0 \]

modified Maxwell's equations

or

\[ \partial_{\mu} \partial_{\mu} A_{\nu} - \partial_{\mu} \partial_{\nu} A_{\mu} + 2\epsilon_{\mu\nu\lambda\rho} g_{a\gamma} \partial_{\mu} a \partial_{\lambda} A_{\rho} = 0 \]
Resonance in homogeneous condensate

For homogeneous condensate, $\psi = \psi_0 = \text{const}$, we have

$$\ddot{A}_k + (k^2 \pm \sqrt{2} g_{a\gamma} f_a m k \cdot \psi_0 \sin(mt)) A_k = 0 \quad \text{– Mathieu equation!}$$

$$A_k \propto c_k e^{\mu t} e^{\pm i \frac{m}{2} (t \pm x)} \quad \text{with} \quad \mu = \frac{g_{a\gamma} f_a m \psi_0}{2 \sqrt{2}}$$

Amplification coefficient:

$$D \equiv \mu \cdot R = \frac{g_{a\gamma} f_a m \psi_0}{2 \sqrt{2}} \cdot R$$

$$D \gtrsim 1 \quad \text{– resonance!}$$
General case

\[ a/f_a = (\psi e^{-imt} + \text{h.c.})/2, \quad \psi(t, x) – \text{weakly depends on } t, x \]

Consider plane waves with frequency \( m/2 \) moving through a star:

\[
\begin{align*}
A_0 &= 0 \quad - \text{gauge} \\
A_i &= e^{i\frac{m}{2}z} \left( c_i^+(t, x)e^{i\frac{m}{2}t} + c_i^-(t, x)e^{-i\frac{m}{2}t} \right) + \text{h.c.}
\end{align*}
\]

Weakly depends on space and time: **eikonal-like approximation**

\[
\begin{align*}
\nu = x & \quad \frac{\partial c_x^+}{\partial t} - \frac{\partial c_x^+}{\partial z} - i g a_\gamma m f_a \frac{\psi^* c_y^-}{\sqrt{2}} = 0 \quad (1) \\
\nu = y & \quad \frac{\partial c_y^-}{\partial t} + \frac{\partial c_y^-}{\partial z} + i g a_\gamma m f_a \psi c_x^+ = 0 \quad (2)
\end{align*}
\]

\[ c_y^+ = c_x^+, \quad c_x^- = -c_y^- \] satisfy another pair of Eqs.

**Boundary conditions:**
no waves coming from infinity!

\[
\begin{align*}
c_{x,y}^+(z = +\infty) &= 0 \\
c_{x,y}^-(z = -\infty) &= 0
\end{align*}
\]
Boundary value problem

Substituting \( c_{x,y}^\pm(t, z) = e^{\mu t} c_{x,y}^\pm(z) \)

we obtain the boundary value problem for \( c_{x,y}^\pm(z) \).

\[
\begin{align*}
\mu c_x^+ - \frac{\partial c_x^+}{\partial z} - i \frac{g a \gamma m f_a}{\sqrt{2}} \psi^* c_y^- &= 0 \\
\mu c_y^- + \frac{\partial c_y^-}{\partial z} + i \frac{g a \gamma m f_a}{\sqrt{2}} \psi c_x^+ &= 0
\end{align*}
\]

For real \( \psi \) and \( \mu = 0 \)

we have analytic solution:

\[
\begin{align*}
c_x^+(z) &= A \cos(S(z)) \\
c_y^-(z) &= A \sin(S(z))
\end{align*}
\]

where \( S(z) = \frac{g a \gamma m f_a}{\sqrt{2}} \int_{-\infty}^{z} \psi(z')dz' \) and

\[
S(+\infty) = D = \frac{g a \gamma m f_a}{\sqrt{2}} \int_{-\infty}^{+\infty} \psi(z')dz' = \frac{\pi}{2}
\]

Resonance condition!

from boundary condition

\( c_{x,y}^+(z = +\infty) = 0 \)

Solutions with \( \mu > 0 \)

exist if \( D > \frac{\pi}{2} \).
Spherically symmetric case

Any direction is in resonance.

One can expand $A_i$ in terms of vector spherical harmonics:

\[
\begin{align*}
\vec{Y}^{(1)}_{lm} &= \frac{r \vec{\nabla} Y_{lm}}{\sqrt{l(l+1)}} ; \\
\vec{Y}^{(2)}_{lm} &= \frac{\vec{\nabla} Y_{lm} \times \vec{r}}{\sqrt{l(l+1)}} ; \\
\end{align*}
\]

$Y_{lm}$ – scalar spherical harmonics. Eigenvectors of the total angular momentum $\hat{J}$

At some distance from the center, $l m r \gg 1$:

\[
\vec{A}(t, x) = \frac{e^{i \frac{m}{2} r}}{r} \left( c^+_{\alpha}(t, r) e^{i \frac{m}{2} t} + c^-_{\alpha}(t, r) e^{-i \frac{m}{2} t} \right) \vec{Y}^{(\alpha)}_{lm} + \text{h.c.}
\]

Weakly depends on space and time: eikonal-like approximation

Very similar equations for $c^+_{\alpha}$ and the same resonance condition.
Bose stars

\[ i \partial_t \tilde{\psi} = -\frac{\Delta \tilde{\psi}}{2} + \left( \tilde{\Phi} - \frac{1}{8} |\tilde{\psi}|^2 \right) \tilde{\psi} \]
\[ \Delta \tilde{\Phi} = 4\pi |\tilde{\psi}|^2 \]

\[ \tilde{t} = mv_0^2 t; \quad \tilde{x} = mv_0 x \]
\[ \tilde{\psi} = g_4 \psi / v_0; \quad \tilde{\Phi} = \Phi / v_0^2 \]
\[ v_0 \equiv f_a / g_4 M_{Pl} \]

Stability criterion \[ dM/d\omega > 0 \] unstable!

\[ \tilde{\psi} = \tilde{\psi}_s(\tilde{r}) e^{+i\tilde{\omega} \tilde{t}} \]

Bose star profile

\[ M_{cr} \approx 10 \frac{M_{Pl} f_a}{g_4 m} \approx 5 \times 10^{-12} M_\odot \]
\[ R_{cr} \approx 0.18 \frac{g_4 M_{Pl}}{m f_a} \approx 70 \text{ km} \]

QCD axion

[N.G. Vakhitov, A.A. Kolokolov, 1973]
Bose stars

Amplification coefficient for Bose stars.

Minimal value for axion-photon coupling constant required for resonance.

Additional axions captured by Bose star will be converted into photons.

Contribution to the radio background!

For minimal axion models $g_{a\gamma} f_a \sim \alpha = 1/137$ Relevant for photofilic axion
**Bose star collapse**

What happens with overcritical Bose star?  
It is collapse!  

[D. Levkov et al, 2016]

\[ i \partial_{\tilde{t}} \tilde{\psi} = -\frac{\tilde{\Delta} \tilde{\psi}}{2} + \Phi \tilde{\psi} - \frac{1}{8} |\tilde{\psi}|^2 \tilde{\psi} \]

The scaling symmetry appears:
\[ \tilde{t} \to \gamma^2 \tilde{t}, \quad \tilde{x} \to \gamma \tilde{x}, \quad \tilde{\psi} \to \tilde{\psi} e^{i\alpha} / \gamma \]

**Finite energy**  
\[ \left\{ \begin{array}{l} \chi_*(0) = 0 \\ \chi_*(+\infty) = \chi_0 \cdot y^{-2i\omega_*} \end{array} \right. \]

**Solution:** \( \omega_* \approx 0.54, \; \chi_0 \approx 2.84 \)
Resonance during Bose star collapse

Numerical solution on collapsing Bose star background:

\[ \text{Re}(\mu) = 0, \; \text{Im}(\mu) \neq 0 \quad \text{due to nonzero axion velocities.} \]

Resonance does not interrupted

New axions fall on the center due to self-similar collapse.

\textbf{Fast radio burst!} [I.I. Tkachev, 2014]
Resonance during Bose star collapse

To check the results we solve numerically relativistic equations in spherically-symmetric case.

\[
\begin{align*}
\partial_\mu \partial_\mu a + V'(a) &= 0 \\
\partial_\mu F_{\mu\nu} + g_{\alpha\gamma} \partial_\mu a \tilde{F}_{\mu\nu} &= 0
\end{align*}
\]

\[\vec{A}(t, x) = a_\alpha(t, r) \hat{Y}_{lm}^{(\alpha)} + \text{h.c.}\]

Weak dependence on \(x, t\) is not assumed!

\[g_{\alpha\gamma} f_\alpha = 0.16\]

\[l = m = 0\]
Collision with black hole

Black hole

$M_{bh} \sim M_\odot$

$R_{bh} \sim 2 \text{ km}$

Destruction by tidal force!

Amplification coefficient increases.

$D \sim 10^2 g_{a\gamma} f_a$ and $\min(g_{a\gamma} f_a) \sim 10^{-2}$

Fast radio burst!

[M. Pshirkov, 2017]
Summary

Bose star
Contribution to the radio background

Bose star collapse, Bose stars collision
Fast radio burst

Bose star collisions with compact astrophysical objects (black holes, neutron stars, etc.)
Fast radio burst

$g_{a\gamma} f_a$

1

0.3

0.15

$10^{-2}$
Thank you for attention!