Dualities in dense baryonic (quark) matter with chiral and isospin imbalance

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Hadronic, quark matter
Methods of dealing with QCD

- First principle calculation – lattice Monte Carlo simulations, LQCD

- Effective models
  - Nambu–Jona-Lasinio model NJL

- Low dimensional models that mimics QCD,
  - (1+1)- dim GN, NJL₂
Lattice QCD
non-zero baryon chemical potential $\mu_B$

sign problem — complex determinant

$$(\text{Det}(D(\mu)))^\dagger = \text{Det}(D(-\mu^\dagger))$$
(1+1)-dimensional GN, NJL model

(1+1)-dimensional Gross-Neveu (GN) or NJL model possesses a lot of common features with QCD

- renormalizability
- asymptotic freedom
- spontaneous chiral symmetry breaking in vacuum
- dimensional transmutation
- have the similar $\mu_B - T$ phase diagrams

NJL$_2$ model
laboratory for the qualitative simulation of specific properties of QCD at arbitrary energies
(3+1)-dimensional NJL model

NJL model can be considered as effective field theory for QCD.

the model is nonrenormalizable
Valid up to $E < \Lambda \approx 1$ GeV

Parameters $G$, $\Lambda$, $m_0$

**chiral limit** $m_0 = 0$

in many cases chiral limit is a very good approximation

dof—**quarks**
no gluons only **four-fermion interaction**
attractive feature — dynamical CSB
the main drawback – lack of confinement (PNJL)

Relative simplicity allow to consider hot and dense QCD in the framework of NJL model and explore the QCD phase structure (diagram).
Unlike the QED, the QCD vacuum has non-trivial structure due to non-perturbative interactions among quarks and gluons.

GOR relation and lattice simulations ⇒ condensation of quark and anti-quark pairs

\[ \langle \bar{q}q \rangle \neq 0, \quad \langle \bar{u}u \rangle = \langle \bar{d}d \rangle \approx (-250\text{MeV})^3 \]
Nambu–Jona-Lasinio model

\[ \mathcal{L} = \bar{q} \gamma^\nu i \partial_\nu q + \frac{G}{N_c} \left[ (\bar{q}q)^2 + (\bar{q} i \gamma^5 q)^2 \right] \]

\[ q \rightarrow e^{i \gamma^5 \alpha} q \]

Continuous symmetry

\[ \tilde{\mathcal{L}} = \bar{q} \left[ \gamma^\rho i \partial_\rho - \sigma - i \gamma^5 \pi \right] q - \frac{N_c}{4G} \left[ \sigma^2 + \pi^2 \right]. \]

Chiral symmetry breaking

1/$N_c$ expansion, leading order

\[ \langle \bar{q}q \rangle \neq 0 \]

\[ \langle \sigma \rangle \neq 0 \quad \rightarrow \quad \tilde{\mathcal{L}} = \bar{q} \left[ \gamma^\rho i \partial_\rho - \langle \sigma \rangle \right] q \]
QCD at finite temperature and nonzero chemical potential plays a fundamental role in many different physical systems. (QCD at extreme conditions)

- neutron stars
- heavy ion collision experiments
- Early Universe
Very brief history and motivation

There has been a lot of activity in this area pion condensation in NJL$_4$
also in (1+1)- dimensional case, NJL$_2$

\[ \downarrow \]

pion condensation in dense matter predicted without certainty

physical quark mass – no pion condensation in dense medium
H. Abuki, R. Anglani, R. Gatto, M. Pellicoro, M. Ruggieri
Very brief history and motivation

There could be parameters that generate pion condensation in dense matter

-Finite volume effects
D. Ebert, T.G. Khunjua, K.G. Klimenko, V.Ch. Zhukovsky,

-Inhomogeneous pion condensate
N. V. Gubina, K. G. Klimenko, S. G. Kurbanov, V. Ch. Zhukovsky,

This is all obtained in (1+1)- dimensional case, NJL$_2$

-Pion Condensation by Rotation in a Magnetic field
Different types of chemical potentials: dense matter with isotopic imbalance

Baryon chemical potential $\mu_B$

Allow to consider systems with non-zero baryon densities.

The corresponding term in the Lagrangian is

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q,$$

where $\mu$ -quark chemical potential

Isotopic chemical potential $\mu_I$

Allow to consider systems with isotopic imbalance.

$$n_I = n_u - n_d \leftrightarrow \mu_I = \mu_u - \mu_d$$

The corresponding term in the Lagrangian is $\frac{\mu}{2}\bar{q}\gamma^0\tau_3 q$
QCD phase diagram with isotopic imbalance

neutron stars, heavy ion collisions have isotopic imbalance
Different types of chemical potentials: chiral imbalance

chiral (axial) chemical potential

Allow to consider systems with chiral imbalance (difference between between densities of left-handed and right-handed quarks).

\[ n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L \]

The corresponding term in the Lagrangian is

\[ \mu_5 \bar{q} \gamma^0 \gamma^5 q \]
Different types of chemical potentials: chiral imbalance

chiral (axial) isotopic chemical potential

Allow to consider systems with chiral isospin imbalance

\[ \mu_{I5} = \mu_{u5} - \mu_{d5} \]

so the corresponding density is

\[ n_{I5} = n_{u5} - n_{d5} \]

\[ n_{I5} \longleftrightarrow \mu_{I5} \]

Term in the Lagrangian — \[ \frac{\mu_{I5}}{2} \bar{q} \gamma^0 \gamma^5 q \]

If one has all four chemical potential, one can consider different densities \( n_{uL}, n_{dL}, n_{uR} \) and \( n_{dR} \)
Chiral magnetic effect

\[ \vec{J} = c \mu_5 \vec{B}, \quad c = \frac{e^2}{2\pi^2} \]

A. Vilenkin, PhysRevD.22.3080,
Chiral separation effect

Chiral imbalance could appear in compact stars

\[ \vec{J}_5 = c \mu \vec{B}, \quad c = \frac{e^2}{2\pi^2} \]

there is current and there is \( n_5 \)
We consider a NJL model, which describes dense quark matter with two massless quark flavors ($u$ and $d$ quarks).

$$\mathcal{L} = \bar{q} \left[ \gamma^\nu i \partial_\nu + \frac{\mu B}{3} \gamma^0 + \frac{\mu I}{2} \tau_3 \gamma^0 + \frac{\mu I^5}{2} \tau_3 \gamma^0 \gamma^5 + \mu_5 \gamma^0 \gamma^5 \right] q + \frac{G}{N_c} \left[ (\bar{q} q)^2 + (\bar{q}i \gamma^5 \tau q)^2 \right]$$

$q$ is the flavor doublet, $q = (q_u, q_d)^T$, where $q_u$ and $q_d$ are four-component Dirac spinors as well as color $N_c$-plets; $\tau_k$ ($k = 1, 2, 3$) are Pauli matrices.
quark masses, chiral limit

light quarks $u, d$

$$m_u = 0.005 \text{ GeV}, \quad m_d = 0.009 \text{ GeV}$$

chiral limit $m_u = m_d = 0$
Equivalent Lagrangian

To find the thermodynamic potential we use a semi-bosonized version of the Lagrangian

$$\tilde{L} = \bar{q} \left[ \gamma^\rho i \partial_\rho + \mu \gamma^0 + \nu \tau_3 \gamma^0 + \nu_5 \tau_3 \gamma^1 - \sigma - i \gamma^5 \pi_a \tau_a \right] q - \frac{N_c}{4G} \left[ \sigma \sigma + \pi_a \pi_a \right].$$

$$\sigma(x) = -2 \frac{G}{N_c} (\bar{q}q); \quad \pi_a(x) = -2 \frac{G}{N_c} (\bar{q}i \gamma^5 \tau_a q).$$

Condensates ansatz $\langle \sigma(x) \rangle$ and $\langle \pi_a(x) \rangle$ do not depend on spacetime coordinates $x$,

$$\langle \sigma(x) \rangle = M, \quad \langle \pi_1(x) \rangle = \Delta, \quad \langle \pi_2(x) \rangle = 0, \quad \langle \pi_3(x) \rangle = 0. \quad (1)$$

where $M$ and $\Delta$ are already constant quantities.
thermodynamic potential

the thermodynamic potential can be obtained in the large $N_c$ limit

$$\Omega(M, \Delta)$$

Projections of the TDP on the $M$ and $\Delta$ axes

No mixed phase ($M \neq 0, \Delta \neq 0$)

it is enough to study the projections of the TDP on the $M$ and $\Delta$

projection of the TDP on the $M$ axis $F_1(M) \equiv \Omega(M, \Delta = 0)$

projection of the TDP on the $\Delta$ axis $F_2(\Delta) \equiv \Omega(M = 0, \Delta)$
Dualities of the TDP

The TDP is invariant with respect to the so-called duality transformations (dualities)

1) The main duality

\[ D : M \leftrightarrow \Delta, \quad \nu \leftrightarrow \nu_5 \]

\[ \nu \leftrightarrow \nu_5 \text{ and PC } \leftrightarrow \text{ CSB} \]

2) Duality in the CSB phenomenon

\[ F_1(M) \text{ is invariant under } D_M : \nu_5 \leftrightarrow \mu_5 \]

3) Duality in the PC phenomenon

\[ F_2(\Delta) \text{ is invariant under } D_\Delta : \nu \leftrightarrow \mu_5 \]

PC phenomenon breaks \( D_M \) and CSB phenomenon \( D_\Delta \) duality
Dualities in different approaches

- Similar dualities between chiral and superconducting condensates have been obtained in (1+1) and (2+1)-dimensional models

  D. Ebert, T.G. Khunjua, K.G. Klimenko, V.Ch. Zhukovsky,
  Phys. Rev. D 90, 045021 (2014),

- Large $N_c$ orbifold equivalences connect gauge theories with different gauge groups and matter content in the large $N_c$ limit.

  M. Hanada and N. Yamamoto,
Phase structure of the (1+1) dim NJL model

Chiral isospin chemical potential $\mu_{I_5}$ generates charged pion condensation in the dense quark matter.

Phase portrait \((\mu, \nu, \nu_5)\) of NJL\(_2\)

**Figure:** \((\mu, \nu, \nu_5)\) phase diagram in homogeneous case.
Phase structure of (3+1)-dim NJL model

Phase structure of the (3+1) dim NJL model

Chiral isospin chemical potential $\mu_5$ generates charged pion condensation in the dense quark matter.
$\nu, \nu_5$) phase portrait of NJL$_4$

Duality between chiral symmetry breaking and pion condensation

$\mathcal{D}: \ M \longleftrightarrow \Delta, \ \nu \longleftrightarrow \nu_5$

$\text{PC} \longleftrightarrow \text{CSB} \ \nu \longleftrightarrow \nu_5$

Figure: $(\nu, \nu_5)$ at $\mu = 0$ GeV

Figure: $(\nu, \nu_5)$ at $\mu = 0.195$ GeV
Comparison of phase diagram of (3+1)-dim and (1+1)-dim NJL models

The phase diagrams obtained in two models that are assumed to describe QCD phase diagram are qualitatively the same.
\((\mu, \nu)\) phase portraits comparison, NJL\(_2\) and NJL\(_4\)
\((\mu, \nu)\) phase portraits comparison, NJL\(_2\) and NJL\(_4\)

**Figure:** \((\mu, \nu)\) phase diagram in the framework of NJL\(_2\) model at \(\nu_5 = 0\) GeV

**Figure:** \((\mu, \nu)\) phase diagram in the framework of NJL\(_4\) model at \(\nu_5 = 0.15\) GeV
comparison of NJL$_2$ and NJL$_4$, slight difference and complementarity

slight difference:
NJL$_2$: $\nu_5$ can generate PC$_d$ phase even at $\nu = 0$
NJL$_4$: $\nu_5$ can generate PC$_d$ phase only at $\nu \neq 0$
NJL$_4$ is more realistic so $\nu_5 \rightarrow$ PC$_d$ only at $\nu \neq 0$

Complementarity
NJL$_2$ is renormalizable, $\nu_5$, $\nu$ etc can have any value
NJL$_4$ is non-renormalizable, effective, $\nu_5$, $\nu < \Lambda \approx 650$ GeV
But if the predictions are the same one can possibly expand the prediction of NJL$_4$ model to the range where its results are not credible.
Consideration of the case with $\mu_B$, $\mu_I$, $\mu_{I5}$ and $\mu_5$ chemical potentials in (3+1)-dimensional NJL model

$$(\mu_B, \mu_I, \mu_{I5}, \mu_5),$$
$$(\nu_5 = \frac{\mu_{I5}}{2}, \nu = \frac{\mu_I}{2})$$

Up to now $(\mu_B, \mu_I, \mu_{I5})$ was considered ($\mu_{I5} \neq 0$ and $\mu_5 = 0$)

Now let us consider $\mu_5$ instead of $\mu_{I5}$ ($\mu_5 \neq 0$, $\mu_{I5} = 0$)

$$(\mu_B, \mu_I, \mu_{I5}) \longrightarrow (\mu_B, \mu_I, \mu_5)$$

How chiral imbalance in the form of chiral $\mu_5$ chemical potential influence PC condensation
Chiral imbalance in the form of $\mu_5$ chemical potential. $(\nu, \mu_5)$ phase diagram

Chiral chemical potential $\mu_5$ generates charged pion condensation in the dense quark matter as well.

$\mu_5 \rightarrow \text{PC}_d$

-Not so prominently as $\mu_I$ does
-But only at comparatively low densities $n_q$

Figure: $(\nu, \mu_5)$ phase diagram at $\mu = 0.23$ GeV.
Figure: $(\nu, \mu_5)$ phase diagram at $\mu = 0.1$ GeV.

Figure: $(\nu, \mu_5)$ phase diagram at $\mu = 0.23$ GeV.
Comparison of non-zero $\nu_5 = \mu_{15}/2$ and $\mu_5$ cases, duality in the PC phenomenon

Duality in the PC phenomenon

PC phenomenon ($F_2(\Delta)$) is invariant under $D_\Delta$

$$D_\Delta : \nu \leftrightarrow \mu_5$$

But CSB does not respect the duality $D_\Delta$ so one has to check that CSB is dynamically suppressed in the duality conjugated regions

CSB is dynamically suppressed $M_0 = 0$
Comparison of non-zero $\nu_5 = \mu_5/2$ and $\mu_5$ cases. Duality in the PC

**Figure:** $(\nu, \nu_5)$ phase diagram at $\mu = 0.55$.

**Figure:** $(\nu, \mu_5)$ phase diagram at $\mu = 0.4$ GeV.
\((\nu, \mu_5)\) phase diagram, duality in the PC phenomenon

If CSB is dynamically suppressed throughout all phase diagram \((\nu, \mu_5)\) then the phase diagram \((\nu, \mu_5)\) is self dual with respect to \(\nu \leftrightarrow \mu_5\).

**Figure:** \((\nu, \mu_5)\) phase diagram at 
\(\mu = 0(0.01)\) GeV.
Consideration of the case $\mu_1 = 0$

Now let us discuss the case of $\mu_1 = 0$

and investigate the influence of $\mu_{15}$ and $\mu_5$

Study the influence of $\mu$, $\mu_{15}$ and $\mu_5$ on the phase diagram
and present the $(\mu, \mu_{15}, \mu_5)$-phase diagram of the model

do not have to calculate anything
one can use duality $\mathcal{D}$ between CSB and PC to get phase diagrams from already depicted ones
$(\mu_5, \nu_5)$ phase diagram, duality in the CSB phenomenon

**Figure**: $(\mu_5, \nu_5)$ phase diagram at $\mu = 0(0.01)$ GeV.

**Duality in the CSB phenomenon**

CSB phenomenon ($F_1(M)$) is invariant under $D_M$

$$D_M : \nu_5 \leftrightarrow \mu_5$$

But PC does not respect the duality $D_M$ so one has to check if CSB is dynamically suppressed

$$M_0 = 0$$

Catalysis of Dynamical CSB by $\mu_5$ Braguta, Kotov Phys. Rev. D 93, 105025 (2016)
Comparison of $\nu$ and $\mu_5$

Figure: $(\nu, \nu_5)$ phase diagram at $\mu = 0.55$ GeV.

Figure: $(\mu_5, \nu_5)$ phase diagram at $\mu = 0.4$ GeV.
Comparison of impact of $\nu$ and $\mu_5$ on PC phenomenon

It was said above that in more realistic NJL$_4$ model PC$_d$ phase can be generated by $\nu_5$ only in isospin asymmetric matter $\nu \neq 0$, but one knows that $\nu$ and $\mu_5$ influence the PC phenomenon exactly the same way and one can guess that PC$_d$ phase can be generated just by $\nu_5$ and $\mu_5$ at $\nu = 0$

NJL$_4$: $\nu_5 \rightarrow$ PC$_d$ only at $\nu \neq 0$

in terms of PC$_d$ the $\nu = \mu_5$

$\nu_5 \rightarrow$ PC$_d$ at $\nu = 0$ and $\mu_5 \neq 0$

Just Chiral imbalance but in both forms generate PC$_d$ phase
Consideration of the general case $\mu, \mu_1, \mu_{15}$ and $\mu_5$

Now let us discuss the general case $\mu, \mu_1, \mu_{15}$ and $\mu_5$
Consideration of the general case $\mu$, $\mu_1$, $\mu_{15}$ and $\mu_5$

In this case the phase diagram even richer

generation of $PC_d$ phase is even more widespread

**Figure:** $(\nu, \nu_5)$ phase diagram at $\mu_5 = 0.5$ GeV and $\mu = 0.3$ GeV.
Charge neutrality condition

the general case \((\mu, \mu_1, \mu_{15}, \mu_5)\)

consider charge neutrality case \(\rightarrow \nu = \mu_1/2 = \nu(\mu, \nu_5, \mu_5)\)
Charge neutrality condition

-pion condensation in dense matter predicted without certainty,
  at $\nu$ there is a small region of $PC_d$ phase

-physical quark mass and electric neutrality - no pion condensation in dense medium
  H. Abuki, R. Anglani, R. Gatto, M. Pellicoro, M. Ruggieri

-Chiral isospin chemical potential $\mu_{I5}$ generates $PC_d$

-can this generation happen in the case of neutrality condition
Charge neutrality condition, just $\nu_5$

Figure: $(\nu, \nu_5)$ phase diagram at $\mu = 0.4$ GeV and $\mu_5 = 0$. 

\[ n_Q = 0 \]
Charge neutrality condition, both $\nu_5$, $\mu_5$

Figure: $(\nu, \nu_5)$ phase diagram at $\mu = 0.4$ GeV and $\mu_5 = 0.4$. 
Conclusions

\[ \mu_B \neq 0 \text{ - dense quark matter} \]
\[ \mu_I \neq 0 \text{ isotopically asymmetric} \]
\[ \mu_5 \neq 0 \text{ and } \mu_{15} \neq 0 \text{ chirally asymmetric} \]

CSB and PC

First NJL$_2$: \[ \mu_{15} \rightarrow \text{PC}_d \quad (\mu_B, \mu_I, \mu_{15}) \]
Duality between CSB and PC: \[ \nu_5 \leftrightarrow \nu \]

NJL$_4$: \[ \mu_{15} \rightarrow \text{PC}_d \quad \text{qualitatively the same picture} \quad (\mu_B, \mu_I, \mu_{15}) \]
Duality between CSB and PC is checked

NJL$_2$ approach is justified
Then $\mu_{I5} \rightarrow \mu_5$: $\mu_5 \rightarrow \text{PC}_d$ but not as widely as $\mu_{I5}$

$(\mu_B, \mu_I, \mu_5)$

Both $\mu_{I5}, \mu_5$: \textbf{wide PC}_d \textbf{ generation even with neutrality condition}

$(\mu_B, \mu_I, \mu_{I5}, \mu_5)$

Found new dualities $\mathcal{D}_M$ and $\mathcal{D}_\Delta$

isotopic and chiral imbalance and baryon density

HIC and NS and early universe
Thanks for the attention

You could wonder what changes will bring finite temperature $T$ and non-zero current quark masses $m_0$, can all this be compared with results of Lattice QCD

next talk by Tamaz Khunjua