Mass deformed super Yang-Mills on spheres

Joseph A. Minahan, Uppsala University

Based on:
Maxim Zabzine, JAM: arXiv:1502.07154
JAM: arXiv:1512.06924
Anastasios Gorantis, Usman Naseer, JAM: arXiv:1711.05669

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Introduction

- Localization of gauge theories on spheres or other compact manifolds has been successfully applied to many situations.
- Examples:
  $\mathcal{N}=4$ and $\mathcal{N}=2$ SYM in 4d Pestun
  $\mathcal{N}=2$ and higher CS/SYM in 3d Kapustin, Willett, Yakov; Jafferis
  $\mathcal{N}=1,2$ SYM in 5d: Källén, Zabzine; Källén, Qiu, Zabzine; Kim, Kim
  $(2,2)$ SYM in 2d: Benini and Cremonisi; Daroud *et. al.*
  $\mathcal{N}=2$ 6d and $\mathcal{N}=1$ 7d SYM JAM, Zabzine
- Different techniques used for the different situations.
  - Odd dimensional spheres have vector fields that act freely.
  - Even dimensional spheres have vector fields with fixed points
  - At the end, the results are very similar
- Is there a more uniform way of doing this for general d?
- Is it possible to continue the value of d to situations where a direct localization procedure is not known?

Outline

- Review and extend a generalized version of Pestun's dimensional reduction and localization of MSYM (and its mass deformations) on round spheres Zabzine & JAM; Gorantis, Naseer, JAM.
- One loop determinants for general d JAM; Gorantis, Naseer, JAM.
- Application: Analytic continuation of mass deformed MSYM with 4 SUSYs from $d = 3$ to $d = 4$ JAM; Gorantis, Naseer, JAM.
- Summary and open questions

Dimensional reduction of MSYM

- 10-dimensional flat-space action: Brink, Scherk & Schwarz

$$S = -\frac{1}{g_{10}^2}\int d^{10}x \,\mathrm{Tr}\left(\tfrac{1}{2}F_{MN}F^{MN} - \Psi \not{D}\Psi\right) .$$

- Action is invariant under the supersymmetry transformations

$$\begin{aligned} \delta_\epsilon A_M &= \epsilon^\alpha \Gamma_{M\alpha\beta}\Psi^\beta , \qquad M = 0,\dots 9 \\ \delta_\epsilon \Psi^\alpha &= \tfrac{1}{2}\Gamma^{MN\alpha}{}_\beta F_{MN}\,\epsilon^\beta , \qquad \alpha,\beta = 1,\dots 16 \end{aligned}$$

  ϵ^α: constant bosonic real chiral spinors; (16 ind. SUSYs)
- Dimensionally reduce to d-dimensional Euclidean gauge theory.

$$A_\mu , \quad \mu = 1\dots, d \qquad \phi_I \equiv A_I , \quad I = 0, d+1, \dots 9 .$$

- Derivatives along compactified directions are zero:

$$F_{\mu I} = [D_\mu, \phi_I] \qquad F_{IJ} = [\phi_I, \phi_J] .$$

- Scalars transform under vector rep. of $SO(1, 9-d)$ R-symmetry in flat Euclidean space. ϕ_0 has wrong-sign kinetic term.
- d-dimensional coupling: $g_{YM}^2 = g_{10}^2 / V_{10-d}$.

The theory on spheres Blau '00, Zabzine and JM

- Put theory on S^d with radius r.
- $d = 4$: MSYM is superconformal, $\Longrightarrow$ conformal mass term

$$S_{\phi\phi} = \frac{1}{g_{YM}^2} \int d^4x \sqrt{-g} \left(\frac{2}{r^2} \,\mathrm{Tr}\phi_I \phi^I \right)$$

- $d \neq 4$: not superconformal, but we include a similar term:

$$S_{\phi\phi} = \frac{1}{g_{YM}^2} \int d^dx \sqrt{-g} \left(\frac{d\,\Delta_I}{2\,r^2} \,\mathrm{Tr}\phi_I \phi^I \right) , \quad [I \text{ is summed over}]$$

  Δ_I is the analog of the dimension for ϕ_I.
- Need further terms to preserve the supersymmetry.

Conformal Killing spinors

- No covariantly constant spinors on the sphere
- There are conformal Killing spinors (CKS)

$$\nabla_\mu \epsilon = \tilde{\Gamma}_\mu \tilde{\epsilon}\,, \qquad \nabla_\mu \tilde{\epsilon} = -\frac{1}{4r^2}\Gamma_\mu \epsilon\,.$$

$\tilde{\epsilon}_\alpha$ has opposite 10D chirality to ϵ^α.

- 32 independent solutions for $d \leq 10$:
- Reduce to 16 spinors by further imposing $(\beta = \frac{1}{2r})$

$$\tilde{\epsilon} \;=\; \beta\Lambda\epsilon\,, \qquad \tilde{\Gamma}^\mu\Lambda = -\tilde{\Lambda}\Gamma^\mu \qquad \tilde{\Lambda}\Lambda = 1$$

$$d \neq 4 \text{ also need } \Lambda^T = -\Lambda \quad \Longrightarrow \quad \Lambda = \Gamma^8\tilde{\Gamma}^9\Gamma^0$$

- This construction can be used for spheres up to $d = 7$.

Modified SUSY Transformations

- SUSY transfs. need to be modified

$$\begin{aligned} \delta_\epsilon A_M &= \epsilon\,\Gamma_M \Psi \\ \delta_\epsilon \Psi &= \tfrac{1}{2}\Gamma^{MN} F_{MN}\,\epsilon + \frac{\alpha_I}{2}\Gamma^{\mu I}\phi_I \nabla_\mu\,\epsilon \end{aligned}$$

$$\alpha_A = \frac{4(d-3)}{d}\,, \qquad A = 8,9,0\,, \qquad \alpha_i = \frac{4}{d}\,, \quad i = d+1,\dots 7$$

- Set $\Delta_A = \alpha_A$, $\Delta_i = 2(d-2)/d$.
- Complete maximally SUSY action:

$$\begin{aligned} S &= -\frac{1}{g_{YM}^2}\int d^d x\sqrt{-g}\,\mathrm{Tr}\Big[\big(\tfrac{1}{2}F_{MN}F^{MN} - \Psi \not{D}\Psi \\ &\quad + \frac{2(d-3)}{r^2}\mathrm{Tr}\phi^A\phi_A + \frac{(d-2)}{r^2}\mathrm{Tr}\phi^i\phi_i + \frac{(d-4)}{2r}\Psi\Lambda\Psi \\ &\quad - \frac{4}{r}(d-4)\mathrm{Tr}(\phi^0[\phi^8,\phi^9])\Big]\,. \end{aligned}$$

- Preserves 16 susys but R-symmetry explicitly broken $(d \neq 4,7)$:

$$SO(1,9-d) \to SO(1,2)\times SO(7-d)$$

$d \leq 5$, can reduce to 8 supersymmetries

- If $d \leq 5$, $\epsilon = +\Gamma^{6789}\epsilon$; $\Psi \to \psi + \chi$

$$\psi = +\Gamma^{6789}\psi \qquad \chi = -\Gamma^{6789}\chi$$

- Vector multiplet: A_μ, ψ, ϕ^I, $I = 0, d+1 \ldots 5$
- Hypermultiplet: χ, ϕ^I, $I = 6 \ldots 9$
- Supersymmetry is less restrictive $\Rightarrow$ hypermultiplet mass m

$$\begin{aligned}
\alpha_I &\Rightarrow \frac{2(d-2)}{d} + \frac{4i\sigma_I\, m\, r}{d} \\
\Delta_I &\Rightarrow \frac{2}{d}\left(mr(mr + i\sigma_I) + \frac{d(d-2)}{4}\right) \qquad I = 6 \ldots 9\,. \\
\sigma_I &= +1\,(-1) \qquad I = 6, 7\ (8, 9)
\end{aligned}$$

$$\frac{(d-4)}{2r}\mathrm{Tr}\Psi\Lambda\Psi \Rightarrow \left(\frac{(d-4)}{2r}\mathrm{Tr}\psi\Lambda\psi - i\, m\, \mathrm{Tr}\,\chi\Lambda\chi\right)$$

Cubic term is also modified.

$d \leq 3$, can reduce to 4 supersymmetries

- If $d \leq 3$, $\epsilon = +\Gamma^{6789}\epsilon$, $\epsilon = +\Gamma^{4589}\epsilon$; $\psi \to \psi' + \chi^{(1)}$; $\chi \to \chi^{(2)} + \chi^{(3)}$

$$\begin{aligned} \psi' &= +\Gamma^{6789}\psi' & \psi' &= +\Gamma^{4589}\psi' \\ \chi^{(1)} &= +\Gamma^{6789}\chi^{(1)} & \chi^{(1)} &= -\Gamma^{4589}\chi^{(1)} \\ \chi^{(2)} &= -\Gamma^{6789}\chi^{(2)} & \chi^{(2)} &= +\Gamma^{4589}\chi^{(2)} \\ \chi^{(3)} &= -\Gamma^{6789}\chi^{(3)} & \chi^{(3)} &= -\Gamma^{4589}\chi^{(3)} \end{aligned}$$

- Vector multiplet: A_μ, ψ', ϕ^I, $I = 0, d+1 \dots 3$
- Chiral multiplets: $\chi^{(\ell)}$, $\phi^{2\ell+2} \pm i\phi^{2\ell+3}$, $\ell = 1, 2, 3$
- Susy less restrictive: Chiral multiplet masses m_ℓ
- Supersymmetry requires

$$\frac{1}{2r}(d-4) + i(m_1 + m_2 + m_3) = 0\,.$$

- The m_ℓ are "real" masses.

A constraint for real masses Gorantis, Naseer and JAM

Consider flat space:

- In 3d $\mathcal{N} = 2$ susy, real masses appear in the superalgebra

$$\{\mathcal{Q}_\alpha, \bar{\mathcal{Q}}_\beta\} = i\,\sigma^\mu_{\alpha\beta}\,P_\mu + i\,m^R\,\varepsilon_{\alpha\beta}$$

- MSYM has a term in the superpotential $\mathrm{Tr}(Q_i Q_j Q_k)\varepsilon^{ijk}$. Acting with $\{\mathcal{Q}_\alpha, \bar{\mathcal{Q}}_\beta\}$ on this gives

$$\sim (m_1 + m_2 + m_3)\,.$$

$\Longrightarrow$ Supersymmetry requires the sum to be zero.

Localization

- Localizing the (off-shell) action ⇒ Modify the path integral to

$$Z(t) = \int \mathcal{D}\Phi e^{-S-t\,QV} \,,$$

Q is a fermionic symmetry generator. QV positive definite.

- If $Q^2 V = 0$ then $dZ/dt = 0$
- Take $t \to \infty$, fields localize onto $QV = 0$.

$$\mathcal{Z} = \sum_{k \in \text{fixed loci}} \int \mathcal{D}\Phi_0 e^{-S_k} \text{Det}_k$$

- For Q choose δ_ϵ, while

$$V = \int d^d x \sqrt{-g}\, \Psi\, \overline{\delta_\epsilon \Psi} \,.$$

Bosonic part of $\delta_\epsilon V$

$$\delta_\epsilon V \Big|_{bos} = \int d^d x \sqrt{-g}\, \mathrm{Tr}(\delta_\epsilon \Psi\, \overline{\delta_\epsilon \Psi}) \,.$$

Localization (continued)

- Fixed locus: (zero instanton sector)

$$\nabla_\mu \phi_0 = 0, \qquad \phi_I = 0 \qquad I \neq 0$$

- Substitute the fixed locus into the action,

$$\begin{aligned} S_{fp} &= \frac{1}{g_{YM}^2} \int d^d x \sqrt{-g}\, \frac{(d-1)(d-3)}{r^2} \mathrm{Tr}(\phi_0 \phi_0) \\ &= \frac{r^{d-4}(d-1)(d-3) S_d}{g_{YM}^2} \mathrm{Tr}\sigma^2 \qquad \sigma = r\phi_0 \end{aligned}$$

- Does not change when breaking the susy's, since only the vector multiplet field ϕ_0 contributes.

One-loop determinants

- Compute quadratic fluctuations about fixed point locus
- Generalization of 5D for 8 susy's (Kim & Kim) and 3D for 4 susy's (Kapustin, Willet & Yaakov)
- Strategy is to find sets of basis vectors for bosons and fermions
- Directly doing 16 susy's is harder this way (But we can find the results indirectly)

One-loop determinants (8 susys) (JAM; Gorantis, JAM, & Naseer)

Vector multiplet:

$$\frac{Det_{f,v}}{Det_{b,v}} = \prod_{\gamma}\prod_{k=1}^{\infty}(k+i\langle\gamma,\phi_0\rangle)^{\frac{\Gamma(k+d-2)}{\Gamma(k+1)\Gamma(d-2)}}\prod_{k=0}^{\infty}(k+d-2+i\langle\gamma,\phi_0\rangle)^{\frac{\Gamma(k+d-2)}{\Gamma(k+1)\Gamma(d-2)}}$$

Hypermultiplet ($\mu = m\,r$):

$$\frac{Det_{f,h}}{Det_{b,h}} = \prod_{\gamma}\prod_{k=0}^{\infty}\left[\left(k+i\langle\gamma,\sigma\rangle+i\mu+\frac{d-2}{2}\right)\left(k-i\langle\gamma,\sigma\rangle-i\mu+\frac{d-2}{2}\right)\right]^{-\frac{\Gamma(k+d-2)}{\Gamma(k+1)\Gamma(d-2)}}$$

- For $d \le 5$ we can combine a vector multiplet with an adjoint hyper with mass $\mu = i(d-4)/2$ to give 16 supercharges.
  - For general d (after shifting some k) the combined determinant factor is (including the Vandermonde determinant)

$$\prod_{\gamma>0}\prod_{k=0}^{\infty}\left(\frac{k^2+\langle\gamma,\sigma\rangle^2}{(k+d-3)^2+\langle\gamma,\sigma\rangle^2}\right)^{\frac{\Gamma(k+d-3)}{\Gamma(k+1)\Gamma(d-3)}}$$

- Analytically continuing to $d > 5$ agrees with $d = 6$ and $d = 7$ cases. JAM, Zabzine

One-loop determinants (4 SUSYs) (JAM; Gorantis, JAM, & Naseer)

Adjoint chiral multiplet with mass parameter $\mu = mr$

$$\frac{Det_{f,c}}{Det_{b,c}} = \prod_{\gamma} \prod_{k=0}^{\infty} \left[\frac{(k - i\langle\gamma,\sigma\rangle - i\mu + \frac{d}{2})}{(k + i\langle\gamma,\sigma\rangle + i\mu + \frac{d-2}{2})} \right]^{\frac{\Gamma(k+d-1)}{\Gamma(k+1)\Gamma(d-1)}}$$

Vector multiplet:

$$\frac{Det_{f,v}}{Det_{b,v}} = \prod_{\gamma} \frac{\prod_{k=1}^{\infty} (k - i\langle\gamma,\sigma\rangle)^{\frac{\Gamma(k+d-1)}{\Gamma(k+1)\Gamma(d-1)}}}{\prod_{k=0}^{\infty} (k + d - 1 + i\langle\gamma,\sigma\rangle)^{\frac{\Gamma(k+d-1)}{\Gamma(k+1)\Gamma(d-1)}}}$$

Mass deformed maximal SYM for $SU(N)$

- Perturbative partition function

$$\mathcal{Z} = \int \prod_i^N d\sigma_i \prod_\gamma \langle \gamma, \sigma \rangle \; e^{-\frac{4\pi^2 r^{d-4} S_{d-4}}{g_{YM}^2} \mathrm{Tr}\sigma^2} \qquad \mu_\ell = m_\ell\, r$$

$$\prod_\gamma \prod_{k=0}^{\infty} \left[\frac{(k - i\langle \gamma, \sigma \rangle)}{(k+d-1+i\langle \gamma, \sigma \rangle)} \prod_{\ell=1}^{3} \frac{(k - i\langle \gamma, \sigma \rangle - i\mu_\ell + \frac{d}{2})}{(k + i\langle \gamma, \sigma \rangle + i\mu_\ell + \frac{d-2}{2})} \right]^{\frac{\Gamma(k+d-1)}{\Gamma(k+1)\Gamma(d-1)}}$$

- What happens when we analytically continue to $d = 4$? Not known how to localize but there is an available supergroup: $Osp(1|4)$

$\mathcal{N} = 4$ Mass corr.

$$\mathcal{Z}_4 = \int \prod_i^N d\sigma_i \; \boxed{e^{-\frac{8\pi^2}{g_{YM}^2} \sum_i \sigma_i^2} \prod_{i<j} (\sigma_i - \sigma_j)^2} \; \boxed{\prod_{i \neq j} \prod_{k=0}^{\infty} \prod_{\ell=1}^{3} Z_k(\sigma_i - \sigma_j, \mu_\ell)}$$

$$Z_k(\sigma, \mu_\ell) \equiv \left[\frac{(k - i\sigma - i\mu_\ell + 2)(k + i\sigma + 1)}{(k + i\sigma + i\mu_\ell + 1)(k - i\sigma + 2)} \right]^{\frac{(k+1)(k+2)}{2}}$$

Mass deformed maximal SYM in $d = 4$

- Assume $N \gg 1$: evaluate by saddle point.
- At strong coupling the saddle point in general has $|\sigma_i - \sigma_j| \gg 1$
- The correction term is divergent and needs to be regularized

$$\sum_{j=1}^{3}\sum_{k=0}^{\infty} \log(Z_k(\sigma-\sigma', \mu_j)_{\mathrm{reg}}) \sim -\frac{i}{2}(\sigma-\sigma')^2 \log(\sigma-\sigma')^2(\mu_1+\mu_2+\mu_3)$$
$$+\frac{1}{4}\log(\sigma-\sigma')^2(\mu_1^2+\mu_2^2+\mu_3^2) - \frac{i}{6}\log(\sigma-\sigma')^2(\mu_1^3+\mu_2^3+\mu_3^3)$$

- Supersymmetry requires

$$\frac{d-4}{2} + i(\mu_1+\mu_2+\mu_3) = 0$$

$\Rightarrow (\sigma-\sigma')^2 \log(\sigma-\sigma')^2$ term is absent.

Mass deformed MSYM in $d = 4$ (JAM; Gorantis, JAM, & Naseer)

- Situation similar to $\mathcal{N} = 2^*$ Russo & Zarembo; Chen, Gordon & Zarembo
- Saddle point equation at strong coupling is approximately:

$$\frac{16\pi^2}{\lambda}\sigma = 2\fint d\sigma' \rho(\sigma') \frac{1 + \frac{1}{2}(\mu_1^2+\mu_2^2+\mu_3^2) - \frac{i}{3}(\mu_1^3+\mu_2^3+\mu_3^3))}{\sigma - \sigma'}$$

Saddle point equation for a Gaussian matrix model

- Free energy for $\mathcal{N} = 4$: $F \approx -\frac{N^2}{2}\log\lambda$

$$F \approx -\frac{N^2}{2}\left(1 + \frac{1}{2}(\mu_1^2+\mu_2^2+\mu_3^2) - \frac{i}{3}(\mu_1^3+\mu_2^3+\mu_3^3)\right) \times \log\left(\lambda\left(1 + \frac{1}{2}(\mu_1^2+\mu_2^2+\mu_3^2) - \frac{i}{3}(\mu_1^3+\mu_2^3+\mu_3^3)\right)\right)$$

Use $\mu_1^3+\mu_2^3+\mu_3^3 = -3\mu_1\mu_2\mu_3$

Expand [dropping quadratic and cubic terms (scheme dep.)]

$$\delta F \approx -N^2\left(\frac{1}{16}(\mu_1^2+\mu_2^2+\mu_3^2)^2 - \frac{i}{4}(\mu_1^2+\mu_2^2+\mu_3^2)\mu_1\mu_2\mu_3 - \frac{1}{96}(\mu_1^2+\mu_2^2+\mu_3^2)^3 - \frac{1}{4}(\mu_1\mu_2\mu_3)^2 + \mathrm{O}(\mu^7)\right),$$

Mass deformed maximal SYM in $d = 4$

Comments:

- Caveat: $N = 1^*$ dimensionally reduced to 3D has complex masses (mass terms in the superpotential). Can we replace m_R with m_C?
- Can't directly compare to a numerical 2016 SUGRA calculation of Bobev, Elvang, Kol, Olson & Pufu. To simplify the analysis they choose $\mu_2 = \mu_3 = 0$ or $\mu_1 = \mu_2 = \mu_3$. Neither case has $\mu_1 + \mu_2 + \mu_3 = 0$.

Evidence:

- Terms that appear in the free energy are consistent with the symmetry analysis of Bobev et. al.: Only combinations of $(\mu_1\bar{\mu}_1)^n + (\mu_2\bar{\mu}_2)^n + (\mu_3\bar{\mu}_3)^n$, $\mu_1\mu_2\mu_3$, or $\bar{\mu}_1\bar{\mu}_2\bar{\mu}_3$ can appear.
- Take $|\mu_i| \to \infty \implies$ reduces to $\mathcal{N} = 1$ pure SYM. One can read off the holomorphic β-function with $|\mu_j|$ acting as a cutoff. ✓
- F is complex – Not surprising as $\mathcal{N} = 1^*$ on S^4 is not reflection positive. Festuccia & Seiberg

Summary

- We have given a uniform description for putting MSYM and its mass deformed cousins on S^d.
- Continuing 4 SUSY case to 4d leads to a possible form for $\mathcal{N} = 1^*$ that passes several tests.

Some questions

- Can we find an analytic continuation to study 4d $\mathcal{N} = 1$ superconformal theories? Free energy is scheme dependent
- Are there more general index theorems to derive the determinant factors directly?
- Can we analytically continue nonperturbative contributions?

Спасибо

Other stuff

Alternative 2D vector multiplet Lagrangian

- In 2 dimensions we can choose a different modification of the flat space Lagrangian.
- $(2,2)$ vector multiplet $[A_\mu, \phi^0, \phi^3, \psi]$:

$$\delta_\epsilon \psi = \tfrac{1}{2}\Gamma^{MN} F_{MN}\,\epsilon + \frac{\alpha_I}{2}\Gamma^{\mu I}\phi_I \nabla_\mu\,\epsilon$$

$$M, N = 0\ldots 3, \quad I, J = 0, 3 \qquad \alpha_3 = 2 \qquad \alpha_0 = 0$$

- The vector multiplet Lagrangian is modified to

$$\begin{aligned}\mathcal{L}_{ss} &= \frac{1}{g_{YM}^2}\mathrm{Tr}\Big[\tfrac{1}{2}F_{MN}F^{MN} - \psi \not{D}\psi + \frac{1}{r^2}\mathrm{Tr}\phi^3\phi^3 - \frac{2}{r}F_{12}\phi^3\Big] \\ &= \frac{1}{g_{YM}^2}\mathrm{Tr}\Big[\left(F_{12} - \frac{\phi_3}{r}\right)^2 + D_\mu\phi_I D^\mu\phi^I + \tfrac{1}{2}[\phi_I, \phi_J][\phi^I, \phi^J] - \psi\not{D}\psi\Big]\end{aligned}$$

  This is the Q-exact Lagrangian
- No change in chiral multiplet Lagrangian

One-loop determinants (8 SUSYs)

- Instead of index theorems we will generalize Kim & Kim for 8 susy's and Kapustin, Willet & Yaakov for 4 susy's
- Directly doing 16 susy's is harder this way
- In this talk we only consider mass deformations of maximal SYM
- Fluctuations about fixed point locus (Bosons)

$$\mathcal{L}^{\text{bos}}_{\text{vm}} = A^{\tilde{M}}\, \mathcal{O}_{\tilde{M}}^{\tilde{N}}\, A_{\tilde{N}} - [A_{\tilde{M}}, \phi^0_{cl}][A^{\tilde{M}}, \phi^0_{cl}] \qquad \tilde{M} = 1 \dots 5$$

$$\mathcal{O}_{\tilde{M}}^{\tilde{N}} = -\delta_{\tilde{M}}^{\tilde{N}} \nabla^2 + \gamma_{\tilde{M}}^{\tilde{N}} + 2\beta(d-3)\epsilon\Gamma_{\tilde{M}}{}^{\nu\tilde{N}89}\epsilon\nabla_\nu .$$

$$\gamma_{\tilde{M}}^{\tilde{N}} = 4\beta^2 \begin{pmatrix} (d-1)\,\delta_\mu^\nu & 0 \\ 0 & \delta_i^j \end{pmatrix} .$$

$$\mathcal{L}_{a,b}(\mu) = \sum_{i=a,b} \left[\phi_i \left(-\nabla^2 + \beta^2 (d-2+2i\mu)^2\right)\phi_i - [\phi^0_{cl}, \phi_i][\phi^0_{cl}, \phi_i]\right]$$
$$- 4\beta\,(1-2i\mu)\,\phi_a v^\mu \nabla_\mu \phi_b \qquad \mu = m\, r$$

$$\mathcal{L}^{\text{bos}}_{\text{hm}} = \mathcal{L}_{67}(\mu) + \mathcal{L}_{98}(-\mu) .$$

One-loop determinants (8 SUSYs)

- Fermion fluctuations

$$\mathcal{L}^{\text{ferm}}_{\text{vm}} = (\psi \nabla\!\!\!/ \psi) - \frac{1}{2}(d-3)\beta v^{\tilde{M}} \left(\psi \Gamma^0 \tilde{\Gamma}_{\tilde{M}} \Lambda \psi\right) - v^0 \left(\psi \Gamma^0 D_0 \psi\right)$$
$$- \frac{1}{4}(d-3)\beta \left(\epsilon \tilde{\Gamma}^{\tilde{M}\tilde{N}} \Lambda \epsilon\right) \left(\psi \Gamma^0 \Gamma_{\tilde{M}\tilde{N}} \psi\right) + \frac{d-1}{2} (\psi \Lambda \psi) \,.$$

$$\mathcal{L}^{\text{ferm}}_{\text{hm}} = (\chi \nabla\!\!\!/ \chi) + \left(\chi \Gamma^0 D_0 \chi\right) - \frac{1}{2}\beta \left(\epsilon \tilde{\Gamma}^{\tilde{M}\tilde{N}} \Lambda \epsilon\right) \left(\chi \Gamma^0 \Gamma_{\tilde{M}\tilde{N}} \chi\right)$$
$$+ 2i\mu\beta v^{\tilde{N}} \left(\chi \Gamma^0 \tilde{\Gamma}_{\tilde{N}} \Lambda \chi\right) .$$

One-loop determinants (8 SUSYs)

Following Kim and Kim we introduce basis vectors for the vm bosons:

$$\mathcal{A}^1_{\tilde{M}} = v_{\tilde{M}} Y^k_m + c^1 \nabla_{\tilde{M}} Y^k_m$$
$$\mathcal{A}^2_{\tilde{M}} = \epsilon \Gamma_{\tilde{M}}{}^{\mu} \Lambda \epsilon \nabla_{\mu} Y^k_m + c^2 \nabla_{\tilde{M}} Y^k_m$$
$$\mathcal{A}^3_{\tilde{M}} = \epsilon \Gamma_{\tilde{M}}{}^{\mu} \Gamma^{079} \epsilon \nabla_{\mu} Y^k_m$$
$$\mathcal{A}^4_{\tilde{M}} = \epsilon \Gamma_{\tilde{M}}{}^{\mu} \Gamma^{069} \epsilon \nabla_{\mu} Y^k_m$$

$$v^{\tilde{M}} v_{\tilde{M}} = 1 \qquad v^{\mu} \nabla_{\mu} Y^k_m = 2im\beta Y^k_m,$$

$$(\mathcal{O}\mathcal{A})^1_{\tilde{M}} = 4\beta^2 \left[k(k+d-1) + (d-1)^2\right] \mathcal{A}^1_{\tilde{M}} - 4\beta \left(\frac{d-1}{2}\right) \mathcal{A}^2_{\tilde{M}}$$
$$(\mathcal{O}\mathcal{A})^2_{\tilde{M}} = -4\beta^3 \, 2(d-1)k(k+d-1)\mathcal{A}^1_{\tilde{M}} + 4\beta^2 k(k+d-1)\mathcal{A}^2_{\tilde{M}}.$$
$$\mathcal{O}\mathcal{A}^I_{\tilde{M}} = 4\beta^2 \left[k(k+d-1) + (d-2)^2 \mathcal{A}^I_{\tilde{M}} + i\epsilon^{IJ}(d-3)m\mathcal{A}^J_{\tilde{M}}\right], \qquad I, J = 3, 4$$

eigenvalues:

$$4\beta^2 k^2, \qquad k \geq 2,\ m \neq \pm k \qquad \text{and} \qquad 4\beta^2 (k+d-1)^2.$$

$$4\beta^2 \left[k(k+d-1) + (d-2)^2 \pm (d-3)m\right]. \qquad (+)\ m \neq +k \quad (-)\ m \neq -k$$

One-loop determinants (8 SUSYs)

Basis spinors for vm fermions

$$\mathcal{X}^1 = Y^k_m \eta \qquad \mathcal{X}^2 = \tilde{\Gamma}_0 \Gamma^{\tilde{M}} \hat{\nabla}_{\tilde{M}} Y^k_m \eta$$
$$\tilde{\mathcal{X}}^1 = Y^k_m \tilde{\eta} \qquad \tilde{\mathcal{X}}^2 = \tilde{\Gamma}_0 \Gamma^{\tilde{M}} \hat{\nabla}_{\tilde{M}} Y^k_m \tilde{\eta}$$

$$\eta = (1 + i\Gamma^{67})\epsilon \qquad \tilde{\eta} = \left(\Gamma^{68} + i\Gamma^{69}\right)\epsilon$$

$$\Gamma^{89}\eta = i\eta, \quad \Gamma^{89}\tilde{\eta} = -i\Gamma^{89}\tilde{\eta}, \quad v^{\tilde{M}}\Gamma_{\tilde{M}}\eta, \quad v^{\tilde{M}}\Gamma_{\tilde{M}}\tilde{\eta} = \tilde{\eta},$$

For $k \geq 1$, $m \neq k$ determinants:
$4\beta^2 k(k + d - 1), \quad 4\beta^2 \left[k(k + d - 1) + (d - 2)^2 + m(d - 3)\right]$
For $k \geq 0$, $m = k$: eigenvalues
$2i\beta(k + d - 1)$, $2i\beta(k + d - 2)$

$$\frac{Det_{f,v}}{Det_{b,v}} = \prod_{\beta \in \mathrm{roots}} \prod_{k=1}^{\infty} (k + i\langle\beta, \phi_0\rangle)^{D(k,k,d)} \prod_{k=0}^{\infty} (k + d - 2 + i\langle\beta, \phi_0\rangle)^{D(k,k,d)}$$

where we have included the contribution of the constant bosonic field ϕ_0