The Possible Lost Anisotropy Of The Unruh Effect

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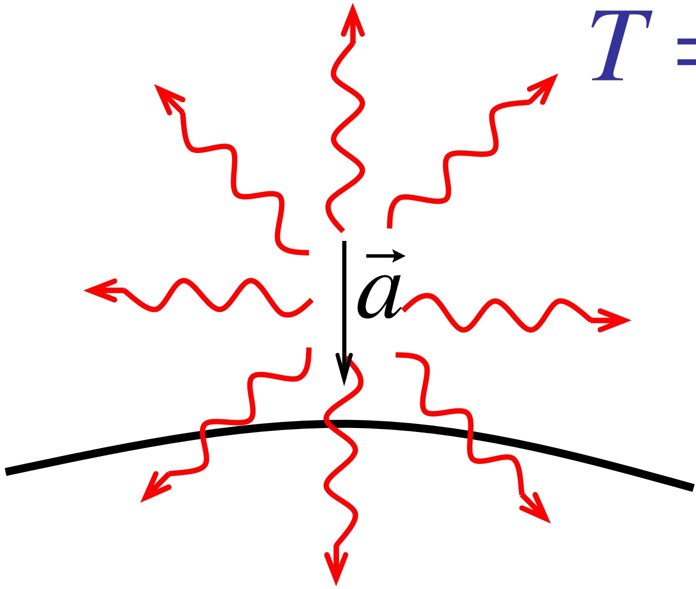
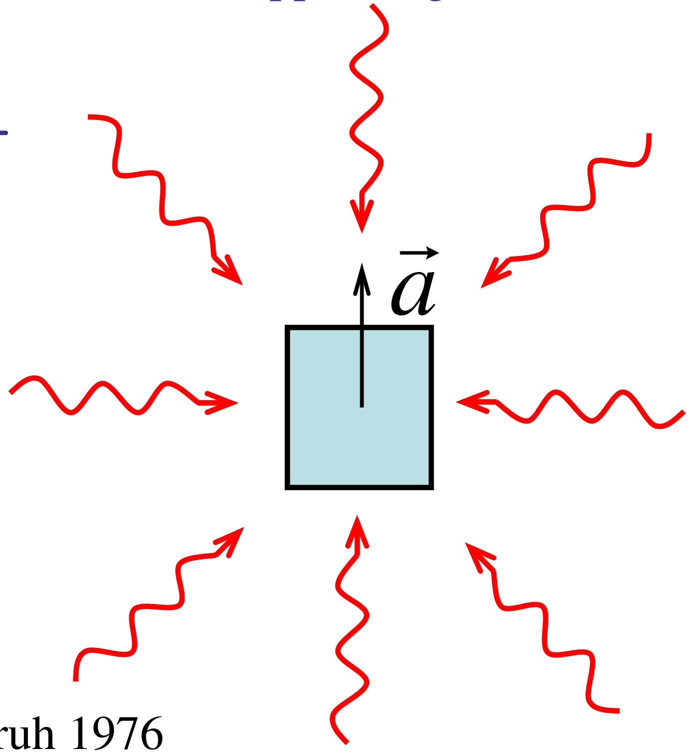
Introduction

More than forty years ago Hawking (1974) and Unruh (1976) discovered theoretically the radiation arising at change of vacuum under the influence of real or apparent gravitation field

$$T = \frac{\hbar a}{2\pi c k_B}$$



Hawking 1974



Unruh 1976

These effects are very subtle:

•for the Black Hole of solar mass the temperature is about $6 \cdot 10^{-8}$ K

•for the acceleration of 1g the temperature is about $4 \cdot 10^{-19}$ K

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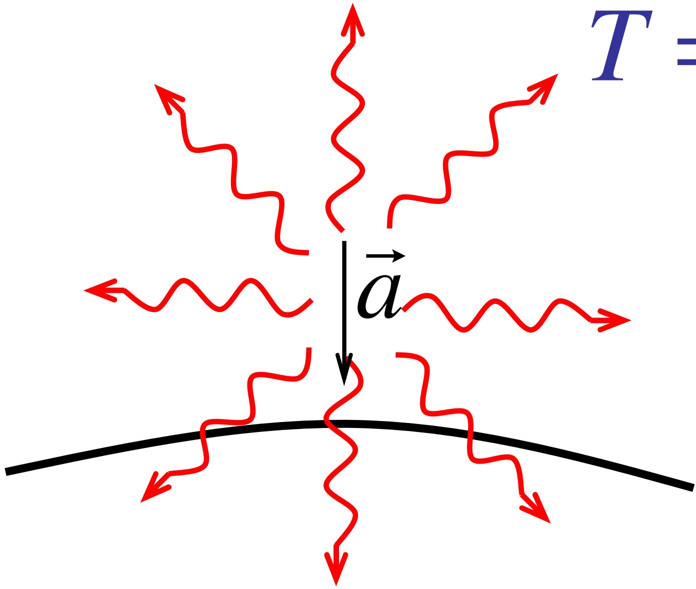

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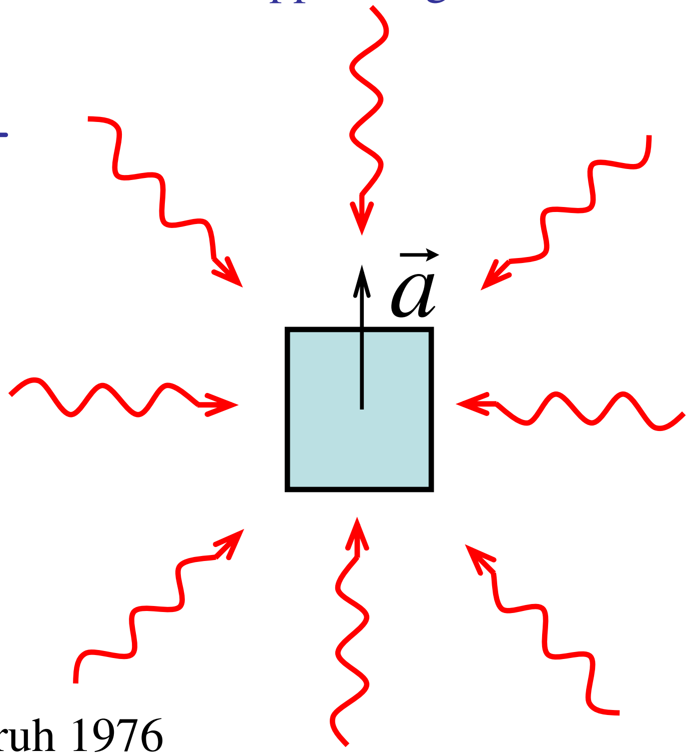

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•for the Black Hole of solar mass the temperature is about $6 \cdot 10^{-8}$ K

•for the acceleration of 1g the temperature is about $4 \cdot 10^{-19}$ K

Despite the presence of preferred direction, ***a***,
these effects are believed to be isotropic!

Previous investigations of anisotropy of the Unruh radiation

- Gerlach, Phys. Rev. D, 27, 2310 (1983)
- Grove and Ottewill, Class. and Quant. Grav., 2, 373, (1985)

Distribution of the Unruh radiation is totally isotropic

- Hinton et al., Physics Letters B, 120, 88 (1983)
- Israel and Nester, Physics Letters A, 98, 329 (1983)
- Sanchez, Physics Letters A, 112, 133 (1985)

Distribution of the Unruh radiation is anisotropic in the following sense: detector response is anisotropic due to aberration effect

Kolbenstvedt [Physics Letters A, 122, 292 (1987); Phys. Rev. D, 38, 1118 (1988)] is cautious in conclusions and points to the possible dependence of final distribution of observed Unruh radiation on detector model (in particular, notable size of detector).

- It seems that full consensus on isotropy/anisotropy of the Unruh radiation has not achieved.

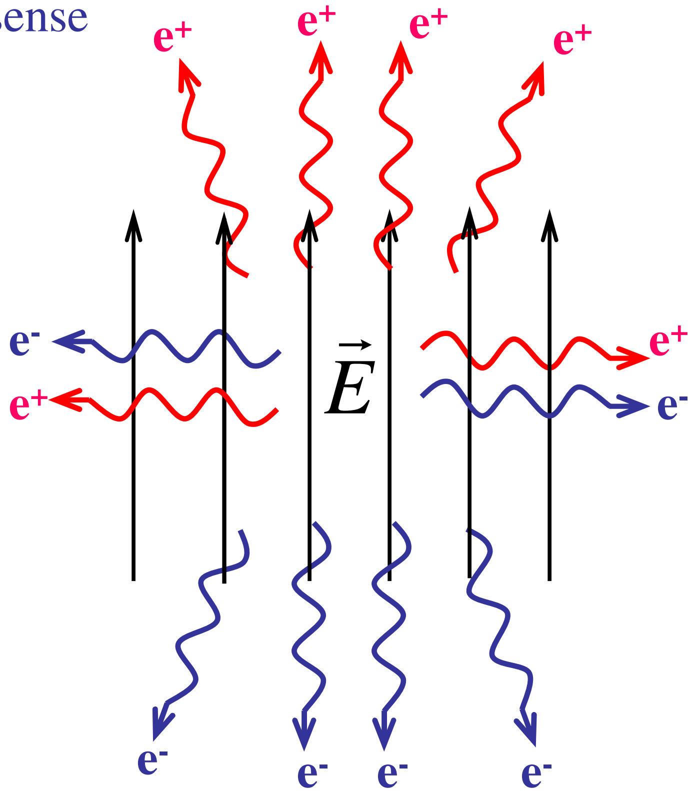
- Commonly accepted point of view: the Unruh radiation is isotropic.

It seems that more fundamental reasons than considered in previous papers should exist and distribution of the Unruh radiation should be anisotropic independently of detector model.

Origin of Doubts: the close-allied effects

The Schwinger effect:

Electron-positron pair production by electric field; analogy of Hawking radiation in some sense

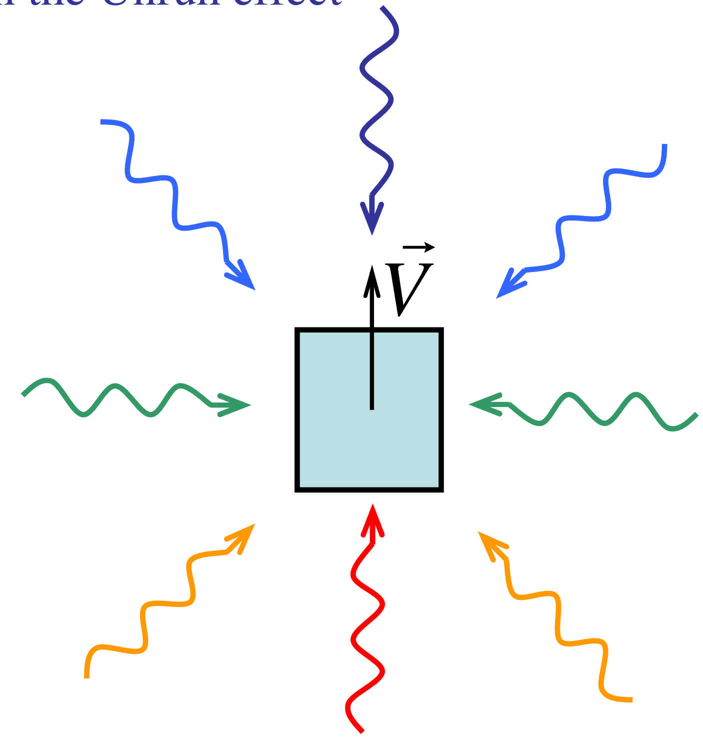


$$W \sim \exp\left(-\frac{\pi E_{cr}}{E}\left(1+\frac{p_{\perp}^{2}}{m_e^2 c^2}\right)\right)$$

e.g. Grib et al. 1994; Popov et al. 2016

The Doppler effect:

Change of quantum frequency due to observer motion; in tight connection with the Unruh effect



$$T \approx T_0\left(1+\frac{V}{c}\cos\theta\right)$$

Both these effects are significantly anisotropic!

Rindler space-time

$$ds^2 = \left(\frac{a\rho}{c}\right)^2 d\tau^2 - d\rho^2$$

a is the observer acceleration directed along $(\partial/\partial\rho)$

ρ is the Rindler coordinate, τ is the Rindler time

Rindler coordinates are related with the coordinates (t,x) in parent Minkowski space-time by the following transforms:

$$\tau = \frac{c}{a}\mathrm{arcth}\frac{ct}{x}\ , \qquad \rho = \sqrt{x^2 - c^2t^2}$$

The inverse transforms are:

$$t = \frac{\rho}{c}\sinh\frac{a\tau}{c}\ , \qquad x = \rho\cosh\frac{a\tau}{c}$$

The object with acceleration, a, has constant position $\rho_0 = c^2/a$ in Rindler space

Massless scalar field in (1+1)D Rindler and Minkowski space-times

$$\hat{\Psi} = \int_{-\infty}^{+\infty} \left(\hat{a}_{\kappa}^{R} \Psi_{\kappa}^{R} + \hat{b}_{\kappa}^{R\dagger} \Psi_{\kappa}^{R*} \right) d\kappa = \int_{-\infty}^{+\infty} \left(\hat{a}_{k}^{M} \Psi_{k}^{M} + \hat{b}_{k}^{M\dagger} \Psi_{k}^{M*} \right) dk$$

where $\hat{a}_{\kappa}^{R}$ is the particle annihilation operator in mode κ for Rindler space, $\hat{b}_{\kappa}^{R\dagger}$ is the anti-particle creation operator in mode κ for Rindler space, $\Psi_{\kappa}^{R}(\tau,\rho)$ are the quantization modes in Rindler space, $\hat{a}_{k}^{M}$ is the particle annihilation operator in mode k for Minkowski space, $\hat{b}_{k}^{M\dagger}$ is the anti-particle creation operator in mode k for Minkowski space, $\Psi_{k}^{M}(t,x)$ are the quantization modes in Minkowski space.

The KFG equations in Minkowski and Rindler space-times

$$\left(\frac{\partial^2}{c^2 \partial t^2} - \frac{\partial^2}{\partial x^2} \right) \Psi_{k}^{M} = 0 \qquad \left(\frac{c^2 \partial^2}{a^2 \partial \tau^2} - \rho \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} \right) \Psi_{\kappa}^{R} = 0$$

The eigen-modes Ψ_{κ}^{R} and Ψ_{k}^{M} are different!

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$$\Psi_\kappa^R(\tau,\rho) = C_\kappa^R \exp\left(-i\left[\omega\tau - \kappa\rho_0 \ln\left(\rho/\rho_0\right)\right]\right)$$

$$= C_\kappa^R \exp\left(i\frac{c^2\kappa}{a}\left[\ln\left(\frac{\rho}{\rho_0}\right) - \frac{a\tau}{c}\operatorname{sgn}(\kappa)\right]\right)$$

$$\Psi_k^M(t,x) = C_k^M \exp\left(-i\left[\epsilon t - kx\right]\right)$$

$$= C_k^M \exp\left(ik\rho\exp\left[-\frac{a\tau}{c}\operatorname{sgn}(k)\right]\right)$$

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"Running plane" waves in Rindler space-time

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$$= C_k^M \exp\left(ik\rho\exp\left[-\frac{a\tau}{c}\mathrm{sgn}\left(k\right)\right]\right)$$

The Bogolyubov transformations

$$\hat{a}_\kappa^R = \int_{-\infty}^{+\infty} \left(\alpha_{\kappa k}^* \hat{a}_k^M + \beta_{\kappa k}^* \hat{b}_k^{M\dagger} \right) dk$$

$$\hat{b}_\kappa^{R\dagger} = \int_{-\infty}^{+\infty} \left(\beta_{\kappa k} \hat{a}_k^M + \alpha_{\kappa k} \hat{b}_k^{M\dagger} \right) dk$$

$$\alpha_{\kappa k} = \left(\Psi_k^M, \Psi_\kappa^R \right) \qquad \beta_{\kappa k} = \left(\Psi_k^{M*}, \Psi_\kappa^R \right)$$

The KFG product [in Rindler space-time]

$$\left(\Psi, \Phi\right) = \frac{i}{2} \int_0^\infty \left(\Psi^* \partial_0 \Phi - \Phi \partial_0 \Psi^* \right) \frac{d\rho}{\rho}$$

The numbers of particles

$$\hat{N}_k^M = \left(\hat{a}_k^{M\dagger}\hat{a}_k^M + \hat{b}_k^{M\dagger}\hat{b}_k^M\right)$$

$$\hat{N}_\kappa^R = \left(\hat{a}_\kappa^{R\dagger}\hat{a}_\kappa^R + \hat{b}_\kappa^{R\dagger}\hat{b}_\kappa^R\right)$$

Mean number of particles

$$\left\langle 0_M \mid \hat{N}_k^M \mid 0_M \right\rangle = 0$$

But

$$\langle 0_M | \hat{N}_\kappa^R | 0_M \rangle = 2\int_{-\infty}^{+\infty} |\beta_{\kappa k}|^2 dk$$

The base of the Unruh effect

The key relations

$$\left\langle 0_M \mid \hat{N}^R_\kappa \mid 0_M \right\rangle = \frac{1}{2}\int_0^\infty \left| \int_0^\infty \left(\Psi^M_k \partial_0 \Psi^R_\kappa - \Psi^R_\kappa \partial_0 \Psi^M_k \right) \frac{d\rho}{\rho} \right|^2 dk$$

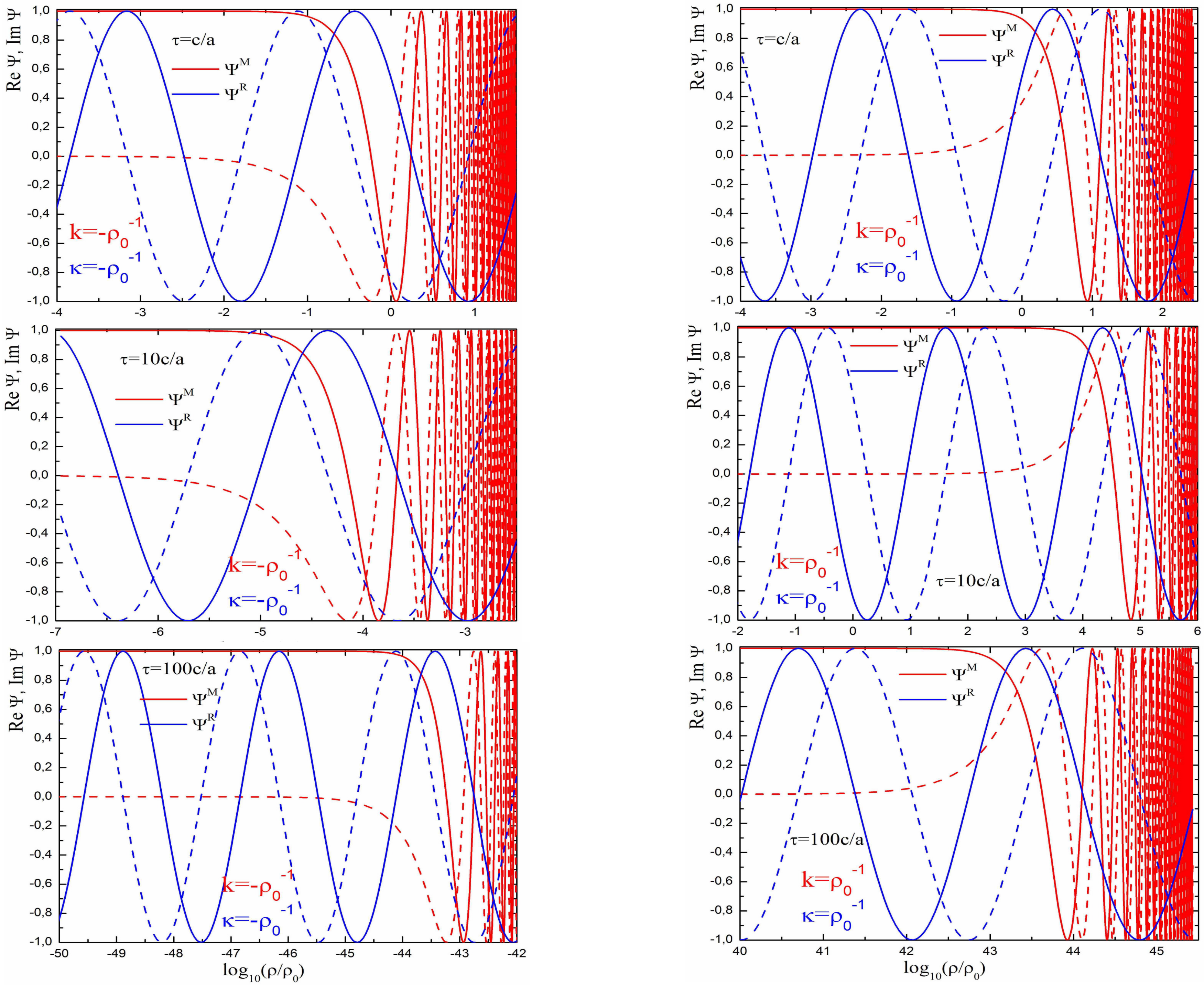
$$\Psi^R_\kappa(\tau,\rho) = C^R_\kappa \exp\left(i\frac{c^2\kappa}{a}\left[\ln\left(\frac{\rho}{\rho_0}\right) - \frac{a\tau}{c}\operatorname{sgn}(\kappa) \right] \right)$$

$$\Psi^M_k(t,x) = C^M_k \exp\left(ik\rho \exp\left[-\frac{a\tau}{c}\operatorname{sgn}(k) \right] \right)$$

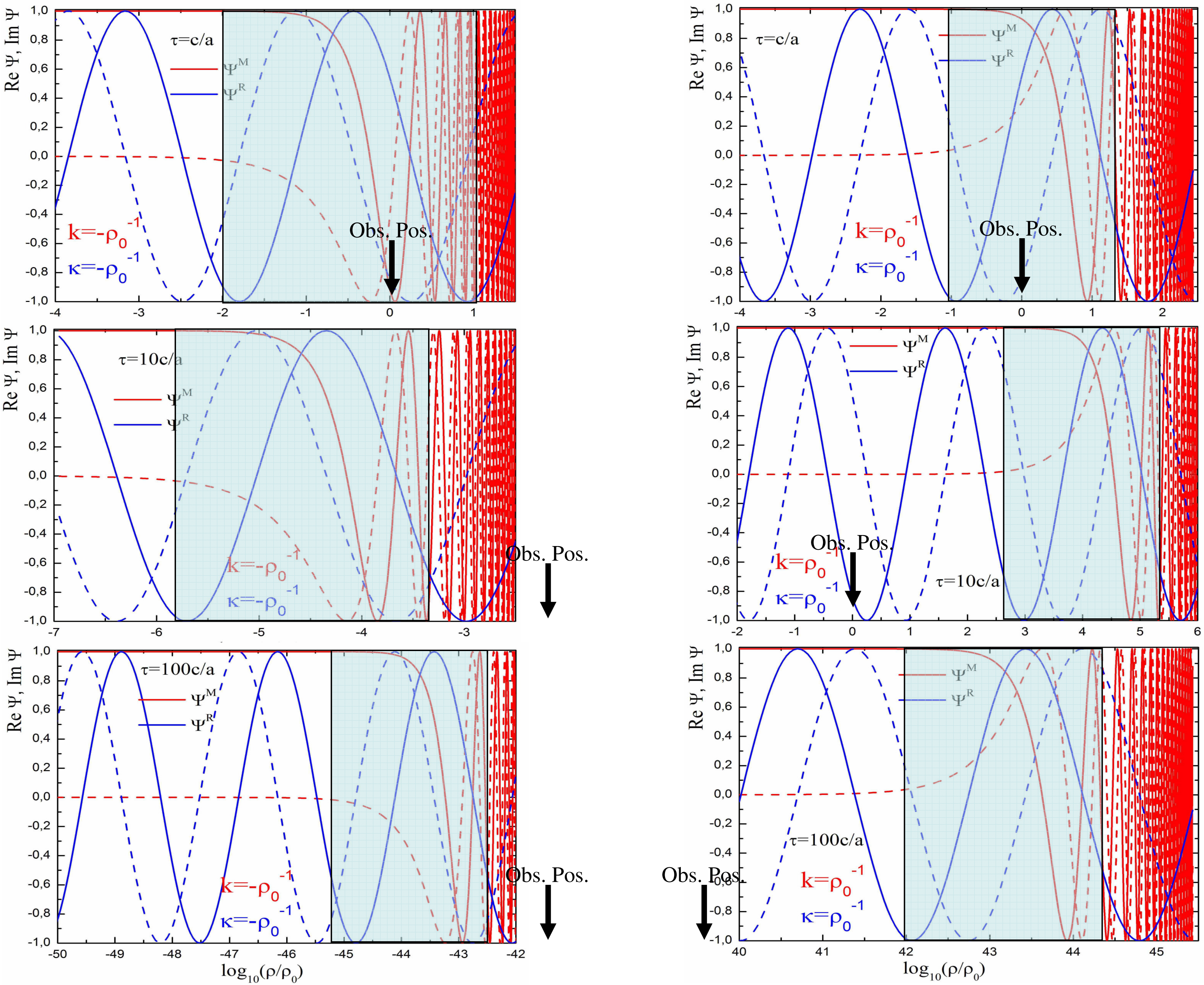
The Unruh result for scalar field

$$\left\langle 0_M \mid \hat{N}^R_\kappa \mid 0_M \right\rangle = 2\left(\exp\left(\frac{2\pi c^2 \mid \kappa \mid}{a} \right) - 1 \right)^{-1} = 2\left(\exp\left(\frac{\hbar c \mid \kappa \mid}{k_B T} \right) - 1 \right)^{-1}$$

It does not depend on quantum propagation direction, i.e. on sign of κ



Re Ψ, Im Ψ
τ=c/a
Ψ^M
Ψ^R
k=-ρ0^-1
κ=-ρ0^-1
k=ρ0^-1
κ=ρ0^-1
τ=10c/a
τ=100c/a
log10(ρ/ρ0)



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Obs. Pos.
log10(ρ/ρ0)

“Rindler” wave - vector operator $\hat{\kappa} = -i \dfrac{\rho}{\rho_0} \dfrac{\partial}{\partial \rho}$

Modes Ψ_κ^{R} are eigen - functions of operator $\hat{\kappa}$

$$\hat{\kappa} \Psi_\kappa^R = \kappa \Psi_\kappa^R$$

Modes Ψ_k^{M} are not eigen - functions of operator $\hat{\kappa}$,

but quasi wave - vector can be defined

$$q = \frac{\left(\hat{\kappa} \Psi_k^M\right)}{\Psi_k^M} = k \frac{\rho}{\rho_0} \exp\left[-\frac{a\tau}{c} \operatorname{sgn}(k)\right]$$

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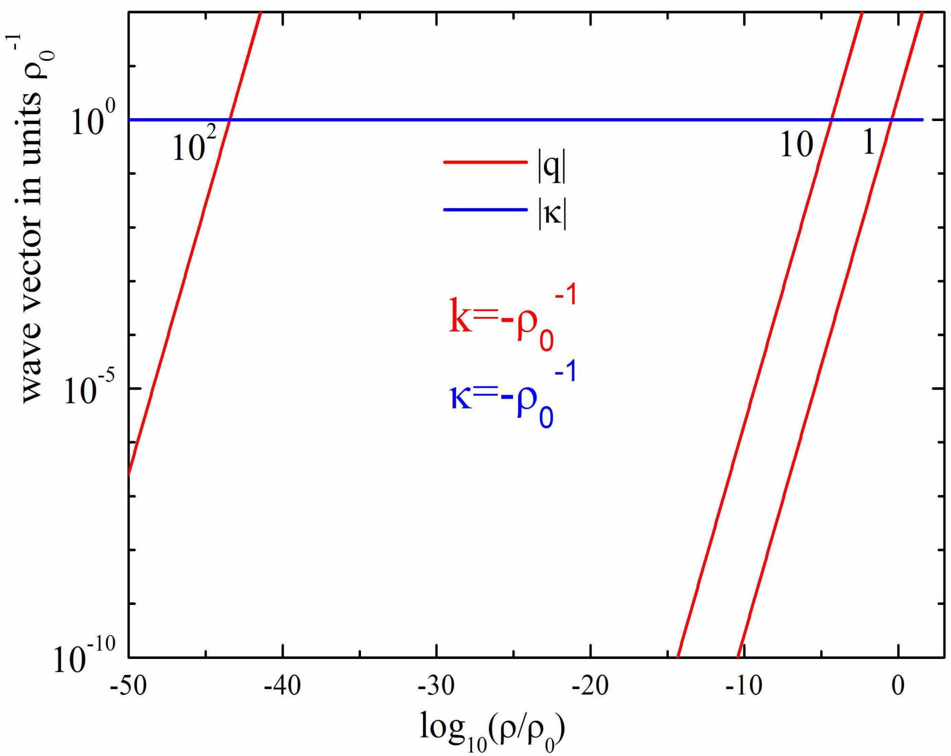
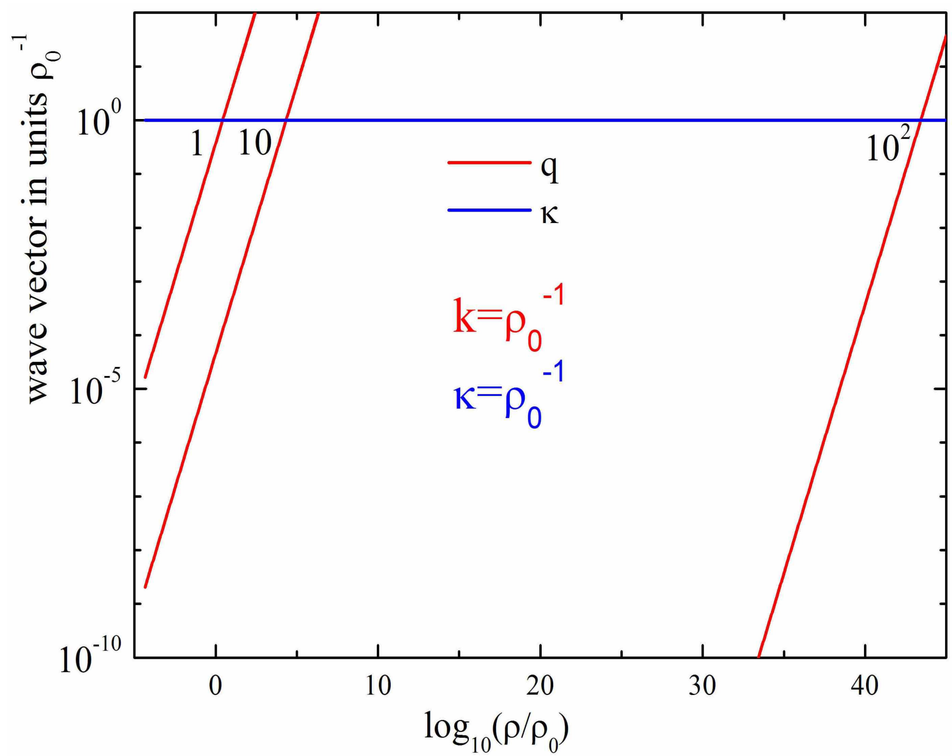
Doppler factor expressed via rapidity

The region of essential contribution to the integral is the vicinity of point where q=κ





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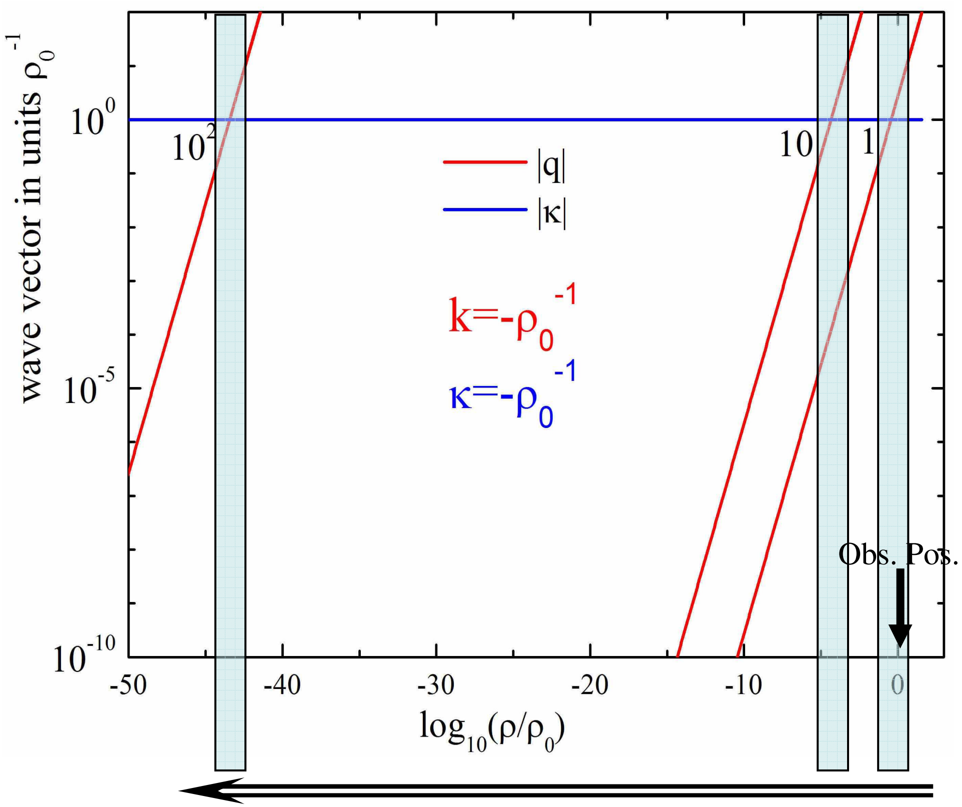

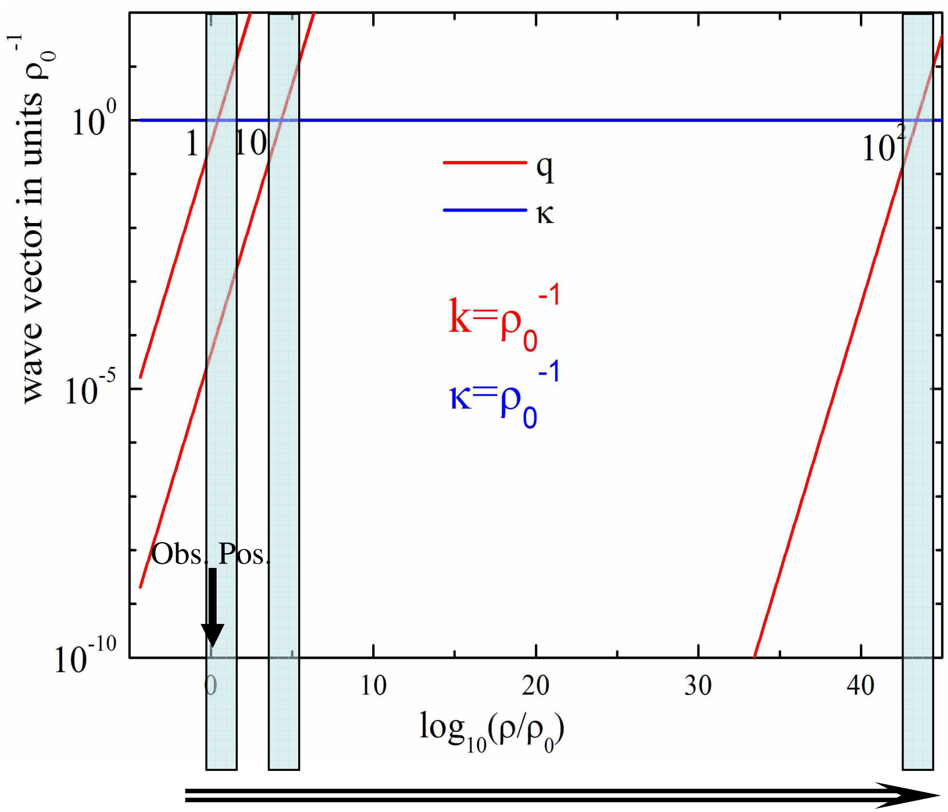

$$\rho_{\text{int}} \sim \rho_0 \exp\left(-\frac{a\tau}{c}\right) \xrightarrow[\tau\to\infty]{} 0$$

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Thus, there is significant asymmetry between positive and negative quantum propagation directions:

- in the behaviour of wave-functions in the vicinity of observer position ρ_0
- in the behaviour of region of main contribution relative to the Rindler coordinate ρ

Let us consider the body accelerated with 1g (9.8 m·s^{-2}) during 20 years of proper time. Size of Rindler horizon ρ_0 is about 1 l.y. The main contribution to the spectrum is expected at wavelengths about ρ_0. After 20 years of acceleration on proper time (that corresponds to about $2.4\cdot 10^8$ years in parent Minkowski space), one will have the following picture:

1) For quanta with $k>0$

$$\rho_{\text{int}} \sim \rho_0 \exp\left(\frac{a\tau}{c}\right) \approx 5\cdot 10^8\, l.y.$$

That corresponds to the region of essential contribution to the integral in parent Minkowski space

$$x_{\text{int}} = \rho_{\text{int}} \cosh\left(\frac{a\tau}{c}\right) \sim \frac{\rho_0}{2} \exp\left(\frac{2a\tau}{c}\right) \approx 1.2\cdot 10^{17}\, l.y.$$

This is about 10^7 times larger than the size of observed Universe!

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2) For quanta with k<0

$$\rho_{\mathrm{int}} \sim \rho_0 \exp\left(-\frac{a\tau}{c}\right) \approx 2\cdot 10^{-9}\, l.y.$$

That corresponds to the region of essential contribution to the integral in parent Minkowski space

$$x_{\mathrm{int}} = \rho_{\mathrm{int}} \cosh\left(\frac{a\tau}{c}\right) \sim const = \frac{\rho_0}{2} \approx 0.5\, l.y.$$

This is also far from the accelerated body (~$2.4\cdot10^8$ l.y. in parent Minkowski space), but this region remains within observable part of the Universe at least.

The doubts presented here partially reflect the following doubts stated by Fulling (1973): ``The quantum theory usually deals with phenomena that happen on a microscopic scale. It is hard to understand how the global structure of the Universe can affect the physics inside a small Cauchy-complete region."

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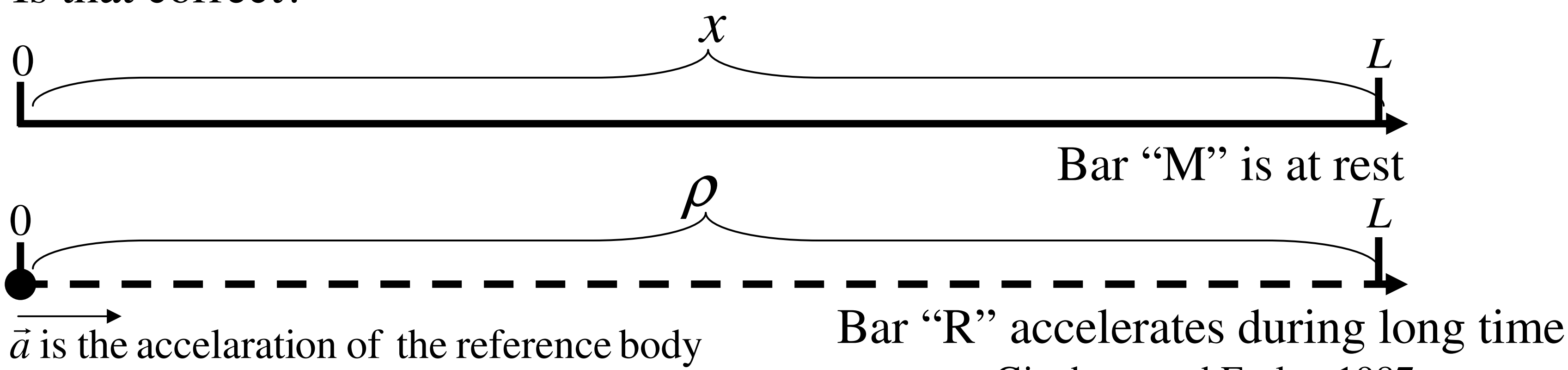

e.g. Ginzburg and Frolov 1987

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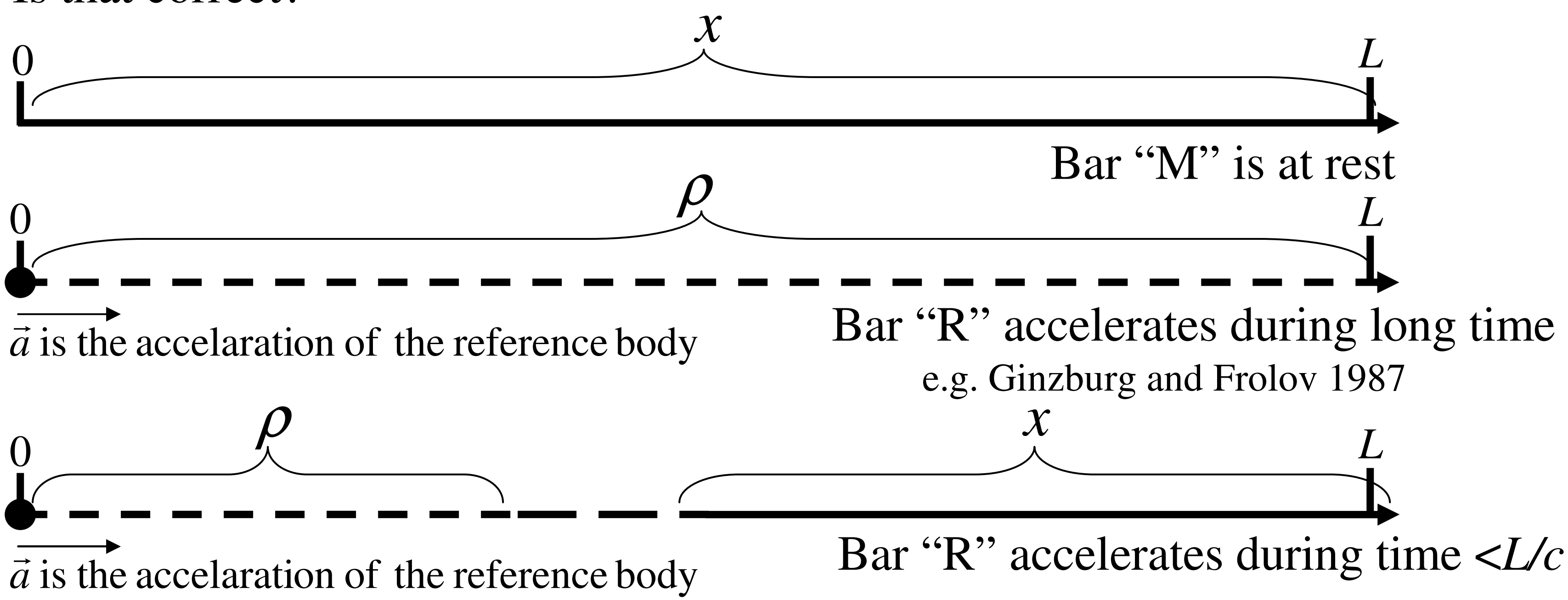

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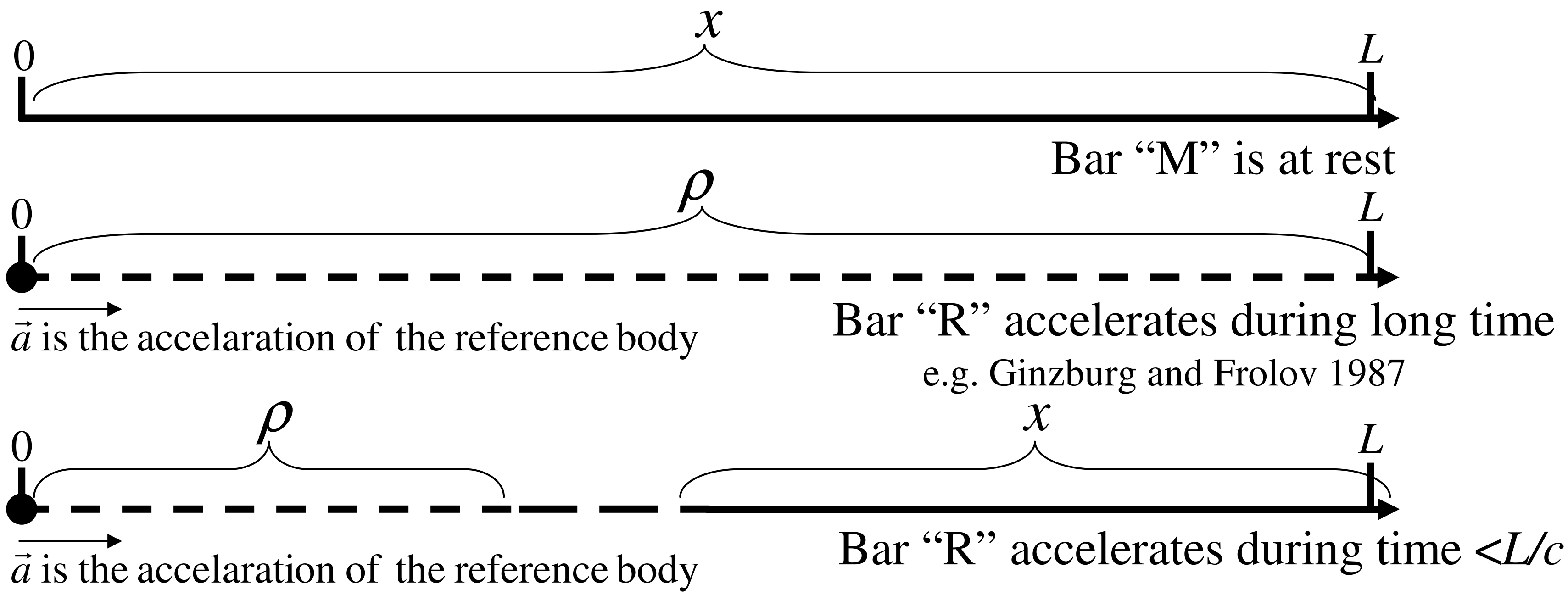

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Thus, to estimate the Unruh effect correctly, one should resolve not only the KFG equation for field Ψ but system of equations for field Ψ and metric g_{ik} even in the case of apparent gravitational field. It may be a difficult problem in general case, but in present case the qualitative answer seems clear.

Draft of correct calculation procedure and Result

$$\left\langle 0_M \mid \hat{N}_\kappa^R \mid 0_M \right\rangle = \frac{1}{2}\int_0^\infty \left| \int_0^L \left(\Psi_k^M \partial_0 \Psi_\kappa^R - \Psi_\kappa^R \partial_0 \Psi_k^M \right) \frac{d\rho}{\rho} \right|^2 dk$$

$L >> \max(k^{-1}, \kappa^{-1}, \rho_0)$ Large but finite value

Then, in stationary limit $\tau \to \infty$

$$\left\langle 0_M \mid \hat{N}_\kappa^R \mid 0_M \right\rangle = \begin{cases} 2\left(\exp\left(\dfrac{2\pi c^2 \mid \kappa \mid}{a} \right) - 1 \right)^{-1}, \kappa < 0 \\ 0, \quad \kappa > 0 \end{cases}$$

Conclusion

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Generalization of obtained result on case of massless and massive scalar particles in (3+1)D spacetime is possible and will be performed soon.

Thank you for your attention!

The key relations

$$\langle 0_M \mid \hat{N}_\kappa^R \mid 0_M \rangle = \frac{1}{2}\int_0^\infty \left| \int_0^\infty \left(\Psi_k^M \partial_0 \Psi_\kappa^R - \Psi_\kappa^R \partial_0 \Psi_k^M \right) \frac{d\rho}{\rho} \right|^2 dk$$

$$\Psi_\kappa^R(\tau,\rho) = C_\kappa^R \exp\left(i\frac{c^2\kappa}{a}\left[\ln\left(\frac{\rho}{\rho_0}\right) - \frac{a\tau}{c}\operatorname{sgn}(\kappa) \right] \right)$$

$$\Psi_k^M(t,x) = C_k^M \exp\left(ik\rho \exp\left[-\frac{a\tau}{c}\operatorname{sgn}(k) \right] \right)$$

The reason of isotropy is the invariance of integral relative to the scaling $\rho \to b\rho'$, $\tau' \to \tau - (c/a)\ln b$ due to the absence of any preferred reference point

For example, observer position is not involved in above expressions directly