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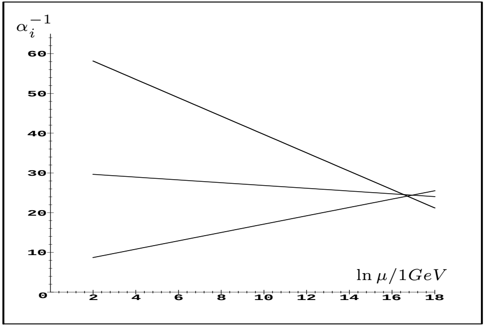
Supersymmetry, quantum corrections, and the higher derivative regularization

$\mathcal{N} = 1$ supersymmetric theories are very interesting for both phenomenology and theory. In supersymmetric extensions of the Standard Model running of the gauge coupling constants agrees with the predictions of Grand Unified Theories. Increasing of the unification mass essentially increases the proton life time, $\tau \sim M_X^4$. There are no quadratically divergent quantum corrections to the Higgs mass.



1. There are no divergent quantum corrections to the superpotential.
2. The β-function of $\mathcal{N}=1$ SYM is related to the anomalous dimensions of the matter superfields by the so called NSVZ β-function,

$$\beta(\alpha,\lambda) = -\frac{\alpha^2\Big(3C_2 - T(R) + C(R)_i{}^j\gamma_j{}^i(\alpha,\lambda)/r\Big)}{2\pi(1-C_2\alpha/2\pi)}, \qquad \text{where}$$

$$\text{tr}\,(T^A T^B) \equiv T(R)\,\delta^{AB}; \qquad (T^A)_i{}^k (T^A)_k{}^j \equiv C(R)_i{}^j;$$

$$f^{ACD} f^{BCD} \equiv C_2\delta^{AB}; \qquad r \equiv \delta_{AA}.$$

V.Novikov, M.A.Shifman, A.Vainshtein, V.I.Zakharov, Nucl.Phys. **B 229** (1983) 381; Phys.Lett. **B 166** (1985) 329; M.A.Shifman, A.I.Vainshtein, Nucl.Phys. **B 277** (1986) 456; D.R.T.Jones, Phys.Lett. **B 123** (1983) 45.

3. The three-point vertices with two lines of the Faddeev–Popov ghosts and one line of the quantum gauge superfield are finite in all orders.

The NSVZ β-function can be compared with the results of calculations in the lowest orders of the perturbation theory. To make such calculations, a theory should be regularized.

The dimensional regularization breaks the supersymmetry and is not convenient for calculations in supersymmetric theories. That is why supersymmetric theories are mostly regularized by the dimensional reduction. However, the dimensional reduction is not self-consistent.

Using the dimensional reduction and $\overline{\mbox{DR}}$-scheme a β-function of $\mathcal{N}=1$ supersymmetric theories was calculated up to the four-loop approximation:

L.V.Avdeev, O.V.Tarasov, Phys.Lett. **B 112** (1982) 356; I.Jack, D.R.T.Jones, C.G.North, Phys.Lett. **B 386** (1996) 138; Nucl.Phys. **B 486** (1997) 479; R.V.Harlander, D.R.T.Jones, P.Kant, L.Mihaila, M.Steinhauser, JHEP **0612** (2006) 024.

The result coincides with the NSVZ β-function only in one- and two-loop approximations. In the higher loops it is necessary to make a special tuning of the coupling constant.

The higher covariant derivative regularization is a consistent regularization, which does not break supersymmetry.

A.A.Slavnov, Nucl.Phys., **B 31** (1971) 301; Theor.Math.Phys. **13** (1972) 1064.

In order to regularize a theory by higher derivatives it is necessary to add a term with higher degrees of covariant derivatives. Then divergences remain only in the one-loop approximation. These remaining divergences are regularized by inserting the Pauli–Villars determinants.

A.A.Slavnov, Theor.Math.Phys. **33** (1977) 977.

The higher covariant derivative regularization can be generalized to the $\mathcal{N}=1$ supersymmetric case

V.K.Krivoshchekov, Theor.Math.Phys. **36** (1978) 745;
P.West, Nucl.Phys. **B 268** (1986) 113.

In this talk we will mostly discuss quantum corrections in SUSY theories regularized by higher covariant derivatives.

Let us consider the (massless) non-Abelian $\mathcal{N}=1$ SYM theory with matter

$$S=\frac{1}{2e_0^2}\,\mbox{Re}\,\mbox{tr}\int d^4x\,d^2\theta\,W^a W_a+\frac{1}{4}\int d^4x\,d^4\theta\,\phi^{*i}(e^{2\mathcal{F}(V)})_i{}^j\phi_j$$
$$+\Big\{\frac{1}{6}\lambda_0^{ijk}\int d^4x\,d^2\theta\,\phi_i\phi_j\phi_k+\mbox{c.c.}\Big\},$$

where the matter superfields ϕ_i belong to a representation R of the gauge group, and Yukawa couplings λ_0 satisfy the condition

$$\lambda_0^{ijm}(T^A)_m{}^k+\lambda_0^{imk}(T^A)_m{}^j+\lambda_0^{mjk}(T^A)_m{}^i=0.$$

Note that usually $\mathcal{F}(V)=V$, but for calculating quantum corrections we should use this function, see below. Then the supersymmetric gauge superfield strength is given by

$$W_a\equiv\frac{1}{8}\bar D^2\Big(e^{-2\mathcal{F}(V)}D_a e^{2\mathcal{F}(V)}\Big).$$

Quantum-background splitting is made by the substitution

$$e^{2\mathcal{F}(V)} \to e^{\boldsymbol{\Omega}^+} e^{2\mathcal{F}(V)} e^{\boldsymbol{\Omega}}.$$

The background superfield $\boldsymbol{V}$ is defined by $e^{2\boldsymbol{V}} = e^{\boldsymbol{\Omega}^+} e^{\boldsymbol{\Omega}}$.
We choose the higher derivative term

$$S_\Lambda = \frac{1}{2e_0^2}\,\mbox{Re}\,\mbox{tr} \int d^4x\, d^2\theta\, e^{\boldsymbol{\Omega}} W^a e^{-\boldsymbol{\Omega}} \Big[R\Big(-\frac{\bar\nabla^2\nabla^2}{16\Lambda^2}\Big) - 1\Big]_{Adj} e^{\boldsymbol{\Omega}} W_a e^{-\boldsymbol{\Omega}}$$
$$+\frac{1}{4}\int d^4x\, d^4\theta\, \phi^+ e^{\boldsymbol{\Omega}^+} e^{2\mathcal{F}(V)} \Big[F\Big(-\frac{\bar\nabla^2\nabla^2}{16\Lambda^2}\Big) - 1\Big] e^{\boldsymbol{\Omega}}\phi$$

and the gauge fixing term

$$S_{\mbox{gf}} = -\frac{1}{16\xi_0 e_0^2}\mbox{tr}\int d^4x\, d^4\theta\, \boldsymbol{\nabla}^2 V K\Big(-\frac{\bar{\boldsymbol{\nabla}}^2\boldsymbol{\nabla}^2}{16\Lambda^2}\Big)_{Adj} \bar{\boldsymbol{\nabla}}^2 V,$$

where the regulators R, F, and K have a rapid growth at infinity.

Actions for the Faddeev–Popov and Nielsen–Kallosh ghosts have the form

$$S_{\mathsf{FP}} = \frac{1}{2}\int d^4x\, d^4\theta\, \frac{\partial \mathcal{F}^{-1}(\widetilde{V})^A}{\partial \widetilde{V}^B}\Big|_{\widetilde{V}=\mathcal{F}(V)} \Big(e^{\mathbf{\Omega}}\bar{c}e^{-\mathbf{\Omega}} + e^{-\mathbf{\Omega}^+}\bar{c}^+e^{\mathbf{\Omega}^+}\Big)^A$$

$$\times\Big\{\Big(\frac{\mathcal{F}(V)}{1-e^{2\mathcal{F}(V)}}\Big)_{Adj}\Big(e^{-\mathbf{\Omega}^+}c^+e^{\mathbf{\Omega}^+}\Big) + \Big(\frac{\mathcal{F}(V)}{1-e^{-2\mathcal{F}(V)}}\Big)_{Adj}\Big(e^{\mathbf{\Omega}}ce^{-\mathbf{\Omega}}\Big)\Big\}^B$$

$$S_{\mathsf{NK}} = \frac{1}{2e_0^2}\mathrm{tr}\int d^4x\, d^4\theta\, b^+\Big[e^{\mathbf{\Omega}^+}K\Big(-\frac{\bar{\mathbf{\nabla}}^2\mathbf{\nabla}^2}{16\Lambda^2}\Big)e^{\mathbf{\Omega}}\Big]_{Adj} b.$$

The total action of the gauge fixed theory is invariant under the BRST transformations and the background gauge transformations.

Below we will see that the quantum gauge superfield V is renormalized nonlinearly. Parameters describing this nonlinear renormalization are included into the function $\mathcal{F}(V)$. Calculating quantum corrections in the lowest loops in the Feynman gauge it is possible to set $\mathcal{F}(V) = V$. However, we will demonstrate that the nonlinear renormalization of the quantum gauge superfield is very essential.

In our notation the renormalization constants are defined by the equations

$$\frac{1}{\alpha_0} = \frac{Z_\alpha}{\alpha}; \qquad \frac{1}{\xi_0} = \frac{Z_\xi}{\xi}; \qquad \boldsymbol{V} = \boldsymbol{V}_R; \qquad \bar{c}c = Z_c Z_\alpha^{-1} \bar{c}_R c_R;$$

$$b = \sqrt{Z_b}\, b_R; \qquad V = Z_V Z_\alpha^{-1/2} V_R + \text{nonlinear terms};$$

$$\phi_i = (\sqrt{Z_\phi})_i{}^j (\phi_R)_j; \qquad \lambda^{ijk} = \lambda_0^{mnp} (Z_\lambda)_m{}^i (Z_\lambda)_n{}^j (Z_\lambda)_p{}^k.$$

The subscript R denotes renormalized superfields, α, λ, and ξ are the renormalized coupling constant, the Yukawa couplings, and the gauge parameter, respectively.

It is possible to impose the following constrains to these renormalization constants:

$$(Z_\lambda)_i{}^j = (\sqrt{Z_\phi})_i{}^j; \qquad Z_\xi = Z_V^{-2}; \qquad Z_b = Z_\alpha^{-1}.$$

The three-point vertices with two ghost legs and a single leg of the quantum gauge superfield are finite in all orders

K.S., Nucl.Phys. **B909** (2016) 316.

There are 4 such vertices, $\bar{c}Vc$, $\bar{c}^+Vc$, $\bar{c}Vc^+$, and $\bar{c}^+Vc^+$.
They have the same renormalization constant $Z_\alpha^{-1/2}Z_cZ_V$. Therefore, the above statement can be rewritten as

$$\frac{d}{d\ln\Lambda}\left(Z_\alpha^{-1/2}Z_cZ_V\right) = 0.$$

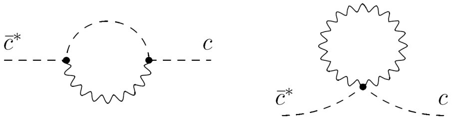
In the one-loop approximation this has first been noted in the paper

S.S.Aleshin, A.E.Kazantsev, M.B.Skopsov, K.S., JHEP **1605** (2016) 014.

Consequently, there is a subtraction scheme in which

$$-\frac{1}{2}\ln Z_\alpha + \ln Z_c + \ln Z_V = 0.$$

The Green functions of the structure $\bar{c}V^nc$ are divergent for $n\ne 1$.

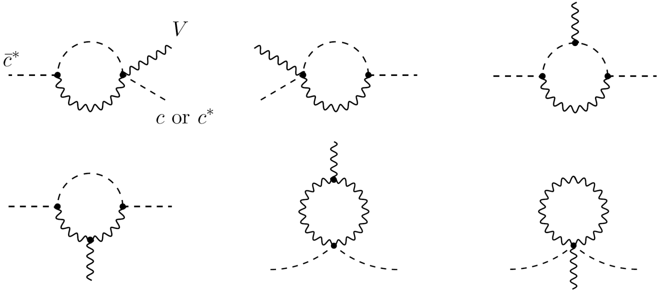


In the Euclidean space after the Wick rotation (for $\mathcal{F}(V) = V$)

$$G_c(p) = 1 + e_0^2 C_2 \int \frac{d^4k}{(2\pi)^4}\Big(\frac{\xi_0}{K_k} - \frac{1}{R_k}\Big)\Big(-\frac{1}{6k^4} + \frac{1}{2k^2(k+p)^2} - \frac{p^2}{2k^4(k+p)^2}\Big) + O(e_0^4, e_0^2\lambda_0^2),$$

where $R_k \equiv R(k^2/\Lambda)$ and $K_k \equiv K(k^2/\Lambda^2)$.
We see that this function is divergent in the ultraviolet region (at infinite Λ).

$$\gamma_c(\alpha_0,\lambda_0) = \frac{d\ln G_c}{d\ln\Lambda}\Big|_{p=0;\ \alpha,\lambda=\text{const}} = -\frac{\alpha_0 C_2(1-\xi_0)}{6\pi} + O(\alpha_0^2, \alpha_0\lambda_0^2).$$



$$\frac{ie_0}{4}f^{ABC}\int d^4\theta\,\frac{d^4p}{(2\pi)^4}\,\frac{d^4q}{(2\pi)^4}\,\bar c^{*A}(\theta,p+q)\Big(f(p,q)\partial^2\Pi_{1/2}V^B(\theta,-p)$$
$$+F_\mu(p,q)(\gamma^\mu)_{\dot a}{}^b D_b\bar D^{\dot a}V^B(\theta,-p)+F(p,q)V^B(\theta,-p)\Big)c^C(\theta,-q);$$
$$\frac{ie_0}{4}f^{ABC}\int d^4\theta\,\frac{d^4p}{(2\pi)^4}\,\frac{d^4q}{(2\pi)^4}\,\bar c^{*A}(\theta,p+q)\widetilde F(p,q)V^B(\theta,-p)c^{*C}(\theta,-q).$$

Calculating these diagrams gives

$$F(p,q) = 1 + \frac{e_0^2 C_2}{4}\int \frac{d^4k}{(2\pi)^4}\Big\{ -\frac{(q+p)^2}{R_k k^2 (k+p)^2 (k-q)^2} - \frac{\xi_0\, p^2}{K_k k^2 (k+q)^2 (k+q+p)^2} + \frac{\xi_0\, q^2}{K_k k^2 (k+p)^2 (k+q+p)^2} + \Big(\frac{\xi_0}{K_k} - \frac{1}{R_k}\Big)\Big(-\frac{2(q+p)^2}{k^4 (k+q+p)^2} + \frac{2}{k^2 (k+q+p)^2} - \frac{1}{k^2 (k+q)^2} - \frac{1}{k^2 (k+p)^2}\Big)\Big\} + O(\alpha_0^2, \alpha_0\lambda_0^2).$$

$$\widetilde{F}(p,q) = 1 - \frac{e_0^2 C_2}{4}\int \frac{d^4k}{(2\pi)^4}\Big\{ \frac{p^2}{R_k k^2 (k+q)^2 (k+q+p)^2} + \frac{\xi_0\, (q+p)^2}{K_k k^2 (k-p)^2 (k+q)^2} + \frac{\xi_0\, q^2}{K_k k^2 (k+p)^2 (k+q+p)^2} + \frac{2\xi_0}{K_k k^2 (k+p)^2} - \frac{2\xi_0}{K_k k^2 (k+q+p)^2} + \Big(\frac{\xi_0}{K_k} - \frac{1}{R_k}\Big) \times \Big(\frac{2q^2}{k^4 (k+q)^2} + \frac{1}{k^2 (k+q+p)^2} - \frac{1}{k^2 (k+q)^2}\Big)\Big\} + O(\alpha_0^2, \alpha_0\lambda_0^2).$$

We see that these expressions are finite in the ultraviolet region.

We will define RGFs in terms of the bare couplings by the equations

$$\beta(\alpha_0,\lambda_0) \equiv \frac{d\alpha_0}{d\ln\Lambda};$$

$$(\gamma_\phi)_i{}^j(\alpha_0,\lambda_0) \equiv -\frac{d\ln(Z_\phi)_i{}^j(\alpha,\lambda,\Lambda/\mu)}{d\ln\Lambda};$$

$$\gamma_V(\alpha_0,\lambda_0) \equiv -\frac{d\ln Z_V(\alpha,\lambda,\Lambda/\mu)}{d\ln\Lambda};$$

$$\gamma_c(\alpha_0,\lambda_0) \equiv -\frac{d\ln Z_c(\alpha,\lambda,\Lambda/\mu)}{d\ln\Lambda}.$$

where the differentiation is made at fixed values of α and λ^{ijk}.
These renormalization group functions are
1. scheme independent at a fixed regularization;
2. depend on a regularization;
2. satisfy the NSVZ relation in all orders for $\mathcal{N}=1$ SQED with N_f flavors, regularized by higher derivatives.

The NSVZ β-function can be equivalently rewritten in the form

$$\frac{\beta(\alpha_0,\lambda_0)}{\alpha_0^2} = -\frac{3C_2 - T(R) + C(R)_i{}^j(\gamma_\phi)_j{}^i(\alpha_0,\lambda_0)/r}{2\pi} + \frac{C_2}{2\pi}\cdot\frac{\beta(\alpha_0,\lambda_0)}{\alpha_0}.$$

Let us express the β-function in the right hand side in terms of the renormalization constant Z_α:

$$\beta(\alpha_0,\lambda_0) = \frac{d\alpha_0(\alpha,\lambda,\Lambda/\mu)}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}} = -\alpha_0\frac{d\ln Z_\alpha}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}}.$$

Then, using the identity $d(Z_\alpha^{-1/2} Z_V Z_c)/d\ln\Lambda = 0$ we obtain

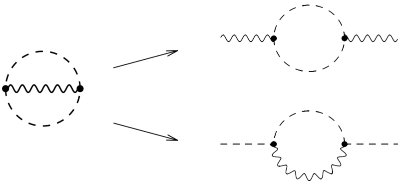
$$\beta(\alpha_0,\lambda_0) = -2\alpha_0\frac{d\ln(Z_c Z_V)}{d\ln\Lambda}\Big|_{\alpha,\lambda=\text{const}} = 2\alpha_0\Big(\gamma_c(\alpha_0,\lambda_0)+\gamma_V(\alpha_0,\lambda_0)\Big),$$

where γ_c and γ_V are anomalous dimensions of the Faddeev–Popov ghosts and of the quantum gauge superfield (defined in terms of the bare coupling constants), respectively.

Substituting this expression into the right hand side of the NSVZ relation we obtain

$$\frac{\beta(\alpha_0,\lambda_0)}{\alpha_0^2} = -\frac{1}{2\pi}\Big(3C_2 - T(R) - 2C_2\gamma_c(\alpha_0,\lambda_0)$$
$$-2C_2\gamma_V(\alpha_0,\lambda_0) + C(R)_i{}^j(\gamma_\phi)_j{}^i(\alpha_0,\lambda_0)/r\Big).$$

From this form of the NSVZ β-function we see that the matter superfields and ghosts similarly contribute to the right hand side.



The graphical interpretation is similar to the Abelian case

A.V.Smilga, A.I.Vainshtein, Nucl.Phys. **B 704** (2005) 445.

Renormalization group functions defined in terms of the renormalized couplings

RGFs are defined in terms of the renormalized couplings by the equations

$$\widetilde{\beta}(\alpha,\lambda) \equiv \frac{d\alpha}{d\ln\mu};$$

$$(\widetilde{\gamma}_\phi)_i{}^j(\alpha,\lambda) \equiv \frac{d\ln(Z_\phi)_i{}^j(\alpha_0,\lambda_0,\Lambda/\mu)}{d\ln\mu};$$

$$\widetilde{\gamma}_V(\alpha,\lambda) \equiv \frac{d\ln Z_V(\alpha_0,\lambda_0,\Lambda/\mu)}{d\ln\mu};$$

$$\widetilde{\gamma}_c(\alpha,\lambda) \equiv \frac{d\ln Z_c(\alpha_0,\lambda_0,\Lambda/\mu)}{d\ln\mu}.$$

where the differentiation is made at fixed values of α_0 and λ_0^{ijk}.
These renormalization group functions are
1. scheme and regularization dependent;
2. satisfy the NSVZ relation only for a special renormalization prescription, called the NSVZ scheme.

The RGFs defined in terms of the renormalized coupling constant are scheme dependent and satisfy the NSVZ relation only in a certain subtraction scheme. Similarly to

A.L.Kataev and K.S., Nucl.Phys. **B875** (2013) 459; Phys.Lett. **B730** (2014) 184.

we see that in the non-Abelian case RGFs defined in terms of the bare coupling constant coincide with ones defined in terms of the renormalized coupling constants if the boundary conditions

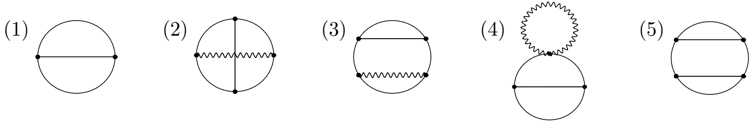
$$Z_\alpha(\alpha,\lambda,x_0) = 1; \qquad (Z_\phi)_i{}^j(\alpha,\lambda,x_0) = \delta_i{}^j; \qquad Z_c(\alpha,\lambda,x_0) = 1,$$

where x_0 is a fixed value of $\ln\Lambda/\mu$, are imposed on the renormalization constants. We also assume that $Z_V = Z_\alpha^{1/2} Z_c^{-1}$.
For $x_0 = 0$ only powers of $\ln\Lambda/\mu$ are included into the renormalization constants. This is very similar to the minimal subtraction scheme. That is why we will call this scheme Higher Derivatives + Minimal Subtractions of Logarithms (HD+MSL). Possibly,

HD+MSL=NSVZ

Three-loop terms containing the Yukawa couplings

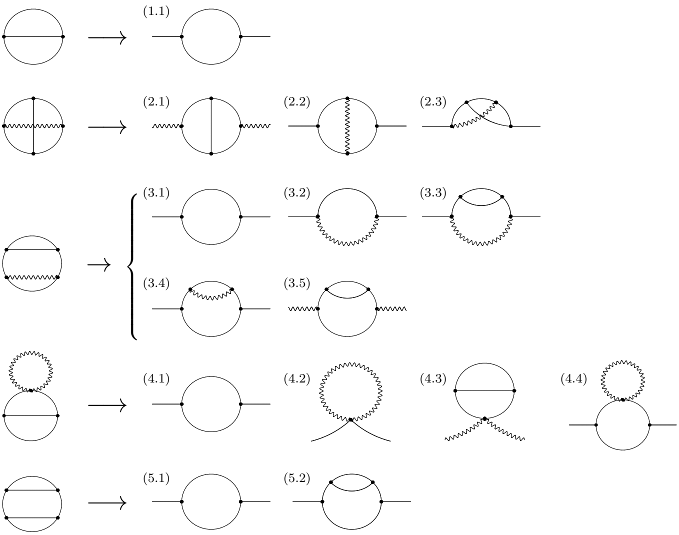
To verify the above results we consider the three-loop terms containing the Yukawa couplings. They correspond to the graphs



V.Yu.Shakhmanov, K.S., Nucl.Phys., **B920**, (2017), 345;
A.E.Kazantsev, V.Yu.Shakhmanov, K.S., JHEP 1804 (2018) 130.

Attaching two external lines of the background gauge superfield by all possible ways we obtain (a large number of) the three-loop superdiagrams contributing to the β-function.
From the other side, cutting internal lines in these graphs we obtain various two-loop contributions to the anomalous dimensions of the quantum gauge superfield and of the matter superfields. For other supergraphs contributions to the anomalous dimension of the Faddeev–Popov ghosts are also possible.

Graphical form of the NSVZ relation



Expressions for all graphs are given by integrals of double total derivatives. This allows to calculate one of the loops integrals analytically and (at the level of loop integrals) obtain the relation

$$\Delta_A\Big(\frac{\beta(\alpha_0,\lambda_0)}{\alpha_0^2}\Big) = \frac{1}{\pi}C_2\Delta_A\gamma_V(\alpha_0,\lambda_0) - \frac{1}{2\pi r}C(R)_i{}^j(\Delta_A\gamma_\phi)_j{}^i(\alpha_0,\lambda_0),$$

which has been checked for all considered graphs.
For example, the terms quartic in the Yukawa couplings have the form

$$\frac{\Delta\beta(\alpha_0,\lambda_0)}{\alpha_0^2} = -\frac{2\pi}{r}C(R)_i{}^j\frac{d}{d\ln\Lambda}\int\frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\,\lambda_0^{imn}\lambda^*_{0jmn}\frac{\partial}{\partial q_\mu}\frac{\partial}{\partial q^\mu}$$
$$\times\Big(\frac{1}{k^2F_k\,q^2F_q\,(q+k)^2F_{q+k}}\Big) + \frac{4\pi}{r}C(R)_i{}^j\frac{d}{d\ln\Lambda}\int\frac{d^4k}{(2\pi)^4}\frac{d^4l}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}$$
$$\times\bigg(\lambda_0^{iab}\lambda^*_{0kab}\lambda_0^{kcd}\lambda^*_{0jcd}\Big(\frac{\partial}{\partial k_\mu}\frac{\partial}{\partial k^\mu}-\frac{\partial}{\partial q_\mu}\frac{\partial}{\partial q^\mu}\Big)+2\lambda_0^{iab}\lambda^*_{0jac}\lambda_0^{cde}\lambda^*_{0bde}\frac{\partial}{\partial q_\mu}\frac{\partial}{\partial q^\mu}\bigg)$$
$$\times\frac{1}{k^2F_k^2\,q^2F_q\,(q+k)^2F_{q+k}\,l^2F_l\,(l+k)^2F_{l+k}} = -\frac{1}{2\pi r}C(R)_i{}^j\Delta\gamma_\phi(\lambda_0)_j{}^i.$$

For the higher derivative regulators $F(x) = 1 + x^n$; $R(x) = 1 + x^m$ the integrals have been calculated:

$$\frac{\beta(\alpha_0,\lambda_0)}{\alpha_0^2} = -\frac{1}{2\pi}\Big(3C_2 - T(R)\Big) - \frac{1}{2\pi r}C(R)_j{}^i\left(\frac{1}{4\pi^2}\lambda^*_{0imn}\lambda_0^{jmn} + \frac{\alpha_0}{8\pi^3}\lambda^*_{0imn}\right.$$
$$\times\lambda_0^{jmn}C_2 - \frac{\alpha_0}{8\pi^3}\lambda^*_{0lmn}\lambda_0^{jmn}C(R)_i{}^l\Big(1-\frac{1}{n}\Big) + \frac{\alpha_0}{4\pi^3}\lambda^*_{0imn}\lambda_0^{jml}C(R)_l{}^n\Big(1+\frac{1}{n}\Big)$$
$$\left.-\frac{1}{16\pi^4}\lambda^*_{0iac}\lambda_0^{jab}\lambda^*_{0bde}\lambda_0^{cde}\right) + O(\alpha_0^2\lambda_0^2, \alpha_0\lambda_0^4, \lambda_0^6) + \text{terms without the Yukawa}$$

couplings;

$$(\gamma_\phi)_i{}^j(\alpha_0,\lambda_0) = -\frac{\alpha_0}{\pi}C(R)_i{}^j + \frac{1}{4\pi^2}\lambda^*_{0imn}\lambda_0^{jmn} - \frac{\alpha_0}{8\pi^3}\lambda^*_{0lmn}\lambda_0^{jmn}C(R)_i{}^l\Big(1-\frac{1}{n}\Big)$$
$$+\frac{\alpha_0}{4\pi^3}\lambda^*_{0imn}\lambda_0^{jml}C(R)_l{}^n\Big(1+\frac{1}{n}\Big) - \frac{1}{16\pi^4}\lambda^*_{0iac}\lambda_0^{jab}\lambda^*_{0bde}\lambda_0^{cde} + O(\alpha_0^2, \alpha_0\lambda_0^4, \lambda_0^6);$$
$$\gamma_V(\alpha_0,\lambda_0) = -\frac{\alpha_0}{4\pi}\Big(3C_2 - T(R)\Big) - \frac{\alpha_0}{16\pi^3 r}\lambda^*_{0jmn}\lambda_0^{imn}C(R)_i{}^j + O(\alpha_0^2, \alpha_0\lambda_0^4).$$

RGFs defined in terms of the renormalized couplings are

$$\frac{\widetilde{\beta}(\alpha,\lambda)}{\alpha^2} = -\frac{1}{2\pi}\Big(3C_2 - T(R)\Big) - \frac{1}{2\pi r}C(R)_j{}^i\Bigg(\frac{1}{4\pi^2}\lambda^*_{imn}\lambda^{jmn} + \frac{\alpha}{8\pi^3}\lambda^*_{imn}\lambda^{jmn}C_2$$

$$+\frac{\alpha}{4\pi^3}\lambda^*_{lmn}\lambda^{jmn}C(R)_i{}^l\Big[b_2 - g_{11} - \frac{1}{2}\Big(1-\frac{1}{n}\Big)\Big] + \frac{\alpha}{2\pi^3}\lambda^*_{imn}\lambda^{jml}C(R)_l{}^n\Big[b_2 - g_{11}$$

$$+\frac{1}{2}\Big(1+\frac{1}{n}\Big)\Big] - \frac{1}{8\pi^4}\lambda^*_{iac}\lambda^{jab}\lambda^*_{bde}\lambda^{cde}\Big[b_2 - g_{12} + \frac{1}{2}\Big] + \frac{1}{16\pi^4}\lambda^*_{iab}\lambda^{kab}\lambda^*_{kcd}\lambda^{jcd}$$

$$\times\Big[g_{12} - b_2\Big]\Bigg) + O(\alpha^2\lambda^2,\alpha\lambda^4,\lambda^6) + \text{terms without the Yukawa couplings;}$$

$$(\widetilde{\gamma}_\phi)_i{}^j(\alpha,\lambda) = -\frac{\alpha}{\pi}C(R)_i{}^j + \frac{1}{4\pi^2}\lambda^*_{imn}\lambda^{jmn} + \frac{\alpha}{4\pi^3}\lambda^*_{lmn}\lambda^{jmn}C(R)_i{}^l\Big[g_{12} - g_{11}$$

$$-\frac{1}{2}\Big(1-\frac{1}{n}\Big)\Big] + \frac{\alpha}{2\pi^3}\lambda^*_{imn}\lambda^{jml}C(R)_l{}^n\Big[g_{12} - g_{11} + \frac{1}{2}\Big(1+\frac{1}{n}\Big)\Big] - \frac{1}{16\pi^4}\lambda^*_{iac}\lambda^{jab}$$

$$\lambda^*_{bde}\lambda^{cde} + O(\alpha^2,\alpha\lambda^4,\lambda^6);$$

$$\widetilde{\gamma}_V(\alpha,\lambda) = -\frac{\alpha}{4\pi}\Big(3C_2 - T(R)\Big) - \frac{\alpha}{16\pi^3 r}\lambda^*_{jmn}\lambda^{imn}C(R)_i{}^j + O(\alpha^2,\alpha\lambda^4).$$

From the above results we see that the new form of the NSVZ relation

$$\frac{\widetilde{\beta}(\alpha,\lambda)}{\alpha^2} = -\frac{1}{2\pi}\Big(3C_2 - T(R) - 2C_2\widetilde{\gamma}_c(\alpha,\lambda)$$
$$-2C_2\widetilde{\gamma}_V(\alpha,\lambda) + C(R)_i{}^j(\widetilde{\gamma}_\phi)_j{}^i(\alpha,\lambda)/r\Big)$$

is not valid for RGFs defined in terms of the renormalized couplings for a general renormalization prescription. However, in the case of using the HD+MSL prescription (that is the Higher covariant Derivative regularization supplemented by Minimal Subtractions of Logarithms) for which

$$g_{11} = 0; \qquad g_{12} = 0; \qquad b_2 = 0$$

the new form of the NSVZ relation is really valid.
The original form of the NSVZ relation is also valid in this case.
Thus, for the considered class of superdiagrams

$$\mathrm{HD} + \mathrm{MSL} = \mathrm{NSVZ}.$$

However, the terms containing the ghost anomalous dimension γ_c in the equation

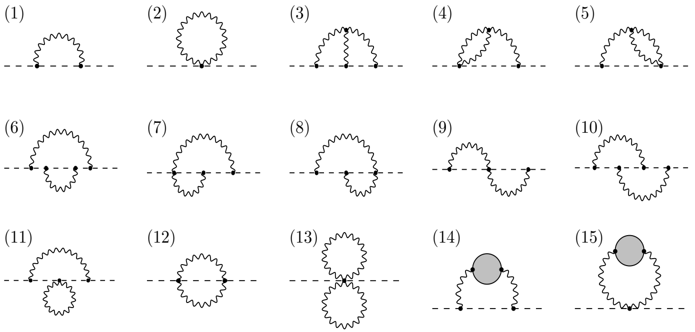
$$\frac{\widetilde{\beta}(\alpha,\lambda)}{\alpha^2} = -\frac{1}{2\pi}\Big(3C_2 - T(R) - 2C_2\widetilde{\gamma}_c(\alpha,\lambda) - 2C_2\widetilde{\gamma}_V(\alpha,\lambda)$$
$$+C(R)_i{}^j\left(\widetilde{\gamma}_\phi\right)_j{}^i(\alpha,\lambda)/r\Big)$$

have been verified only for the two-loop β-function and the one-loop anomalous dimensions

V.Yu.Shakhmanov, K.S., Phys.Lett. B **776** (2018) 417.

However, a nontrivial check can be obtain only by comparing the two-loop ghost anomalous dimension and the three-loop β-function, because only stating from this approximation the scheme dependence becomes essential. That is here we describe the calculation of the two-loop anomalous dimension of the Faddeev–Popov ghosts.

The two-loop anomalous dimension of the Faddeev–Popov ghosts is contributed to by the superdiagrams



where the gray circle denotes insertion of the one-loop polarization operator calculated in

A.E.Kazantsev, M.B.Skoptsov, K.S., Mod.Phys.Lett. A **32** (2017) no.36, 1750194.

The ghost anomalous dimension can be obtained from the function G_c which is constructed according to the prescription

$$\Gamma_c^{(2)} = \frac{1}{4}\int \frac{d^4p}{(2\pi)^4}\, d^4\theta\, \Big(c^{*A}(-p,\theta)\bar c^A(p,\theta) + \bar c^{*A}(-p,\theta)c^A(p,\theta)\Big) \times G_c(\alpha_0,\lambda_0,\xi_0,y_0,\ldots,\Lambda/p).$$

In terms of bare couplings the ghost anomalous dimension is defined by

$$\gamma_c(\alpha_0,\lambda_0,\xi_0,y_0) \equiv -\frac{d\ln Z_c}{d\ln\Lambda}\Big|_{\alpha,\lambda,\xi,y=\text{const}} = \frac{d\ln G_c}{d\ln\Lambda}\Big|_{\alpha,\lambda,\xi,y=\text{const};\, p=0}.$$

Note that for calculating this expression in the considered approximation we have to take into account nonlinear terms inside the function $\mathcal{F}(V)$,

$$\mathcal{F}^A(V) = V^A + e_0^2\, y_0\, G^{ABCD} V^B V^C V^D + \ldots, \qquad \text{where}$$

$$G^{ABCD} = \frac{1}{6}\Big(f^{AKL} f^{BLM} f^{CMN} f^{DNK} + \text{permutations of } B,\ C, \text{ and } D\Big)$$

The nonlinear renormalization of the quantum gauge superfield V was first discussed in

O.Piguet and K.Sibold, Nucl.Phys. B **196** (1982) 428; B **197** (1982) 257; 272; B **248** (1984) 301.

Explicit calculations (for gauge superfield four-point Green function)

J.W.Juer and D.Storey, Phys.Lett. **119B** (1982) 125; Nucl.Phys. B **216** (1983) 185.

demonstrated that the nonlinear terms really appear. In our notation this implies that

$$y_0 = y + \frac{\alpha}{90\pi}\Big((2+3\xi)\ln\frac{\Lambda}{\mu} + k_1\Big) + \ldots$$

Also we need the one-loop renormalization of some other parameters:

$$\alpha_0 = \alpha - \frac{\alpha^2}{2\pi}\Big[3C_2\Big(\ln\frac{\Lambda}{\mu} + b_{11}\Big) - T(R)\Big(\ln\frac{\Lambda}{\mu} + b_{12}\Big)\Big] + O(\alpha^3,\alpha^2\lambda^2);$$

$$\alpha_0\xi_0 = \alpha\xi + \frac{\alpha^2 C_2}{3\pi}\Big(\xi(\xi-1)\ln\frac{\Lambda}{\mu} + x_1\Big) + O(\alpha^3,\alpha^2\lambda^2).$$

The result for the ghost anomalous dimension defined in terms of the bare coupling constant has been obtained in

A.E. Kazantsev, M.D. Kuzmichev, N.P. Meshcheriakov, S.V. Novgorodtsev, I.E. Shirokov, M.B. Skoptsov, K.S., arXiv:1805.03686 [hep-th]

for the higher derivative regulators $R(x) = K(x) = 1+x^m$; $F(x) = 1+x^n$:

$$\gamma_c(\alpha_0,\lambda_0,\xi_0,y_0) = \frac{\alpha_0 C_2(\xi_0-1)}{6\pi} - \frac{5\alpha_0 y_0 C_2^2(\xi_0-1)}{4\pi} - \frac{\alpha_0^2 C_2^2}{24\pi^2}\Big(\xi_0^2-1\Big)$$
$$-\frac{\alpha_0^2 C_2^2}{4\pi^2}\Big(\ln a_\varphi + 1\Big) + \frac{\alpha_0^2 C_2 T(R)}{12\pi^2}\Big(\ln a + 1\Big) + \ldots,$$

where $a \equiv M/\Lambda$; $a_\varphi \equiv M_\varphi/\Lambda$. Note that we keep the one-loop y-dependence, but omit the dependence on the nonlinearity parameters in the two-loop terms. (In the two-loop approximation it is necessary to take into account other parameters describing the nonlinear renormalization.)
In agreement with the general arguments the result is scheme-independent.

In terms of the renormalized couplings the ghost anomalous dimension is defined as

$$\widetilde{\gamma}_c(\alpha,\lambda,\xi,y) = \frac{d\ln Z_c}{d\ln\mu}\Big|_{\alpha_0,\lambda_0,\xi_0,y_0=\text{const}}.$$

The result is (h_1 is finite constant inside Z_c)

$$\widetilde{\gamma}_c(\alpha,\lambda,\xi,y) = \frac{\alpha C_2(\xi-1)}{6\pi} - \frac{5\alpha y C_2^2(\xi-1)}{4\pi} - \frac{\alpha^2 C_2^2}{4\pi^2}\Big(\ln a_\varphi + 1 + 6h_1 - b_{11}\Big) + \frac{\alpha^2 C_2 T(R)}{12\pi^2}\Big(\ln a + 1 + 6h_1 - b_{12}\Big) - \frac{\alpha^2 C_2^2}{24\pi^2}\Big(\xi^2 - 1\Big) + \frac{\alpha^2 C_2^2}{72\pi^2} \times\Big(4x_1 - (\xi-1)k_1\Big) + \ldots$$

We see that the result is scheme dependent. Also it can be easily verified that in the HD+MSL scheme (for which $b_{11} = b_{12} = 0$, $h_1 = 0$, $k_1 = 0$, $x_1 = 0$) it coincides with γ_c after a formal substitution

$$\alpha \to \alpha_0; \qquad \xi \to \xi_0; \qquad y \to y_0.$$

Conclusion

- ✓ In the non-Abelian case the perturbative calculations seem to produce the new form of the NSVZ equation which relates the β-function to the anomalous dimensions of the quantum gauge superfield, of the Faddeev–Popov ghosts, and of the chiral matter superfields.
- ✓ The new form of the NSVZ relation can be obtained from the non-renormalization theorem for the triple ghost-gauge vertices and has the same graphical interpretation as in the Abelian case.
- ✓ For RGFs defined in terms of the bare couplings the NSVZ relation is possibly valid with the higher derivatives regularization in an arbitrary subtraction scheme. Then the NSVZ scheme for RGFs defined in terms of the renormalized couplings is produced by the HD+MSL prescription.
- ✓ Renormalization of supersymmetric theories in higher orders requires the nonlinear renormalization of the quantum gauge superfield.

Thank you for the attention!