Quantum field-theoretical description of neutrino behavior in dense matter.

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Abstract

We propose a modification of the electroweak theory, where the fermions with the same electroweak quantum numbers are combined in multiplet and are treated as different quantum states of a single particle. Thereby, in describing the electroweak interactions it is possible to use four fundamental fermions only. In this model, the mixing and oscillations of the particles arise as a direct consequence of the general principles of quantum field theory.

Calculations of higher-order processes including the computation of the contributions due to radiative corrections can be performed in the framework of perturbation theory using the regular diagram technique.

Hence, it is possible to make an explicitly covariant description of neutrino interaction with fermions of the medium, based on an equation similar to the Dirac-Schwinger equation of quantum electrodynamics. Such an equation describes neutrino oscillations and its spin rotation in dense matter.

INTRODUCTION

The Standard Model of electroweak interactions which is based on the non-Abelian gauge symmetry of the interactions, the generation of particle masses due to the spontaneous symmetry breaking mechanism, and the philosophy of mixing of particle generations is universally recognized. Its predictions obtained in the framework of perturbation theory are in a very good agreement with the experimental data, and there is no serious reason, at least at the energies available at present, for its main propositions to be revised.

However, in describing such an important and firmly experimentally established phenomenon as neutrino oscillations, an essentially phenomenological theory based on the pioneer works by B. Pontecorvo and Z. Maki et al. is used. This theory is well developed and is consistent with the experimental data.

The primary assumption of this theory is that the neutrinos are massive, and moreover, there are three neutrino types with different masses. It is also postulated that the neutrinos produced in reactions are in the states which are superpositions of the states with fixed masses. Such states form the so-called flavor basis. The transformation to this basis from the mass basis is given by a unitary mixing matrix. Initially, the mass basis elements are described by plane waves with the same (three-dimensional) momentum. The time evolution of the flavor states is described by the solution of the corresponding Cauchy problem. For this reason it is taken for granted that the mass and the flavor states can be connected by a unitary transformation.

However, this supposition is not correct. For example, it is discussed in paper

K.C. Hannabuss and D.C. Latimer, J. Phys. A: Math. Gen., **44**, 1369 (2000).

So, it is impossible to construct the Fock space for flavor states and, as a consequence, it is impossible to calculate the transition probabilities for flavor states in the framework of the perturbation theory using the conventional approach.
However, it is possible to circumvent this obstacle as it was made in the following papers

A. E. Lobanov, arXiv:1507.01256 [hep-ph]

A. E. Lobanov, Teor. Mat. Fiz., **192**, 70 (2017). [Theor. Math. Phys. **192**, 1000 (2017)].

Due to this fact, a modification of electroweak theory has been proposed in which all fermions with equal electroweak quantum numbers are united into SU(3)-multiplets, and such fermions can be considered as different quantum states of a single particle.

MODIFIED MODEL OF ELECTROWEAK INTERACTION

The Lagrangian of such a theory is the Lagrangian of the Standard Model supplemented by the singlets of the right-handed neutrinos with changes in the fermion sector only. The Lagrangian for the physical fermion fields in our model is written as follows.

$$\mathcal{L}_f = \mathcal{L}_0 + \mathcal{L}_{int}. \tag{2.1}$$

The Lagrangian of free fields is determined by the formula

$$\mathcal{L}_0 = \sum_{i=\nu,e,u,d} \frac{\mathrm{i}}{2} \left[\left(\bar{\Psi}^{(i)} \gamma^\mu (\partial_\mu \Psi^{(i)}) \right) - (\partial_\mu \bar{\Psi}^{(i)}) \gamma^\mu \Psi^{(i)} \right] \mathbb{I} - \bar{\Psi}^{(i)} \mathbb{M}^{(i)} \Psi^{(i)}. \tag{2.2}$$

Here the field functions for the fermion fields are 12-component objects. These field functions satisfy the modified Dirac equations

$$\left(\mathrm{i}\gamma^{\mu}\partial_{\mu}\mathbb{I} - \mathbb{M}^{(i)}\right)\Psi^{(i)}(x) = 0. \tag{2.3}$$

In these equations $\mathbb{M}^{(i)}$ are the Hermitian mass matrices of the multiplets.

The interaction Lagrangian between the fermion fields, the vector boson fields $W_\mu^\pm, Z_\mu, A_\mu$, and the Higgs field H is determined by the next formula

$$\begin{aligned}\mathcal{L}_{int} = -\sum_{i=\nu,e,u,d} \bar{\Psi}^{(i)}\mathbb{M}^{(i)}(H/v)\Psi^{(i)} \\ -\frac{g}{2\sqrt{2}}\left(\bar{\Psi}^{(e)}\gamma^\mu(1+\gamma^5)\mathbb{I}\,\Psi^{(\nu)}W_\mu^- + \bar{\Psi}^{(\nu)}\gamma^\mu(1+\gamma^5)\mathbb{I}\,\Psi^{(e)}W_\mu^+\right) \\ -\frac{g}{2\sqrt{2}}\left(\bar{\Psi}^{(d)}\gamma^\mu(1+\gamma^5)\mathbb{I}\,\Psi^{(u)}W_\mu^- + \bar{\Psi}^{(u)}\gamma^\mu(1+\gamma^5)\mathbb{I}\,\Psi^{(d)}W_\mu^+\right) \\ -e\sum_{i=\nu,e,u,d} Q^{(i)}\bar{\Psi}^{(i)}\gamma^\mu\mathbb{I}\,\Psi^{(i)}A_\mu \\ -\frac{g}{2\cos\theta_{\mathrm{W}}}\sum_{i=\nu,e,u,d}\bar{\Psi}^{(i)}\gamma^\mu\mathbb{I}\left(T^{(i)} - 2Q^{(i)}\sin^2\theta_{\mathrm{W}} + T^{(i)}\gamma^5\right)\Psi^{(i)}Z_\mu.\end{aligned} \tag{2.4}$$

Here θ_{W} is the Weinberg angle, $e = g\sin\theta_{\mathrm{W}}$ is the positron electric charge, $T^{(i)}$ is the weak isospin ($T^{(\nu)} = T^{(u)} = 1/2,\ T^{(e)} = T^{(d)} = -1/2$), $Q^{(i)}$ is the electric charge of the multiplet in the units of e. The value v is the vacuum expectation of the Higgs field.

Thus, this Lagrangian formally coincides with the Lagrangian of the Standard Model. However, each fermion wave function $\Psi^{(i)}$ describes not the individual particle, but the multiplet as a whole. So it is not necessary to introduce the mixing matrices into $\mathcal{L}_{int}$ explicitly.

The action defined by the Lagrangian of free fields (2.2) is explicitly invariant with respect to $SU(3)$, so when we quantize the model, the multiplet can be considered as a single particle. Therefore, the one-particle states in the Fock space are defined as usual, the creation and annihilation operators satisfy the canonical commutation relations. However, these operators carry an additional discrete quantum number that is associated with the mass of the state.

Each multiplet can be either in one of the three mass states, or in a pure quantum state that is a superposition of the states with fixed masses. In a certain sense we may say that there are only four fundamental fermions in this model.

Due to this fact it is possible to obtain the Dyson decomposition, which enables one to construct the perturbation theory in the interaction representation. The probabilities of detecting neutrinos with one or another flavor obtained in this framework depend on the distance from the source. So both mixing and oscillations of particles arise as direct consequences of general principles of quantum field theory.
We can also calculate radiative corrections using the regular diagram technique. Therefore it is rather natural to use this model to describe the interaction of neutrinos with matter.

NEUTRINO OSCILLATIONS IN DENSE MATTER

It is well known that if matter density is high enough for considering the weak interaction of neutrino with the background fermions as coherent, it is possible to describe neutrino interaction with matter by an effective potential. The origin of this effective potential is the forward elastic scattering of neutrino on the fermions of the matter.

L. Wolfenstein, Phys. Rev. D., **17**, 2369, (1978).

In the considered model, calculations of higher-order processes, including radiative corrections, can be performed using regular diagram techniques. Hence, we can get an explicitly covariant description of neutrino interaction with fermions of the medium, based on an equation similar to the Dirac-Schwinger equation of quantum electrodynamics.

The form of such an equation is well known. See, for example,

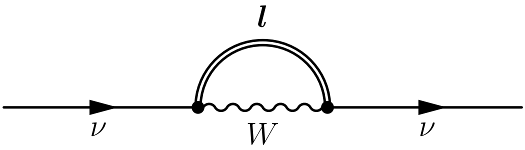
N.N. Bogoliubov and D.V. Shirkov, *Introduction to Theory of Quantized Fields* (John Wiley, New York, 1979).

$$\left(\mathrm{i}\gamma^{\mu}\partial_{\mu}\mathbb{I}-\mathbb{M}^{(\nu)}\right)\Psi^{(\nu)}(x)=\int d^4yM(x,y|\mathrm{g})\Psi^{(\nu)}(y), \tag{3.1}$$

where $M(x,y|\mathrm{g})$ is the mass operator of a neutrino in matter.
Here and what follows the notation

$$\gamma^{\mu}\mathbb{A}\equiv\gamma^{\mu}\otimes\mathbb{A} \tag{3.2}$$

is a tensor product of the Dirac matrices and 3×3 matrices acting in the space of fundamental representation of $SU(3)$.
For greater clarity, consider a neutrino propagation in the environment consisting of electrons, protons and neutrons (e, p,n) , assuming that the density of neutrino flux is small. Then in the lowest order of the perturbation theory, only two diagrams shown on the following slides contribute to the mass operator.



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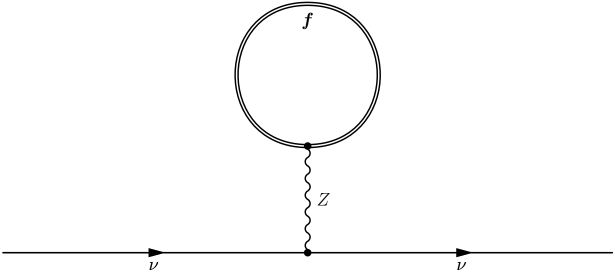

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Thus, we obtain the following expression

$$\begin{aligned}
&\left(\mathrm{i}\gamma^{\mu}\partial_{\mu}\mathbb{I}-\mathbb{M}^{(\nu)}\right)\Psi^{(\nu)}(x)+\\
&\quad+\mathrm{i}\,\frac{g^2}{8}\int d^4y\,\gamma^{\mu}(1+\gamma^5)S^{(e)}(x,y|\mathrm{g})\gamma^{\nu}(1+\gamma^5)D^{W}_{\nu\mu}(y-x)\Psi^{(\nu)}(y)\\
&\qquad-\mathrm{i}\frac{g^2}{8\cos^2\theta_{\mathrm{W}}}\sum_{i=e,u,d}\int d^4y\,\gamma^{\mu}(1+\gamma^5)D^{Z}_{\mu\nu}(x-y)\\
&\quad\times\mathrm{Sp}\left\{\gamma^{\nu}\left(T^{(i)}-2Q^{(i)}\sin^2\theta_{\mathrm{W}}+T^{(i)}\gamma^5\right)S^{(i)}(y,y|\mathrm{g})\right\}\Psi^{(\nu)}(x)=0.
\end{aligned}\tag{3.3}$$

The meaning of the introduced notations is obvious. Here $D^{W}_{\mu\nu}(x-y)$ and $D^{Z}_{\mu\nu}(x-y)$ are the causal Green functions of W and Z bosons respectively, and $S^{(i)}(x,y|\mathrm{g})$ are the causal Green functions of the fermion multiplets in the real-time formalism taking into account the external conditions g, i.e., the temperature and the chemical potential of the background.

For relatively small neutrino energies when $\mathcal{E}_\nu \ll M_W^2/\mathcal{E}_F \lesssim M_W^2/T_f$, $\mathcal{E}_F \lesssim T_f \ll M_W$, where $\mathcal{E}_F$, T_f are the Fermi energy and the temperature of the background fermions, it is possible to use the Fermi approximation. Then

$$D_{\mu\nu}^W(x-y) \approx \frac{g_{\mu\nu}}{M_W^2}\delta(x-y), \quad D_{\mu\nu}^Z(x-y) \approx \frac{g_{\mu\nu}}{M_Z^2}\delta(x-y) \tag{3.4}$$

and equation (3.3) takes the form

$$\begin{aligned}&\left(\mathrm{i}\gamma^\mu\partial_\mu\mathbb{I} - \mathbb{M}^{(\nu)}\right)\Psi^{(\nu)}(x) + \mathrm{i}\frac{G_\mathrm{F}}{\sqrt{2}}\gamma^\mu(1+\gamma^5)\Big\{S^{(e)}(x,x|\mathrm{g})\gamma_\mu(1+\gamma^5)\\ &-\sum_{i=e,u,d}\mathrm{Sp}\left\{\gamma_\mu\left(T^{(i)} - 2Q^{(i)}\sin^2\theta_\mathrm{W} + T^{(i)}\gamma^5\right)S^{(i)}(x,x|\mathrm{g})\right\}\Big\}\Psi^{(\nu)}(x) = 0.\end{aligned} \tag{3.5}$$

The imaginary parts of the Green functions are, in fact, the density matrices of the fermions of the external medium, i.e.

$$S^{(i)}(x,x|\mathrm{g}) \Rightarrow -\mathrm{i}\varrho^{(i)}(x,x|\mathrm{g}).$$

After summation over the quantum numbers of the background fermions, the density matrices take the form that is well known from general considerations

L. Michel and A. S. Wightman, Phys. Rev., **98**, 1190, (1955).

Assuming that it is now necessary to consider constituent parts of the medium as the components of multiplets, for the density matrices we have the expressions

$$\varrho^{(i)}(x,x|\mathrm{g}) = \sum_{l=1,2,3} \mathbb{P}_l^{(i)} \frac{\mathrm{n}_l^{(i)}}{4p_l^{0(i)}}(\gamma_\alpha p_l^{\alpha(i)} + m_l^{(i)})(1-\gamma^5\gamma_\alpha s_l^{\alpha(i)}), \qquad (3.6)$$

where $\mathrm{n}_l^{(i)}$ is the number density of the multiplet components, $p_l^{\alpha(i)}, s_l^{\alpha(i)}$ are averaged 4-momentum and 4-polarization of the multiplet components respectively and $\mathbb{P}_l^{(i)}$ are the projectors on the l state of the multiplet i.

Let us introduce effective four-potentials. The potential

$$f^{\alpha(e)} = \sqrt{2} G_{\mathrm{F}} \left(j^{\alpha(e)} - \lambda^{\alpha(e)} \right) \tag{3.7}$$

determines the interaction of neutrino with electrons via the charged currents, while the potential

$$f_{\mathrm{N}}^{\alpha} = \sqrt{2} G_{\mathrm{F}} \sum_{i=e,u,d} \left(j^{\alpha(i)} \left(T^{(i)} - 2Q^{(i)} \sin^2 \theta_{\mathrm{W}} \right) - \lambda^{\alpha(i)} T^{(i)} \right) \tag{3.8}$$

determines the contribution of the vacuum polarization.

Here

$$j^{\alpha(i)} = n^{(i)} \frac{p^{\alpha(i)}}{p_0^{(i)}} = \{\bar{n}^{(i)} u_0^{(i)}, \bar{n}^{(i)} \mathbf{u}^{(i)}\} \tag{3.9}$$

are the currents, and

$$\lambda^{\alpha(i)} = n^{(i)} \frac{s^{\alpha(i)}}{p_0^{(i)}} = \left\{ \bar{n}^{(i)}(\zeta^{(i)}\mathbf{u}^{(i)}), \bar{n}^{(i)} \left(\zeta^{(i)} + \frac{\mathbf{u}^{(i)}(\zeta^{(i)}\mathbf{u}^{(i)})}{1 + u_0^{(i)}} \right) \right\} \tag{3.10}$$

are the polarizations of the background fermions. It is significant that only $j^{\alpha(i)}$ and $\lambda^{\alpha(i)}$ characterize a medium as a whole.
Here $\bar{n}^{(i)}$ and $\zeta^{(i)}$ $(0 \leqslant |\zeta^{(i)}|^2 \leqslant 1)$ are the number density and the mean value of the polarization vector of the background fermions in the center-of-mass system of matter, respectively. In this reference frame the mean momentum of the fermions (i) is equal to zero. The 4-velocity of this reference frame is denoted as $u_\mu^{(i)} = \{u_0^{(i)}, \mathbf{u}^{(i)}\}$.

Using these potentials, after elementary calculations we can write the equation under consideration in the form

A. E. Lobanov, Izvestiya Vysshikh Uchebnykh Zavedenii, Fizika, **59**, No. 11, 141, (2016). [Russ. Phys. J., **59**, No. 11, 1891, (2016)].

$$\Big(i\gamma^{\mu}\partial_{\mu}\mathbb{I} - \frac{1}{2}\gamma_{\alpha}f^{\alpha(e)}(1+\gamma^{5})\mathbb{P}^{(e)} - \frac{1}{2}\gamma_{\alpha}f^{\alpha}_{\mathrm{N}}(1+\gamma^{5})\,\mathbb{I} - \mathbb{M}^{(\nu)}\Big)\,\Psi^{(\nu)}(x) = 0. \quad (3.11)$$

This equation generalizes the equation, which was used in the following papers

A. E. Lobanov, Dokl. Akad. Nauk Ser. Fiz., **402**, 475, (2005); [Dokl. Phys. **50**, 286, (2005)].

A. Studenikin and A. Ternov, Phys. Lett. B, **608**, 107, (2005).

A. E. Lobanov, Phys. Lett. B, **619**, 136, (2005).

in order to describe spin precession of a neutrino mass state.

CONCLUSION

Neutrino oscillations including oscillations in a dense medium can be considered in a mathematically rigorous way in the framework of a slightly modified Standard Model.

You will hear about how to solve the neutrino evolution equation in the next talk.

Thank you for your attention!