Skewed Sudakov Regime

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Sudakov Form Factor

V.V. Sudakov, JETP, 1956

Vertex Parts

Γ_µ(p, q) in Minkowski kinematics

(The plane of external momenta p, q, l = p − q contains two light-like lines crossing at the origin)

\[
\lim_{l^2 \to \infty} \Gamma_\mu(p, q) = -ie\gamma_\mu \exp\left(-\frac{\alpha}{2\pi} \log \left| \frac{l^2}{p^2} \right| \log \left| \frac{l^2}{q^2} \right| \right)
\]
Sudakov at One Loop

Scalar

$$\lim_{l^2 \to \infty} \frac{l^2}{4\pi^2} \int \frac{d^4k}{(p - k)^2k^2(q - k)^2} \approx \frac{i}{4} \log \left| \frac{l^2}{p^2} \right| \log \left| \frac{l^2}{q^2} \right|$$

Sudakov variables and logarithmic regions
Sudakov Results

- Coefficients by the doubly logarithmic corrections do not depend on the UV scale
- No new tensor structures
- Explicit Form Factor
Skewed Sudakov at One Loop

\[
\lim_{q^2 \to 0} \frac{l^2}{4\pi^2} \int \frac{d^4k}{(p - k)^2k^2(q - k)^2} \approx \frac{i}{4} \log \left| \frac{l^2}{q^2} \right| C\left(\frac{l^2}{p^2}\right)
\]

\[C\left(\frac{l^2}{p^2}\right) = ?\]

Relation to Sudakov:

\[
\lim_{p^2 \to 0} C\left(\frac{l^2}{p^2}\right) \approx \log \left| \frac{l^2}{p^2} \right|
\]
Tensor Structure at One Loop

Loop momentum in the denominators of the propagators can be neglected

\[ \psi \left( \Gamma_\mu(p,q) \right) \gamma_\lambda \approx 2 \psi \left( -e^3 \gamma_\lambda \gamma_\mu \right) \gamma_\lambda \times \frac{\text{Scalar}}{4\pi^2 l^2} \]

\[ \approx \psi \left( -ie\gamma_\mu \right) \gamma_\lambda \times \left( -\frac{\alpha}{2\pi} \log \left| \frac{l^2}{p^2} \right| \log \left| \frac{l^2}{q^2} \right| \right) \]

Agrees with the result

\[ \Gamma_\mu(p,q) \approx -ie\gamma_\mu \exp \left( -\frac{\alpha}{2\pi} \log \left| \frac{l^2}{p^2} \right| \log \left| \frac{l^2}{q^2} \right| \right) \]
Definition of Leading Logs

On dimensional grounds, there are three types of logs:

\[ \log \left| \frac{l^2}{q^2} \right|, \log \left| \frac{l^2}{p^2} \right|, \text{ and } \log \left| \frac{l^2}{\mu^2} \right| \]

\[ t \equiv \frac{\alpha}{2\pi} \log \left| \frac{l^2}{q^2} \right| \]

**Leading log:** \( t^n F_n \left( \frac{l^2}{p^2}, \frac{l^2}{\mu^2} \right) \)

**Subleading logs:** \( \alpha^n t^m, n > 0 \)

**Result #1:** \( F_n \) depends only on \( \frac{l^2}{p^2} \)

No dependence on \( \mu \) in the leading logarithms
Tensor Structure

Result #2:

\[ \Gamma_\mu(p, q) \approx -ie\gamma_\mu F^{sk}(p, q) + ie\frac{q_\mu p}{qp} \left( F^{sk}(p, q) - 1 \right) \]

Compare to classical Sudakov:

\[ \Gamma_\mu(p, q) \approx -ie\gamma_\mu F^{S}(p, q) \]

\[ F^{S}(p, q) = \exp \left( -t \log \left| \frac{l^2}{p^2} \right| \right) \]
The Skewed Form Factor

\[
\lim_{p^2 \to 0} F^{sk}(p, q) = F^S(p, q) \Phi_s(t)
\]

\[
s = 0, \pm
\]

\[
\frac{l^2}{p^2} > 0 \iff s = 0
\]

\[
\Phi_0(t) = \frac{1}{1-t^2-t \exp(-t)}
\]
The Skewed Form Factor ...

\[
\frac{l^2}{p^2} < 0 \iff s = \frac{l^2}{|l^2|}
\]

\[
\Phi_{\pm}(t) = \exp(\pm i\pi t)\Phi_0(t)
\]
Longitudinal and Transverse Momenta of Virtual Particles

\[ \Gamma_1 \equiv \frac{l^2}{4\pi^2} \int \frac{d^4k}{(p - k)^2 k^2(q - k)^2} = \frac{l^2}{4\pi^2} \int \frac{d^2k_\parallel d^2k_\perp}{(p - k)^2 k^2(q - k)^2} \]

\( k_\parallel \) is a linear combination of external momenta, and \( k_\perp q = k_\perp p = 0 \)

Transverse plane is Euclidean

\( k_\perp^2 \leq 0, \ t \equiv -k_\perp^2 \)

Integrating out the orientation of \( k_\perp \) one obtains

\[ \Gamma_1 = \frac{l^2}{4\pi} \int \frac{d^2k_\parallel dt \theta(t)}{(p - k)^2 k^2(q - k)^2} \]
We will manipulate with the $\theta$-function. As a result, integration over negative $t$ will appear.

It will be advantageous to consider doubly virtual particles with not only nonzero $k^2$ but also with positive $k^2_\perp$ (with purely imaginary components of $k_\perp$).
Inclination of Doubly Virtual Particles

Instead of $t \equiv -k_{\perp}^2$ it will be convenient to use another variable—the inclination $\nu$

$$\nu \equiv \frac{k_{\parallel}^2}{k_{\parallel}^2 - t} = \frac{k_{\parallel}^2}{k^2}$$

In terms of $\nu$

$$\theta(t) = \theta(k_{\parallel}^2) - \frac{k_{\parallel}^2}{|k_{\parallel}^2|} \theta(\nu(1 - \nu))$$
The Integral

\[ \int d^2k_\parallel dt \theta(t) G(k_\parallel, t) = \int d^2k_\parallel dt \theta(k_\parallel^2) G(k_\parallel, t) - \int_0^1 \frac{d\nu}{\nu^2} \int d^2k_\parallel k_\parallel^2 G(k_\parallel, k_\parallel^2(1 - 1/\nu)) \]

The part with the integration over time-like longitudinal momentum vanishes because of Feynman’s \( i\epsilon \)
Inclination Representation

As a result

\[ \Gamma_1 = - \int_0^1 d\nu \iint \frac{d^2k_\parallel}{(k_\parallel - q\nu)^2 + q^2\nu(1 - \nu) + i\epsilon} G(k_\parallel, \nu) \]

\[ G(k_\parallel, \nu) = \left( \frac{l^2}{4\pi} \right) \frac{1}{(p - k_\parallel)^2 - k_\parallel^2(1 - 1/\nu)} \]
Leading Logarithm

Leading logarithm comes from a vicinity of \( k_\parallel = q\nu \) To extract it, one uses the formula

\[
\int_{A} \frac{d^2k}{k^2 + q^2 + i\epsilon} \approx -i\pi \log \left| \frac{A^2}{q^2} \right|
\]

\( A_\pm \) are limits of integration around zero \( k_\pm = k_0 \pm k_1 \)
The Answer

\[ \Gamma_1 \approx \frac{i}{4} \log \left| \frac{l^2}{q^2} \right| \int_0^1 \frac{l^2 d\nu}{p^2(1 - \nu) + l^2\nu + i\epsilon} \]

\[ C(l^2/p^2) = \int_0^1 \frac{l^2 d\nu}{p^2(1 - \nu) + l^2\nu + i\epsilon} \]

As expected

\[ C(l^2/p^2) \approx \log \left| \frac{l^2}{p^2} \right| \]
In all Orders...

\[ G_\mu(p, q) = \int_0^{2\pi} d\phi \int_0^1 d\nu \int d^2k_\parallel \frac{I_{\mu,\lambda}(p, k; q-k) k^\lambda}{(k_\parallel - \nu q)^2 + q^2 \nu (1 - \nu) + i\epsilon} \]

\[ k_\perp = \sqrt{-k_\perp^2 (\cos(\phi), \sin(\phi))}, \quad k_\perp^2 = k_\parallel^2 \left( \frac{1}{\nu} - 1 \right) \]

after radiating the photon the incoming fermion is “inclined” to carry away the fraction of the initial momentum \( \nu \)
Conclusions

- Skewed Sudakov regime was introduced
- Doubly virtual particles and their inclination were defined
- Consideration of the Skewed Sudakov using incination gives simultaneously simplification and generalization of the classical Sudakov treatment
- Skewed Sudakov may have phenomenological applications